INTRODUCTION

In no branch of learning can an author disregard the results of honest research with so much impunity as he can in Philosophy and Logic.

Wittgenstein

"One way of viewing mathematics is in terms of number. I guess you know what the other way is. I'll say the word in a more expressive language so there'll be no doubt exactly what it is we're talking about." "I wish you wouldn't." "Logik," softly said.

Don De Lillo

What Is a Logical Constant?

But the English...having such varieties of incertitudes, changes and Idioms, it cannot be in the compas of human brain to compile an exact regular Syntaxis thereof.

James Howell (1662)

Is it possible to say, in a clear and precise way, just what constitutes the distinction between logical expressions (formatives, syncategoremata, particles, constants) and nonlogical expressions (material expressions, categoremata, terms, variables)? Pessimism concerning this seems to be the rule among most modern logicians. For example, Tarski: "No objective grounds are known to me which permit us to draw a sharp boundary between [logical and extralogical] terms" (1956: 418-9); Mates: "...unfortunately the question as to which words should be considered logical and which not involves a certain amount of arbitrariness" (1965: 14); Quine: "Each such word [i.e., particle] is in a class fairly nearly by itself; few words are interchangeable with it *salva congruitate*. . . . Instead of listing a construction applicable to such a word and to few if any others, we simply count the word an integral part of the construction itself. Such is the status of particles" (1970: 29); Allwood, Anderson, and Dahl: "In the last instance it is a matter of decision whether a word belongs to the logical vocabulary or not" (1977: 24); and, of course, Russell: "The logical constants themselves are to be defined only by enumeration, for they are so fundamental that all the properties by which the class of them might be defined presupposes some terms of the class. But practically, the method of discovering the logical constants is the analysis of symbolic logic" (1903: 37).

In the face of this overwhelming pessimism, we must remind ourselves that there have been earlier times of optimism. For example, Leibniz: "So just as there are two primary signs of algebra and analytics, + and −, in the same way there are, as it were, two copulas, 'is' and 'is not' " (1966: 3); De Morgan: "I think it reasonably probable that the advance of symbolic logic will lead to a calculus of opposite relations, for mere inference, as general as that of + and − in algebra" (1966: 26); Sommers: "All formatives—including propositional 'constants'—are analogous to plus and minus signs of arithmetic" (1973: 249).

If these optimists are correct, not only is it possible to draw a clear and precise distinction between the logical and extralogical expressions of a language, it is also possible to give a very simple characterization of the nature of logical expressions—they are all signs of opposition, analogous to the oppositional signs of mathematics. Such prospects are surely attractive. If so, however, why have most modern logicians turned pessimistic? The answer seems to lie in the shift from a traditional account of logical syntax to a Fregean account. We will examine that shift more closely below. But we should recognize now that there are important consequences of the traditionalists' optimism with regard to the logical/extralogical distinction and the nature of logical expressions. Any substantiation of the attractive prospects offered by the traditionalist view must inevitably cast some doubt on the generally accepted modern, Fregean, view and its concomitant pessimism.

The systematic ambiguity of plus and minus expressions in mathematical language (an ambiguity that we will explore below) is not only benign, it is a source of great expressive power for the mathematician. Leibniz, De Morgan, and Sommers have suggested that natural language has a logic that, like arithmetic and algebra, makes use of two kinds of basic expressions, the signs of opposition. In fact, their common position seems to be that all expressions of natural language that carry the responsibility for determining logical form are either positive signs or negative signs or signs definable in terms of these. If this is so, it means, among other things, that one could build an artificial formal language that would model natural language by using the mathematician's opposition signs for all formatives. The result would be an algorithm for natural language that would model natural statements as arithmetical formulae and inference as arithmetical calculation. There is little doubt that this was Leibniz's goal throughout his logical studies, and Sommers has come very close to reaching that goal in his own logical work. It would seem, therefore, that the idea of using signs of opposition to model natural-language formatives is a good one, leading, as it seems to have done, to rich programmes of logical investigation and to viable systems for logical reckoning.

One of the consequences of this idea has been great optimism among those who have felt that a clear and precise account of the nature of logical formatives, and of their distinction from nonlogical expressions, can be provided. In a sense, their account is quite simple: logical formatives, unlike other expressions, are oppositional in just the way that plus and minus are oppositional in mathematics. But to appreciate fully this kind of account we will look closely, in later chapters, at the oppositional character of formatives, their roles in inferences, and the kind of algorithm that could model those inferences.

The Problem of Sentential Unity

> *One of the 'insights' of modern logic has been that nouns and verbs are basically the same sorts of things; both may be symbolized as predicates of the same sort.*
>
> *T. Parsons*

The problem of sentential unity is simply the problem of determining what accounts for the fact that some strings of expressions form sentences while others do not. Just what is it that ties words of a sentence together to form a single linguistic unit, a sentence? The problem is an ancient one. The earliest clear attempt to solve it was Plato's.

While Plato cannot properly be called a logician (who before Aristotle could be?) he did demonstrate in the *Dialogues* an ability to use very subtle argument forms, and he had an unschooled but intuitive sense of the form/content distinction. Yet he did exhibit a tendency to make elementary logical mistakes and seems to have believed that an accumulation of arguments, however weak, strengthened one's position.[1] The problem of sentential unity is one that he tried to solve in a careful and clear way. In the *Sophist*, especially, he sought an appropriate way to analyse simple statements that could serve as premises or conclusions of arguments. The view he took was that such statements have a *binary* logical structure. At 261d, Plato asked "whether all names can be connected with one another, or none, or only some of them." He then concluded (262c) that a sentence is a string of words in which "verbs are mingled with nouns," for "the first and smallest" phrase is a combination of a noun with a verb—that is, the binary analysis. According to this view, the minimal requirement for a string of terms to constitute a statement is that the string consist of two terms, and, furthermore, that one of the terms be an *onoma* (noun) and the other be a *rhema* (verb). So, for Plato, there were at least two kinds of terms, nouns and verbs, and a statement required one of each. One cannot form a statement from a pair of nouns, nor from a pair of verbs; one of each is required. One cannot form an axe by combining a pair of axe-heads, nor by combining a pair of axe-handles; one of each is required. According to

Plato's binary analysis, the two terms that form a simple statement are logically heterogeneous. They are unfit for each other's sentential roles.

When Plato said that a simple sentence was properly constituted just by a noun and a verb, he did not demand that some other expression or sign be present to connect them. He believed that they could just naturally "mix," or "blend," or "combine," or "mingle," all on their own. Moreover, his theory of logical syntax was tied to his theory of Forms. Indeed, he wanted to hold that the blending of a noun and a verb to form a statement was a reflection of the blending of the two Forms signified by those terms. Naturally, his theory of syntax inherited many of the criticisms aimed at his metaphysics.[2]

As we will see later, the idea of a binary analysis for logical syntax has had a strong and lasting appeal throughout the history of logic and is well entrenched in today's standard system. Aristotle was the first to offer an alternative analysis, but only after an initial period of devotion to his teacher's binary theory. In both the *Categories* and *De Interpretatione*, Aristotle held that a sentence was a combination of a noun and a verb.[3] However, by the time he wrote *Prior Analytics* he had come to adopt a quite different view. He invented syllogistic in the *Analytics*. A key requirement for syllogistic inference is that at least one term occur as subject-term in one statement but as predicate-term in another. In other words, Aristotle's syllogistic requires that terms be logically homogeneous, fit for playing more than one role in different statements. Thus, he had to abandon the binary theory, with its distinction between nouns and verbs. The distinction may have been good grammar, but it had no place in (syllogistic) logic. In *Prior Analytics,* Aristotle introduced for the first time the idea that a simple sentence must consist not only of a pair of terms (*horos*) but of something else as well. What is required is a logical *copula*. While the terminology is Abelard's, the idea is Aristotle's. He says (24b16), "I call *term* (*horos*) that into which the premise is resolved, viz., the predicate and that of which it is predicated, with *be* or *not be* added."

In effect, Aristotle had adopted a *ternary* theory of logical syntax. According to this view, simple statements consist of pairs of terms connected by a third expression. Since the sole function of this third expression is to *connect* the two terms to form a statement, it is appropriate to call it a *copula*, for it literally copulates (connects, joins, fuses, binds, links, unites) the two terms. While Abelard is usually given credit for introducing the word 'copula' into logic, what he had in mind was primarily the (Latin equivalent of) 'is' in such statements as 'Socrates is wise'. But as we will see, 'is' here is no copula—it is a qualifier.

Plato's logical insight was supplied by his recognition of the grammatical noun/verb distinction. He took it to be a logical distinction as well—thus the binary theory. Aristotle, under the pressure of building a theory of formal deductive inference, came to see that the grammatical distinction was of no logical import. From a logical point of view, terms are

homogeneous. But this makes the question of sentential unity all the more urgent, for now grammar cannot be appealed to in order to unite pairs of terms (as it could be in order to unite a noun and a verb). Aristotle took the simple, commonsense view that any pair of items must be linked by some third thing in order to form a unit. As we saw above, early on in *Prior Analytics* Aristotle claimed that forms of 'to be' and its negation could be taken as the third item, the link or copula. But as the work progressed, he came to realize that this was too simple a view. What was required was a single expression that could be used to link the terms of *any* (statement-making) sentence. There were four kinds of "Aristotelian" logical copulae. Their English versions are: 'belongs to every', 'belongs to no', 'belongs to some' and 'does not belong to some'. Thus 'Every man is rational' would be regimented so that the copula expression stood between the two terms (making the terms quite literally *termini*, endpoints): 'Rational belongs to every man'. The result of connecting a pair of terms by one of Aristotle's copulae was, of course, a categorical statement. Note that the copula determined both the quantity and quality of the categorical, for it should be kept in mind that the features of quantity and quality were properly applied to a statement as a whole—not to terms alone.

The binary and ternary theories represent two quite different solutions to the problem of sentential unity. The former solves the problem by claiming that sentential unity is the result of two expressions naturally fitting together, mixing, blending, and so on, by virtue of the fact that they are formally different but nonetheless complementary to one another, so that together they form a unit. The second theory, having denigrated the grammatical differences between expressions (at least for logical purposes), solves the problem by positing a third expression, the sole duty of which is logical copulation. Unity is the result of the connection (via a connector) of a pair of expressions. Versions of each of these theories are still offered today. After a long detour, we shall return to the problems of logical constants and sentential unity. We hope that by then the best choice of solutions will be obvious.

Notes for Introduction

[1] See Patzig (1972) and Bochenski (1968), pp. 14-18.
[2] See Ackrill (1957), Kahn (1972), and Kneale and Kneale (1962).
[3] *Categories* 1a16ff and *De Interpretatione* 16a1-17a37 (in Ackrill, 1963).

Part I

CHAPTER ONE

THE GOOD OLD DAYS OF THE BAD OLD LOGIC (or, Adam's Fall)

Logic, n. The art of thinking and reasoning in strict accordance with the limitations and incapacities of the human misunderstanding. Understanding, n. A cerebral secretion that enables one having it to know a house from a horse by the roof on the house. Its nature and laws have been exhaustively expounded by Locke, who rode a house, and Kant, who lived in a horse.

Ambrose Bierce

Aristotle's Syllogistic

L'invention de la forme des syllogismes [est] l'une des plus belles de l'esprit humain, et même des plus considérables.

Leibniz

Sweet Analytics, 'tis thou hast ravish'd me.

Marlowe

Bachelors and Masters of Art who do not follow Aristotle's philosophy are subject to a fine of 5 shillings for each point of divergence.

—fourteenth-century rule, Oxford University

Aristotle's syllogistic logic rests four-square on the theory of logical syntax worked out in *Prior Analytics*. As we have already seen, this theory is ternary, taking simple sentences to consist (logically) of a pair of terms connected by a logical copula. Aristotle had adopted the ternary theory of logical syntax only after abandoning the earlier binary theory, which he had shared with Plato. At least one prominent philosopher has likened Aristotle's shift here to Adam's fall.[1] According to Geach, the binary theory represents an earlier state of grace, which Aristotle foolishly abandoned after being blinded by the false promises of syllogistic. The *De Interpretatione* view that a noun and a verb are different in logically important ways (e.g., only the latter is tensed, only the latter can be negated) should never have been forsaken. But the development of syllogistic in *Prior Analytics* required the terms of a statement to be such that they could occur in either of the two term positions ("Aristotle's thesis of interchangeability"), and this thesis led directly to what Geach calls "the two-term theory," namely, the ternary analysis: statement-making sentences are logically formalizable as term-copula-term.

The real sin of the two-term theory was its commitment to the logical homogeneity of terms, its failure to preserve in logic the grammatical noun/verb distinction. Geach concurs with most modern logicians in holding such a distinction to be foundational. Referring to nouns as "names" and verbs as "predicates," Geach claims (48), "It is logically impossible for a term to shift about between subject and predicate position without undergoing a change of sense as well as a change of role. Only a name can be a logical subject; and a name cannot retain the role of name if it becomes a logical predicate." Geach is defending the view most commonly held nowadays by logicians, according to which there are two quite distinct roles to be played by terms: referential and predicative. Names (viz., proper names and personal pronouns) are the only terms suited for the first role; only (verbalized) general nouns and adjectives and verbs are suited for the second.[2] In *Prior Analytics,* the referential/predicative distinction, a semantic distinction that even Aristotle could not deny, is not reflected in a syntactical distinction, and it is this failure that seems most to exercise Geach. According to Geach's reading of logical scripture-history, this failure, when coupled with the ternary theory (the two-term theory), unavoidably led to the "two-name theory." The two-name theory was a transgression even more grave than the two-term theory. It was, says Geach, the theory of logical syntax held by the Scholastics, and later Mill, and even later Lesniewski. By Geach's description, the two-name theory parses statements as pairs of names (referential expressions) linked by a copula (viz., a form of 'to be'). And this copula is a mark of identity!

Still, Adam's fall was not complete. It ended, for Geach, only with the coupling of the two-name theory with the view that the referent of a term is the *class* of individuals denoted by that term. When conjoined, these two theories led, finally, to the ultimate apostasy: "the two-class theory." This theory, with its doctrine of distribution based on a confusion concerning reference and denotation, is what is usually called "Traditional Logic." And Geach's condemnation of the heresy of traditional logic is uncompromising: "Between such logic and genuine logic there can only be war" (54). And to think this all started with Aristotle's innocent attempt to build a formal logic, and thus to shift from a binary to a ternary analysis.

Did Aristotle really eat the forbidden fruit when he opted for the term-copula-term theory? One way to look for an answer is to look at

syllogistic, the logic made possible by the adoption of that theory. Those who believe that syllogistic logic (especially in its guise as Traditional Logic) is fit to be taught only in "Colleges of Unreason," that it is hopelessly inadequate to the demands of genuine logic, that its rival (modern mathematical logic) has shown itself to be superior in every way, will, with Geach, happily reject the ternary analysis, syllogistic and, perhaps, Aristotle himself.[3] I do not share this belief.

Aristotle's syllogistic is the first systematic, fully articulated formal logic.[4] It is not "traditional logic," but it does serve as the historical foundation for that logic. The crucial elements of Aristotle's syllogistic are its underlying ternary theory of logical syntax; its recognition of two kinds of negation; its use of the perfect first-figure forms as basic; its construal of proof as reduction to perfect forms; its explicit use of a small number of immediate inference patterns as rules of proof; and its implicit use of the *dictum de omni et nullo* as a rule of proof. There are certain important limitations on Aristotle's version of syllogistic: it ignores, for all practical purposes, the logic of compound statements; it deals with relational expressions only in a brief and superficial way; and it generally excludes singular statements from the domain of logical analysis.

We have already seen how he abandoned the binary theory of logical syntax for the ternary theory. Aristotle never gave up the notion that statement-making sentences (statements), the kinds of expressions that enter into inferences as either premises or conclusions (as opposed to prayers, commands, etc.), are essentially copulated pairs of terms. Since he wanted to reject the grammatical noun/verb distinction for logic, thus allowing any term to occupy any term-place in a statement, he no longer had available to him the grammatical mechanism that permitted nouns and verbs to combine without an intermediary to form a linguistic unit. Once the two terms of a statement are seen as *logically* undifferentiated, no grammatical device can unite them. What Aristotle needed was a logical device for connecting the pairs of terms to form statements—the logical copula.

In fact, Aristotle found four logical copulae. Throughout *Prior Analytics* he repeatedly refers to statements simply by listing their terms,—for example, 'AB', 'MN', and so on. I call these "proto-propositions." In such cases a connecting logical copula is understood but left implicit for various purposes.[5] For example, when determining syllogistic figure all one needs to pay attention to is the arrangement of terms. For such purposes, proto-propositions are sufficient. However, no statement consists of just a pair of terms; some expression, the sole duty of which is to connect, is required. At 24a16-23, Aristotle defines quantity in terms of his four copulae. From our post-traditional point of view, this seems to make no sense. We have learned to think of both quantity and quality as primitive logical concepts in traditional logic. And they are. But they were not in Aristotle's syllogistic. Statements, according to Aristotle in Book I, chapter 1, of *Prior Analytics,* are either affirmative or negative and either universal, particular, or indefinite. But this is so not because, as the Scholastics would tell us, one term is quantified and the other qualified in certain ways, but because the term pairs are connected by one kind of copula or another, by virtue of which the entire statement is quantified and qualified. For Aristotle, every statement has the general logical form:

A applies to (*hyparchei*) some/every B

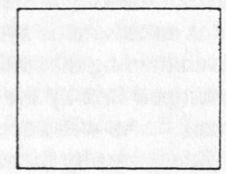
Here 'applies' is generic for either 'belongs' or 'does not belong'. So we have the four Aristotelian logical copulae: 'belongs to some', 'belongs to every', 'belongs to no' (= 'does not belong to some'— i.e., 'fails to belong to some'), and 'does not belong to every' (i.e., 'fails to belong to every'). The idea that a statement is the result of combining a quantified term and a qualified term is not Aristotle's. As we will see, it belongs to the traditional logic built first by the Scholastics.[6] The original syllogistic required that in each syllogistic inference at least one term occur in a statement as the first term and in another statement as the second term. This meant that term positions were undetermined with respect to either grammatical distinctions or semantic features. Any term was logically fit for any term position. The true logical work of a statement was determined by the expression connecting the two terms.

I said above that Aristotle's recognition of two kinds of negation was a crucial element for his syllogistic. In the next chapter, we will see that modern logicians generally take all logical negation to apply to entire statements and never to terms alone. In other words, for modern logicians, generally speaking, all negation is sentential, never terminal. Yet Aristotle, and indeed virtually all logicians before Frege, recognized two kinds of negation.[7] Aristotle's two kinds of negation are term negation and term denial. The negation of a term, say P, results in a new term, nonP, which is (logically) contrary to the original term. Aristotle's discussion of contrariety (e.g., at *Categories* 12a26-12b5, *De Interpretatione* 23b23-24 and 24b7-10 and *Metaphysics* 1055a34) is far from clear. What is clear is that, at the very least, Aristotle wished to connect his view of logical contrariety with his notion that accidents but not substances have contraries. Two accidents are contrary whenever, with respect to a given primary substance, at most one can be truly applied. Thus, Socrates cannot be both white and red. Aristotle also holds that a substance can be privative with respect to some quality, in the sense that it is the sort of thing that could sensibly be said to have that quality but actually has some contrary quality. Thus, given that Socrates is white he is not also red. But he is the sort of thing which could sensibly be said to be red. So, Socrates is privative with respect to red. He is nonred. Of course, he is also nonblue, nongreen, nonorange, and so on. At *Categories* 12a26-12b5, Aristotle gives the example of a stone, which is not sighted but also not nonsighted (=blind) since it is not the sort of thing to be either sighted or privative with respect to sight.

Aristotle's second kind of negation is term denial. Two of his logical copulae are negative, used to deny rather than affirm one term of some or all of another. Let A and B be two terms. One way to form a statement from these is to connect them with the expression 'belongs to some': 'A belongs to some B.' Aristotle would say that this is a case of affirmation. 'A belongs to every B' would be another affirmation involving the same terms. Denial is achieved by replacing an affirmative copula with a negative one. Thus, 'A belongs to no B' denies what 'A belongs to some B' affirms. The logical relation between such pairs of statements is contradiction. Two statements are contradictory only if exactly one is true and one is false. The use of 'belongs to some' and 'belongs to no' to connect the same pair of terms results in a contradictory pair of statements. The use of 'belongs to every' and 'does not belong to every' to connect the same pair of terms results in a contradictory pair of statements as well.

How are Aristotle's two kinds of negation related? Suppose that two terms are connected by the same copula in two statements except that the first term is negated in one, for example, 'A belongs to every B' and 'nonA belongs to every B.' It is clear that it is impossible for both statements to be true at the same time. Still, the two statements are not contradictory, since it is surely possible for both to be false. Consider 'Bald belongs to every logician' and 'Nonbald belongs to every logician,' neither of which happens to be true. Let's take another look at the negative copulae. We said that 'belongs to no' could be paraphrased by the expression 'does not belong to some.' Now, if A does not belong to B, it follows that B is nonA; thus, if 'bald' does not belong to some logicians, then 'nonbald' must belong to every logician. In other words, to deny a term of some/every X is to affirm its negation of every/some X. To summarize, a genuine "Aristotelian square of opposition" would look like this.

A belongs to every B — A does not belong to some B (= nonA belongs to every B)



A belongs to some B — A does not belong to every B (= nonA belongs to some B)

This is not familiar to us because it is not the traditional square of opposition that was developed by the Scholastic logicians.[8]

Aristotle's favoured way of proving a valid syllogistic inference was reduction. In Book I, chapter 1, of *Prior Analytics,* Aristotle makes an important distinction between *perfect* and *imperfect* syllogisms. A perfect syllogism is one such that the conclusion can immediately be seen to follow of necessity from the premises (cf. *Prior Analytics* 25b32-35); an imperfect syllogism requires one or more statements in addition to the premises before the implication of the conclusion can be seen. Perfect syllogisms need no proof of their validity, which is obvious, easily grasped by any rational being. But the validity of imperfect syllogisms must be demonstrated by a proof.[9]

Aristotle had divided syllogisms into three "figures" (Book I, chapters 4-6). He made it clear there that he thought of a syllogism as a linear arrangement of terms, with the terms of the conclusion at the endpoints (viz., the "extremes") and the term occurring in each premise but not the conclusion in the middle (thus the "middle term"). When the terms of a syllogism are so arranged, and when the generalities of the terms are in descending order (e.g., 'animal', 'man', 'logician'), the syllogism is perfect—it is in the first figure. A valid syllogism in another, imperfect, figure can be proved by manipulating the positions of premises and of terms so as to produce a first-figure syllogism. Every syllogism is either perfect or can be reduced to one that is (29b1-2). However, sometimes this reduction is not actually to a first-figure syllogism but to one that premises the denial of the conclusion of a first-figure syllogism and concludes the denial of one of its premises. This kind of reduction is called by later logicians *reductio per impossible* or *reductio ad absurdum.*

The rearrangement of premises or terms in imperfect syllogisms that results in a reduction to perfect syllogisms must be carried out systematically, according to rules. The rules were not always stated explicitly by Aristotle, but he did make use of such now-familiar rules as subalternation, simple conversion, accidental conversion, contraposition, and obversion.

Many modern logicians, following the lead of Lukasiewicz (1957), think of Aristotle's syllogistic as an axiomatic system modelled on Russell and Whitehead (1910-13), even holding that the syllogistic itself is based upon the statement logic developed first by the Stoics and then again by modern mathematical logicians.[10] As will become increasingly obvious throughout this essay, I have little sympathy for such a view. I see Aristotle as using the rules of conversion as natural deduction rules governing immediate (single premise) inference. Mediate (syllogistic, two-premised) inference is governed by what the Scholastics called the *dictum de omni et nullo*. Proof—that is, reduction—is achieved, according to Aristotle (29a30ff), by applying the conversion rules to statements in imperfect syllogisms. The result will be a perfect syllogism. Perfect syllogisms are

valid by virtue of the dictum. Thus the dictum governs *all* valid syllogisms. Consider the four perfect syllogisms.

1. A belongs to every B, B belongs to every C,
   so A belongs to every C
2. A belongs to no B, B belongs to every C,
   so A belongs to no C
3. A belongs to every B, B belongs to some C,
   so A belongs to some C
4. A belongs to no B, B belongs to some C,
   so A does not belong to every C
   (= nonA belongs to some C)

A single principle can be formulated to describe each of these inferences: *What does or does not belong to all or none of something likewise does or does not belong to what that something belongs to*—that is, the dictum. And since every valid syllogism is reducible to a perfect syllogism, the principle applies to all valid syllogisms. Aristotle himself came close to formulating this principle (*Categories* 1b9-15; *Prior Analytics* 24b27-31, 32b38-33a5). Needless to say, we will return to the dictum on several occasions.

Aristotle thought of his syllogistic as a tool for the teaching of theoretical sciences. Unlike logic itself, the theoretical sciences were taken to be axiomatic. A syllogistic inference was used to deduce theorems from the universally and necessarily true principles of a given theoretical science. In *Posterior Analytics,* he set out to show just how this is so. Note, however, that this view of logic and its relation to science places an important limitation on the logic. Aristotle's logic is not universal; it is incomplete *qua* formal logic.

One way in which the Aristotelian syllogistic is incomplete is in its failure to give any real account of inferences involving compound statements (the so-called hypotheticals). The Stoics invented just such a logic. In *Prior Analytics,* Aristotle was surely not unaware of such inferences (45b15-20), but held them to be in some sense irreducible and thus inferior to categorical syllogisms (50a16-24).[11] Later Peripatetic logicians, such as Theophrastus, Alexander of Aphrodisias, and Boethius, did develop a syllogistic incorporating compound statements. But the ancients, including both Peripatetics and Stoics, were never clear about just what the relationship was between a logic of categoricals (a term logic) and a logic of compounds (a sentential logic). All of the textual evidence seems to suggest that the latter was patterned after the former. This, in turn, suggests that the logic of compounds may not be a part or derivative of the logic of categoricals, but a parallel, isomorphic, logic.

A second limitation of Aristotle's syllogistic is its lack of a way of analysing inferences involving relational expressions (the so-called oblique terms, because they are terms in oblique grammatical cases, cases other than nominal). Aristotle did say something about such inferences,[12] but he failed to incorporate them systematically into his syllogistic. Serious attempts to do so were eventually made by Leibniz and, still later, by De Morgan. But a satisfactory logic of relationals was not available until the advent of mathematical logic. My survey of Sommers's work in chapter three will show that a properly formulated logic of terms can indeed incorporate relationals.

It is the third limitation of Aristotle's syllogistic that is most directly the result of his view of the relationship between logic and theoretical science. Since the axioms of any theoretical science must be universal and necessary, they cannot make reference to individuals. Thus it would seem that no principle (axiom or theorem) of theoretical science can be stated in the form of a singular proposition. Various arguments to this effect can be found in the literature.[13] Nonetheless, Aristotle himself never actually bars singulars from syllogistic: indeed, he gives examples of syllogisms with singulars (e.g., *Prior Analytics* 43a35ff and *Posterior Analytics* 90a5-25). Still it must be admitted that Aristotle does not thoroughly and systematically show how singulars are incorporated into syllogistic. This, too, is be a topic that we will pursue more than once in this essay.

Scholastic Additions

> *There is another use of syllogistic, namely, that it enables one in a learned dispute to vanquish an uncautious adversary. But as this only belongs to the athletics of the learned, an art, however useful it may be otherwise, and does not contribute much to the advancement of truth, I pass it over in silence.*
>
> *Kant*

The Scholastic period in logic, especially from Abelard in the eleventh century to Ockham in the fourteenth, was a long, complex, often confusing series of attempts to recover and strengthen Aristotle's syllogistic. Of course, by that time much of what passed as Aristotelian contained material that belonged not only to later Peripatetics, but to Stoics and Megarians as well. Scholastic logic, a logic that still thrives in certain quarters, became what most philosophers now think of as Traditional Logic. While it was clearly based upon and inspired by Aristotle, its additions took it far beyond the Master.

The Scholastic logicians amended and emended the original syllogistic in a number of ways, one of the most important of which was to add to it very elaborate semantic theories (usually under the heading of theories of "supposition"). These logicians tended to see logic as dealing

with what were later referred to as the "acts of the intellect." The first such act (simple apprehension) would deal with terms *per se*, the second (composition and division) would deal with statements (terms joined to form statements), and the third (reasoning) would deal with inferences (statements joined to form syllogisms). Each such branch of logic was supposedly inspired by a work of Aristotle's *Organon* in which he laid down the fundamental elements. Thus *Categories* was the inspiration for apprehension, *De Interpretatione* for composition and division, and *Prior Analytics* for reasoning. It is in their discussions of apprehension that the Scholastics developed their semantic theories.[14]

Medieval logicians generally took terms to have first *signification* and then *supposition*. The signification of a term was that by virtue of which it supposited (stood for) objects. The theory of supposition was intended to give a full account of just how a term could *stand for* objects. Some Scholastics distinguished between supposition and *appellation*, where the latter was the function of predicate-terms and the former the function of subject-terms. (I will discuss the notions of subject and predicate below.) Terms used in their normal sense (i.e., literally as opposed to metaphorically) were said to have *proper* (or sometimes *natural*) supposition. Here the term stood for, supposited, every object that does or could satisfy its signification. Proper supposition is what, after Mill, came to be called "extension" (just as signification came to be called "intension"). A term with proper supposition can be either mentioned in a sentence or used in a sentence. When a term is mentioned it supposits itself (as 'red' does in 'Red is three-lettered') and is said to have *material* supposition. When a term is not mentioned but used it is said to have *formal* supposition. In the normal case, the terms of any statement all have formal supposition. Such terms are also said to have either *simple* or *personal* supposition. Terms have simple supposition, it seems, when they denote the properties, concepts, ideas, or universals determined by the term's signification. Thus, in 'Man is a species' and 'Wisdom is rare' the terms 'man' and 'wisdom' denote species or properties, and so are said to have simple supposition. A term has personal supposition when it is used to denote objects (Aristotelian primary substances) rather than universals (Aristotelian secondary substances). Terms used with personal supposition are either singular or general. A singular term used with personal supposition is said to have *discrete* supposition. A general term so used is said to have *common* supposition. There are two kinds of common supposition. The subject-term of a particular sentence is said to have *determinate* supposition; all other terms are said to have *confused* supposition. Terms with confused supposition consist of predicate-terms of affirmations, said to have *pure* supposition, and subject-terms of universals as well as the predicate-terms of negations, said to have *distributive* supposition. As it turns out, singular terms (those with discrete supposition) and terms with distributed supposition are logically distributed terms. The rest, those with determinate or pure supposition, are logically undistributed terms. The above is, at best, a sketch of an enormously large, complex topic. But what will be of most interest to us is the doctrine of distribution just alluded to above. Post-Scholastic logicians have tended to divide themselves between friends and foes of distribution. Soon, I will enlist on the side of distribution's friends.

The Scholastics' theory of logical syntax is found in their discussions of *composition* and *division*. A statement is seen as either the composition or the division of pairs of terms. Terms are composed in affirmations, divided in denials. It would be a mistake, however, to think that these logicians simply bought wholesale Aristotle's theory of logical syntax, the ternary theory. As we have seen, an "Aristotelian" copula is a single (though many-worded) expression, namely, 'does/does not belong to some/every,' which, upon logical regimentation, comes between a pair of terms and unites or binds them to form a sentence. But the technical term 'copula' (or its Greek equivalent) was never used by Aristotle. That innovation belongs to Abelard[15] (here I follow Kahn, 1972).

At *De Interpretatione* 21a25, Aristotle raises the question of how we can attribute a property to what does not exist. For example, how can we say of Homer that he is a poet without thereby asserting as well that he is (i.e., is real, exists)? Aristotle's solution is to claim that in a statement such as 'Homer is a poet', 'poet' is predicated of 'Homer' but 'is' is only "accidentally predicated" of 'Homer'. The notion of accidental predication, at least as used here, is far from clear. It was in his attempt to shed light on this notion that Abelard introduced the technical expression "copula." Abelard's theory of logical copulation is summarized by Kahn in the following theses.

> 1. Every simple declarative sentence can be rewritten in the form *X is Y* and in particular every sentence of the form *NV* can be rewritten in the form *N is Ving* (where "*N*" stands for a noun form and "*V*" for a verb).
>
> 2. In a sentence of the form *X is Y*, *X* and *Y* are terms (in the sense of the terms of a syllogistic premise), whereas *is* is a meaningful third part which is not a term.
>
> 3A. In such a sentence, the meaning of *is* is that of a sign of affirmation, signifying that the predicate *Y* is affirmed of—said to belong to or to be true of—the subject *X*. Similarly, *is not* is a sign of denial.
>
> 3B. (The same point otherwise expressed:) in *X is Y*, the verb *is* serves to link *Y* to *X* and thus to combine them in a complete sentence (or proposition), i.e. one which can be true or false.
>
> 4. In such a sentence *is* serves merely as a link or copula (in the sense of 3A-B) and not also as a predicate which asserts the existence of the subject.
>
> 5. In the ordinary *NV* sentence the verb form serves twice: first as predicate term (like *Y* in *X is Y*) and again as *copulans* or linking element.

> The rewriting of *NV* sentences as *N is Ving* according to 1. above (e.g. rewriting *John runs* as *John is running*) serves precisely to bring out this double role of the verb. (Kahn, 1972: 146)

Notice first that Abelard has solved Aristotle's problem with accidental predication by denying that 'is' in a sentence such as 'Homer is a poet' is any kind of predicate at all. It is always, in such sentences, performing the logical task of linking. Also, in sentences with finite verbs other than 'is', the verb is forced to take on both the task of serving as one of the two terms and the task of linking (Abelard called these "copulative verbs" [*verbum copulativum*]). Compare this with the Platonic theory of the *Sophist*. For Plato, the basic form of a statement is noun-verb (*NV*). Abelard held that in such sentences the verb is not only a term of the sentence but also, implicitly, a copula. In effect, such verbs contain the logical copula within themselves. So, for logicians after Abelard, the basic form of a statement is term-copula-term. Noun-verb statements must be regimented (the verb split into copula and term) to yield the logically basic form.

This theory put an end to the Platonic binary theory of logical syntax. The idea that the basic form of elementary sentences could be viewed as noun-verb did not return to logic until the late nineteenth century. Where Abelard analysed, say, 'John runs' as 'John is running', these later logicians analyse 'John is running' as 'John runs'.

Still, the theory proposed by Abelard is not quite Aristotle's ternary theory. Specifically, Abelard's copula is not Aristotle's. While Aristotle does, especially in *De Interpretatione*, sometimes cite 'is' and 'is not' as logical copulae, his idea of such a copula is always that it is some kind of expression that serves solely to link or unite pairs of terms to form sentences. For him, every expression used to link terms can be rewritten as one of his four logical copulae.

There is another reason that Abelard's copula is not Aristotle's, and this goes to the heart of the Scholastics' theory of logical syntax. Logicians of the Middle Ages could hardly have ignored, much less been ignorant of, Aristotle's ternary analysis of statements. In fact, they eventually came to adopt agreed-upon symbols for each of his four copulae, writing each of his four categorical forms as, for example, 'AaB', 'AeB', 'AiB', and 'AoB'. Here the 'a', 'e', 'i', and 'o' were seen literally to link the term pairs. But later Scholastic logicians also recognized that an Aristotelian copula always gives two kinds of information about the sentences in which it operates, indicating both the quantity of the sentence and its quality. It indicates whether the sentence is universal or particular (indefinite sentences being implicitly one or the other), and whether the sentence is affirmative or negative. Sentences formed using Aristotelian copulae are as unnatural in our English as they were in the Latin of the Schoolmen, so they decided to cast Aristotle's regimented sentences into more natural Latin, and in so doing split the two roles of the copula. Aristotle's copulae became the

Scholastics' quantifier/qualifier pairs, the *syncategoremata*, which literally go 'with the *categoremata* (terms)' to form a sentence.

In Aristotle's terminology, the term belonging or not belonging is the *predicate*; the other is the *subject*. This distinction is logical rather than grammatical (e.g., the noun/verb distinction). For the Scholastic logicians a sentence of the form 'A belongs to some B' is parsed first as 'A belongs to/some B', with the copular expression now split, with one part, the quantifier, going with the subject-term and the rest, the qualifier, going with the predicate-term. Then, applying the restrictions of Latin grammar, the order of subject and predicate is reversed. Finally, the qualifier is replaced by Abelard's copula. Thus the logical form of any statement for the Scholastic becomes quantifier-term-qualifier-term (or quantifier-term-[Abelard's]copula-term), the standard *categorical* form. But I said above that the basic sentence form for logicians after Abelard was term-copula-term. Which is it? The difference here turns on the presence of singular subject-terms. The example from Aristotle that Abelard first had in mind was 'Homer is a poet'. Such a sentence, having a singular subject-term and thus no explicit quantity, could be easily construed as a singular term, 'Homer', and a general term, '(a) poet' (no articles needed in Latin), linked by the copula, 'is'. Such a sentence was a model for Abelard's further analysis. But singular sentences are not logically typical (and certainly were not for Aristotle). General statements are more typical, and they always have a quantity (though its quantifier is sometimes implicit). Such sentences are analysed as having the logical form quantifier-term-copula-term. We will see that most medieval logicians then fit singular sentences to this form by taking them to have implicit universal quantity (thus 'Homer is a poet' would be analyzed according to the pattern: [quantity]-term-copula-term).

But note now that the copula is no longer a logical copula; it is merely a qualifier. It serves not to link or unite the two terms, but merely to indicate the quality of the sentence. In one sense we could say that the Scholastic analysis (quantifier-term-qualifier/copula-term), now the standard categorical analysis of Traditional Logic, is neither binary nor ternary but *quaternary*. But Traditional Logic has tended to view the analysis as binary. It has done this by taking the quantity-term expression to constitute a single logical unit, the Subject, and the quality/copula-term expression to be a second logical unit, the Predicate. As we shall see, more than a little mischief has been caused during the past century by the tendency to confuse a Subject-Predicate theory of logical syntax with a noun-verb theory. In a more proper sense, however, the Scholastic analysis is ternary. The two terms of a sentence are indeed connected by a third, nonterminal expression—a logical copula. But, unlike Aristotle's copula, the logical copula (not just the qualifier construed as copula) is simply split into two parts. The quantifier and the qualifier of a sentence are merely the two discontinuous parts of a single logical expression. The quantifier alone

clearly does not link terms into sentences. Nor does the qualifier (even if mistaken for a copula). The uniting of pairs of terms to form sentences is accomplished jointly by the quantifier and the qualifier. Viewed in this way, the traditional theory of logical syntax is ternary.

In an introductory philosophy text written by an anonymous Parisian master in 1245, one finds this simple claim: "The art of disputation is logic."[16] For the men of the schools this was a commonplace. In the Middle Ages, schools were the product of the Church; naturally, the methods of the Church became the methods of the schools. In theological matters, disputes concerning the proper reading of a biblical passage were determined, or settled, by rigorous argument. In the school setting, disputation became the means of leading a student to the truth. A master raised a *quaestio* concerning the reading of a given text (say, one by Aristotle). Students then presented arguments from reason or authority for various ways of reading and interpreting the text. The various arguments were, of course, incommensurate. The master then determined, or settled, the question by producing a demonstration in the form of a rigorous argument establishing the proper, true interpretation.[17] This method of teaching, the practice of disputation, depended heavily on familiarity with a set canon of doctrines and methods concerning argumentation—a logic—shared by both master and students. The logic was, naturally, Scholastic. Since during this period the students entering the schools were boys generally aged twelve and thirteen, and since logic was seen as the essential tool for learning, methods were constantly being devised to aid the learning of logic. Keep in mind that logic was not a topic of disputation itself in the schools—it was the art, the very method, of disputation. Its terminology, rules, and practice were already established. Students first learned them and then learned theology, medicine, or law by applying them. The teaching of logic to young adolescents relied especially on a variety of mnemonic devices. By the early thirteenth century, William of Sherwood had offered the verse containing the now-familiar names of the valid syllogistic moods in the four figures.[18] These traditional names were a boon to students, since they constituted, in effect, recipes for the reduction of imperfect to perfect syllogism.

In *Prior Analytics* (41b6-36), Aristotle had already laid down some necessary conditions for syllogistic validity—for example, at least one premise must be universal, at least one premise must be affirmative. To these the Scholastic logicians added rules drawn from the doctrine of distribution to yield a set of necessary and sufficient conditions for validity. Such a set could then be used (e.g., by students) as a decision procedure to be applied to any inference under examination. Distribution was seen as a property that a term had relative to a sentence in which it was used. Aristotle seems to have had at least a primitive notion of such a property when he talked in *De Interpretatione* (esp. 17b12-13 and 18a1-2) of terms being used in their "fullest extension."

The traditional laws of distribution demanded that in any valid syllogism the middle term must be distributed at least once, and that any term distributed in the conclusion must be distributed as well in the premise. Formulating these laws was easy. What was never easy, from then until the demise of Traditional Logic after the nineteenth century, was an adequate explication of the notion of distribution itself. Traditional logicians drew their doctrine of distribution in part from their semantics. Thus, any lack of clarity in their semantic theories was inherited by distribution theory. (I hope to show in the second part of this essay that a distribution theory based on syntax is viable and explicable, and thus preferable to the traditional theory.)

The logic of the Schoolmen drew its texts and inspiration from Aristotle's *Organon*. Did it suffer from the same limitations that restricted the original syllogistic system? Could Scholastic logic offer adequate analyses of inferences involving singulars, relationals, and compounds? For Aristotle, as we have seen, syllogistic was a tool for the teaching of and research in the theoretical sciences, such as physics and theology. This seems also to have been the attitude of the Scholastic logicians. Syllogistic was a tool, an organon, for carrying out and teaching the theoretical sciences. But, of course, the first theoretical science, theology, was now something different. Aristotle could virtually ignore the role of singular terms in his syllogistic, but the Scholastics could not. Teaching theology in Paris in 1250 or Oxford in 1450 meant dealing with such statements (and thus any inferences in which they might occur) as 'The Apostle Peter is a man', 'God is good', 'Christ was born of a virgin', 'Invisible God created the visible world', and so on. Singular statements can find a place in syllogistic only if they can be seen to have a logical form appropriate to the theory of logical syntax at the base of that logic. Since for the Scholastics statements involved in syllogisms must have the general logical form 'quantifier-term-qualifier/copula-term', singular statements having the grammatical form 'singular term-verb' must be reparsed. The verb, as usual now, was easily rewritten as 'qualifier/copula-term'. The singular term was then rewritten as a universal quantifier plus the term. The justification for this new move was provided by the doctrine of distribution, for a term was, generally, said to be used distributively in a statement when it was used to refer to every individual for which that term had personal supposition. Since a singular term such as 'Socrates' in 'Socrates is wise' is being used to refer to every individual for which 'Socrates' has personal supposition (viz., just Socrates), it is distributed. Universal quantity is the mark of any distributed subject-term, so singular subject-terms were given an implicit universal quantity. This done, singulars could be incorporated into syllogistic on all fours with general terms.

By the thirteenth century it had become customary to include in texts and compendia of logic sections dealing with what had come to be called *consequentia*—the logic of propositions.[19] Scholastic thought

concerning propositional, or sentential, logic was deeply influenced by two ancient sources: the Stoic and Megarian theories of implication and Theophrastus's work on hypothetical syllogisms. Scholastic logicians included a wide variety of sentence forms under the heading "hypothetical." These included not only conditionals, conjunctions, and disjunctions, but non-truth-functionals such as causal sentences. Some logicians, such as Ockham, tended to keep the logic of hypotheticals separate from the logic of categoricals (syllogistic), but later logicians tended to incorporate all logic into a general syllogistic theory.

It seems that a fully articulated general theory of logic for compound sentences is not to be found among extant Scholastic literature, and this in spite of the enormous amount of attention paid to the subject, especially during the thirteenth and fourteenth centuries. This is partly due to the fact that these logicians were never fully clear about just what the relationship between a term logic and a sentential logic should be. As well, much of the discussion of so-called hypotheticals involved modal issues. Ivan Boh (1982) has suggested that the etymology of "consequentia" might provide another clue. The relationship of "following along" was commonly felt by medieval thinkers to hold between concepts, terms, propositions, and premise-conclusion pairs. Scholastic logicians seemed to take little care in distinguishing between saying, for instance, that the concept *male* is a consequence of the concept *bachelor*, the term 'male' is a consequence of the term 'bachelor', the consequent of the conditional 'If he is a bachelor then he is a male' is a consequence of the antecedent, the proposition 'He is a male' is a consequence of the proposition 'He is a bachelor', and the conclusion 'He is a male' is a consequence of the premise 'He is a bachelor'. We will soon see that Leibniz intentionally tried to eliminate some of these distinctions, and later we will question whether such distinctions (now made almost instinctively by today's logicians) are the results of confusion or of keen logical insight.

Propositions containing relational terms were said to be "oblique" because their object terms were in non-nominal—that is, oblique—grammatical cases. Scholastic logicians often tried desperately to account for such sentences. But their commitment to parsing statements as categoricals of the general form, quantifier-term-qualifier-term, made it difficult to formulate relational statements, which always had at least two quantified (or, at least, quantifiable) terms.[20]

Cartesian Interlude

> *Logick without Oratory is drye and unpleasing and Oratory without Logick is but empty babling.*
>
> *Richard Holdworth*

By the sixteenth century, Scholastic logic was scarcely to be found as a subject of inquiry or teaching in most European universities.[21] Indeed, Scholasticism itself (and with it Scholastic grammar, logic, theology, etc.) was in shambles. It would be a very long time before logic would again hold the pride of place in the academic curriculum that it enjoyed during the High Middle Ages. By then, both logic and the academy had changed beyond measure: most of the gains in logic made by the Scholastics were lost. Leibniz rediscovered or reinvented some, and modern mathematical logicians have built a cottage industry producing textual evidence that the Scholastics were precursors for many key elements of mathematical logic.

While the rise of Humanism in the fifteenth and sixteenth centuries was not the direct cause of the decline in Scholastic logic, it did very rapidly replace the Scholastic curriculum with its own.[22] And its view of logic was at odds in almost every way with that of its predecessor. Two names in particular are to be associated with the change in attitude toward logic—Peter Ramus and René Descartes.

Humanism itself was the result of the rediscovery of large portions of the literature of ancient Greece and Rome. Cicero soon became a greater authority than Aristotle. Humanist grammar (with its emphasis on the linguistic customs found in Roman authors) replaced Scholastic grammar (with its careful attention to parsing and syntax). Rhetoric replaced logic. Eloquence replaced logic-chopping. Dialectic (i.e., logic) continued to hold a central place in the curricula of arts faculties in most European universities; however, the subject itself was gradually extended to cover topics that came to characterize Humanist logic. Ramus (1515-72), unlike many other Humanists, had been trained in the old Scholastic logic. But his contribution was not due to his logical abilities; he was the great popularizer of Humanist logic and its consequences for the universities. Having made a reputation for himself as a critic of Aristotle and Aristotelians, he went on to argue strenuously for the key theses of Humanist logic: that the logicians should primarily be interested in the conditions of *good* arguments—arguments that are persuasive and well presented—rather than *valid* arguments; that the logician/dialectician should view logic as a *means* for discovering new knowledge, rather than as a *goal* (i.e., the Scholastics' goal of valid inferences); and that the logician/dialectician should abandon the useless attempts to find a formal logic of language (viz., the artificial Latin of the Schoolmen), concentrating instead on ways to use effectively ordinary language, which is too slippery for the grip of formal syllogistic. Ramists saw the old school logic as unnecessarily difficult to teach and learn and as generally useless. The Scholastics' philosophical attitude toward logic was replaced by the Humanists' pragmatic attitude.

Humanist logic was able to prevail in the university curricula during the fifteenth and sixteenth centuries partly because Aristotelian logic was no longer the area of research for significant numbers of first-rate thinkers. The best minds of the age were turned toward literature, Platonism, rhetoric, mathematics, and, eventually, the new science. The

only logic texts available were often poor and attenuated summaries, sometimes mixing Scholastic with Humanist theories. Nonetheless, by the end of this period certain elements of traditional logic had begun to reappear.[23] At least two events account for this revitalization. First, new Greek editions of the *Organon* became available and replaced the Latin translations. Logicians already influenced by the Humanists were disposed to favour Greek over medieval Latin. Thus, Aristotle's logic in Greek gave a new respectability to the entire field of syllogistic. Second, logicians began to pay particular attention to problems of semantics and to recognize themes common to both Scholastic and Humanist philosophies of language. In particular, philosophers of language of both the medieval and Renaissance periods continued to hold that spoken language is purely conventional, and that spoken language corresponds to *mental* language. The Aristotelian inspirations for these beliefs came from *De Interpretatione* 17a1-2 and 16a3-8, respectively. By the late sixteenth century, most debate centred on the second statement and concerned the question of whether words signify concepts (in the mind) or things (in the world) or both, and the question of whether a mental proposition is a single unit or a complex of united parts, each corresponding to a mental term, combined by mental syncategorematic "acts." The interest of seventeenth-century philosophers of language in universal language, mental language, and artificial languages can be traced back in part to these earlier debates, which themselves harken back to the even older question of sentential unity.

It was in this intellectual climate that Descartes opened the age of modern philosophy. Having rejected all that he had learned in school, including formal logic, Descartes made a typical Humanist turn. He decided that since logic could not provide knowledge (indeed, nothing taught in the schools could), a new method must be sought for discovery—an *inventio,* as Ramus had called it—a method for finding new knowledge, rather than *iudicium*, a tool for making judgments (viz., of truth and validity).

Descartes's attack (especially in his *Discourse on Method*, *Regulae ad Directionem In genii*, and *La Recherche de la Vérité par Lumière Naturelle)* was aimed at the Scholastic institutions (such as the Jesuit College Henri IV de la Flèche, which he had attended) in general, and at their logic in particular. Knowledge of truth could be obtained by the proper application of a method (shades of Ramus). And this method, which Descartes devised for himself, required no master, no text, no school. The light of reason, common to all, is what allows one to see truth. The man who would seek knowledge need not ask entrance to the school. The right-thinking man (*honnête homme*) need merely follow the light of reason concerning things that are useful and least taxing to the memory (i.e., no need for Barbara, Celarent, Darii, Ferio, and the rest). Logic as taught in the schools, according to Descartes, is unnatural and forced upon the young before their natural reason has matured. Indeed, logic itself is unnecessary, because once natural reason has developed in the mind, all the useful truths can be seen or deduced by simple, self-evident chains of reasoning. No rules of logic are required. The logic of the schools is at best useless, at worst corrupting.

> Mais d'aucuns s'étonneront peut-être que, cherchant ici les moyens de nous rendre plus aptes à déduire les vérités les unes des autres, nous omettions tous les préceptes par lesquels les dialecticiens pensent gouverner la raison humaine, en lui prescrivant certaines formes de raisonnement qui aboutissent à une conclusion si nécessaire que la raison qui s'y confie, bien qu'elle ne se donne pas la peine de considérer d'une manière évidente et attentive l'inférence elle-même, peut cependant quelquefois, par la vertue de la forme, aboutir à une conclusion certaine. C'est qu'en effet nous remarquons que souvent la vérité échappe à ces chaînes, tandis que ceux-là mêmes qui s'en servent y demeurent engagés. Cela n'arrive pas si fréquemment aux autres hommes; et l'expérience montre qu'ordinairement tous les sophismes les plus subtils ne trompent presque jamais celui qui se sert de la pure raison, mais les sophistes eux-mêmes. (1956: 71)

In contrast with the obscurity of the old logic, the light of pure reason is nearly foolproof and, perhaps most importantly, so much easier.

> ...car comme il ne suit aucun maître que le sens commun, et comme sa raison n'est gatée par aucun faux préjugé, il est presque impossible qu'il se trompe, ou du moins il s'en apercevra facilement et pourra être ramené sans peine dans la bonne voie. (901)[24]

This Cartesian attitude toward logic dominated philosophy well into the nineteenth century. Locke's denigration of formal logic (especially in the fourth book of the *Essay*[25]) is just one example of this attitude. Almost everywhere, the counsel was to relax and let the unschooled, natural light of reason illuminate the truth; the tedious and difficult task of learning the rules of syllogistic logic was unnecessary and unilluminating. Moreover, this new attitude was further encouraged by the developments taking place in mathematics during the sixteenth and seventeenth centuries. Given the current relationship between logic and mathematics, this may seem surprising. Nonetheless, during that period mathematicians began to abandon their traditional view of geometry (an axiomatic, *logical* system) as central to mathematics and began concentrating on algebra and analysis. In contrast to geometry, algebra and analysis were viewed as ways of discovering new knowledge. Thus mathematics was beginning to be seen

as a method of discovery rather than as a logical system. This shift, coupled with the new science of Galileo and Newton, helped to strengthen the Renaissance emphasis on method over reason, and contributed to the marginalization of traditional logic.

Given Descartes's general animosity toward formal logic (and its role in the curriculum), it is ironic that one of the most influential logical works of the seventeenth and eighteenth centuries was produced by a group of Cartesians, the so-called Port Royal logicians.[26] The Port Royal logic is often best known for its revolution in semantics (its replacement of the Scholastic signification/supposition distinction by the comprehension/extension distinction), but it is the Port Royal theory of grammar and logical syntax that is of most interest to us. While they were Cartesians, mixing theories of method and epistemology with logic and grammar, the Port Royal logicians were not Ramists. They took Aristotle as the chief authority on formal logic, and took logic to involve both the effective use of language and correct reasoning.

The Port Royal theory is expounded in three important works, which were written separately and independently but can be treated "as one grammatico-logical work in three volumes" (Padley, 1976: 256). The three works are *Nouvelle Méthode pour apprendre facilement et en peu de temps la langue latine* by C. Lancelot (Paris, 1644), *Grammaire générale et raisonnée* by A. Arnauld and Lancelot (Paris, 1660), and *La Logique, ou l'art de penser* by Arnauld and P. Nicole (Paris, 1662).[27] The theory of grammar and logical syntax found there formed part of a long tradition in linguistics, combining ideas from Aristotelian logic, ancient grammar, the *grammaticae speculativae* of the Scholastics, seventeenth-century philosophical grammars, and rationalism. That tradition was obviously fragmented and incoherent, including philosophers like Descartes and Leibniz as well as Bacon, Hobbes, and Locke. In contrast to the Humanists, the Port Royal logicians were sober, practical Jansenists, seeing logic, rather than rhetoric, as the source of syntactical insights. Indeed, they completely subordinated grammar to logic. The tradition to which they belonged posited a level of language—a universal language—underlying and common to all of the various natural languages. The grammar of this universal language was a deep grammar, in that it was a grammar of concepts (which are shared by all persons) rather than of mere words (which are relative to each natural language). In fact, Lancelot's Latin grammar was meant to be a tool for teaching Latin by instruction not of Latin grammar rules but of universal rules. Thus it can be seen as an attempt to apply the grammatico-logical theory of the *Logique* and *Grammaire*.

Cartesian linguists, such as those of Port Royal, recognized the frequent occurrence of natural-language sentences that differed markedly from their logical, deep forms. Most of the time, this is due to ellipsis. Certain terms or phrases essential to the deep sentence are omitted (for various reasons) from the surface, or "figurative," sentence. An important initial task of the logician, then, is to "resolve" surface sentences into deep sentences; they are natural in the sense that they carry the meaning of the surface sentence. Deep sentences have a *logical* syntax. The basic theses of this theory of logical syntax can be summarized as follows.[28]

(i) Every judgment or proposition (i.e., statement-making sentence) is a predication.
(ii) Predication takes place in categorical sentences.
(iii) Every categorical sentence consists of a subject and a predicate.
(iv) Every subject is a universal or particular quantifier plus a term.
(v) Every predicate is a copula (i.e., qualifier) plus a term.
(vi) Terms may be simple or complex.
(vii) Predication takes place between the subject-term and the predicate-term.
(viii) Predication is effected by the syncategoremata (i.e., the quantifiers and copulae).
(ix) Predication is symmetric (i.e., the terms of a predication are logically homogeneous, fit for either a subject-term role or a predicate-term role).

It is obvious that these logicians were trying hard to preserve the generally Scholastic view of logical syntax.

The Port Royal logicians (*Logique*, II, 7) chastised logicians who taught merely that sentences consist of two parts—subject and predicate—without indicating anything more than that the subject is the first part and the predicate the last part of a sentence. As the above theses show, they saw predication as a joining (or separating) of two concepts expressed by terms (categoremata). The job of syncategoremata is to do the joining (or separating). Indeed, where Aristotle and his followers marked the difference between categoremata and syncategoremata by saying that the former are independent and the latter dependent, the Port Royal logicians added that the former result from the first act of the intellect (conceptualization, or simple apprehension), while the latter result from the second act (judgment, or composition and division—i.e., the formation of statements).

Their theory of logical syntax is best seen in their account of verbs. Verbs are of two sorts: adjective and substantive. A "verb adjective" was analysed as an affirmation plus an attribute—that is, as a copula plus a term (this recalls Abelard's theory). A "verb substantive" was simply an affirmation—i.e., a copula. Consider the sentence 'Some man runs'. An alternative theory (viz., a binary one) might analyse this as a subject, 'some man', and a predicate, 'runs'. But, good Aristotelian ternarists that they were, the Port Royal logicians demanded a connecting link between the two terms. Predication occurs not between a subject and a predicate, but

between a subject-term and a predicate-term. So, the verb 'runs' must be analysed into a copula (always in the qualifier sense) plus a term: 'Some man is running'. The two terms 'man' and 'running' are connected by the syncategorematic expression 'some . . . is'. Where another analysis might take quantifiers and copulae (qualifiers) as optional, this analysis demands them. Sentences like the following all must be resolved into sentences containing syncategoremata.

1. Socrates runs.
2. Men reason.
3. No men fly.
4. Amamus.

Their corresponding deep sentences are:

1a. Every Socrates is running.
2a. Every man is reasoning (or, is rational).
3a. It is not the case that some man is flying.
4a. Nos summus amantes (or, better, Omnes nostrum sunt amantes).

In 3a it must be noted that the phrase 'it is not the case' was seen as modifying the copula—changing it from one of joining to one of separating. Thus, 3a was taken as a sentence that negatively connects (separates) the concept of man and the concept of flying. It was not seen, as it is by modern logicians, as the negation of an entire sentence.

An essential feature of language, often recognized now by linguists, is its creative aspect. An infinite number of sentences can be generated from a finite number of terms connected in a finite number of ways. A recursive formation rule, like (vi) above, permits this. To see how this is so, it is necessary to realize that the Port Royal logicians implicitly treated entire sentences as complex terms and resolved all complex terms as predications. A noun-adjective combination is an example of a complex term. Consider 'wise man'. This is resolved as 'man who is wise', where the phrase 'who is wise' is a predication between 'wise' and the "principal word," which is antecedent for the relative pronoun. In this case, the principal word is 'man'. In fact, they took all adjectives to be resolvable into predications. A substantive may be absent in the surface sentence but must occur in the deep sentence. For example, 'Some man is white' resolves into 'Some man is a thing which is white' (*Logique*, I: 8 and II: 1).

By admitting into their deep sentences predications that act as terms, the Port Royal logicians had to allow that not all predications are assertions. When I assert (to use one of their own well-known examples) 'Invisible God created the visible world,' I predicate 'invisible' of 'God', 'visible' of 'the world' and 'creating the visible world' of 'God'. But only the latter predication is asserted. The unasserted predications are sentences "contained implicitly" in the surface sentence. The result of all this for logical syntax is simply that all logical complexity, for sentences and for complex terms, is accounted for in terms of predication. Syntactically, complex terms are sentences—sentences are complex terms.

The Port Royal logicians were acutely aware of the temptation to make logic a simple science of simple inferences. Consequently, they took care not to avoid complicated or difficult cases. But the kind of sentence that posed the greatest difficulty for their theory was one that they scarcely seemed bothered by—the relational sentence. Clues to how relationals might be incorporated into the theory are found in the Port Royalists' notion that all verbs connect subject-terms with attributes (predicate-terms), and the idea that complex terms are implicit predications. Consider 'Some boy loves some girl'. In the following section, we will see that Leibniz took this to be a conjunction of 'Some boy is loving' and 'Some girl is *eo ipso* loved' in order to account for the fact that 'loves' somehow attaches to both 'some boy' and 'some girl'. However, by treating relational terms as complex, all complex terms as predications, and the predicate-term of a complex relational term as predicated as well of the main subject-term, one can incorporate relationals into the theory of logical syntax laid out by the Port Royal logicians without recourse to Leibniz's splitting procedure. Thus the Port Royal logicians, though they did not, could have analysed 'Some boy loves some girl' as a predication between two terms: a simple term, 'boy', and a complex term, 'loves some girl'. This latter term, being complex, can itself be analysed as a predication between two terms: 'loves' and 'girl'. Though some of the ideas concerning logical syntax put forward by the Port Royal logicians either are found in Leibniz or are similar to his, few, if any, have survived into our own day. In part 2 of this essay I will try to retrieve a few of them (along with other Scholastic and Leibnizian ideas).

Leibnizian Insights

> *For Reason, in this sense, is nothing but reckoning, that is adding and subtracting, of the consequences of general names agreed upon for the marking and signifying of our thoughts.*
>
> *Hobbes*

> *We do well to analyse matters most industriously and reduce everything to the simplest and most easily grasped inferences, so that even the most insignificant student cannot fail to see what follows and what does not.*
>
> *Leibniz*

In the seventeenth century, there were a few who simply refused to abandon the logic of Aristotle and Abelard, of Sherwood and Ockham. Foremost among them was, of course, Leibniz. But there were many important figures

who influenced Leibniz's logical work. Naturally, Aristotle was a strong influence, with Leibniz going beyond Arnauld's claim in the *Logique* that all precepts of logic belong to Aristotle, and holding that Aristotle's syllogistic system was correct but incomplete. Plato was another influence, as he was on most philosophers after the Humanist revival of Greek.[29] But the most immediate influence on Leibniz's thinking concerning logic and language was Hobbes.

That Leibniz, one of the greatest mathematicians and logicians of all time, should have been influenced by someone who was no logician and a bad mathematician may be surprising, but the fact is that Hobbes's work made a significant impression on the younger philosopher as early as the 1660s. Leibniz's famous letter to Hobbes of 1670 displays his deep respect for the ideas of the aged Englishman. But most important for our purposes is Leibniz's remark in "Of the Art of Combination":

> Thomas Hobbes, everywhere a profound examiner of principles, rightly stated that everything done by our mind is a *computation*, by which is to be understood either the addition of a sum or the subtraction of a difference (*De Corpore*, Part I, Chap. I, art. 2). So just as there are two primary signs of algebra and analytics, + and -, in the same way there are as it were two copulas, 'is' and 'is not'; in the former case the mind compounds, in the latter it divides. In that sense, then, 'is' is not properly a copula, but part of the predicate; there are two copulas, one of which, 'not', is named, whilst the other is unnamed, but is included in 'is' as long as 'not' is not added to it. This has been the cause of the fact that 'is' has been regarded as a copula. We could use as an auxiliary the word 'really'; e.g. 'Man is *really* an animal', 'Man is *not* a stone'. (1966: 2-4)

I will have more to say about Leibniz's view of the copula below. For now, I want to look at the two core logical ideas that Leibniz seems to be borrowing from Hobbes. The first is that all reasoning consists of computation—adding and subtracting; the second is that all statements consist of pairs of terms connected by a copula. Hobbes stated the first thesis on a number of occasions. In *De Corpore,* I, he wrote,

> By reasoning, however, I understand computation. And to compute is *to collect the sum of many things added* together at the same time, or to know the remainder *when one thing has been taken from another*. To reason therefore is the same as *to add* or *to subtract*, . . . Therefore, all reasoning reduces to these two questions of the mind, *addition* and *subtraction*. (1981: 177)

And later in that work he defined a syllogism as "a collection of two propositions into a sum" (1981: 255). The same thesis is propounded in books IV and V of Hobbes's *Leviathan* (1904). Hobbes's second thesis is stated in his attempt to account for propositions (i.e. statement-making sentences) in *De Corpore, I:* "Proposition is speech consisting of two copulated names by which the one who is speaking signifies that he conceives the name which occurs second to be the name of the same thing as the name which occurs first" (1981: 225). He adds that the copula may be explicit (e.g., by the use of "is") or may be indicated by an inflection. And he continues, "Therefore, in every *proposition* three things occur that have to be considered, namely, two names, *subject* and *predicate*, and *copulation*" (1981: 227).

Leibniz did very clearly borrow the first idea from Hobbes. Throughout his logical studies, Leibniz persisted in his view of reasoning as computation. But Hobbes's second thesis is found only in a modified form in Leibniz. Hobbes viewed a proposition as a copulation of two names. However, he took names to be a fairly heterogeneous collection, including proper, common, and abstract terms as well as quantified terms. Though he made use of the Scholastic vocabulary of "subject," "predicate," and "copula," Hobbes had no great respect for the logic of the schools, so "subject" and "predicate" tended to be simply terms for the first and second names of a proposition. There is little syntactical insight here. Leibniz, on the other hand, had great respect for the accomplishments of the Scholastic logicians, thinking only that they had not gone far enough. Like them, he believed that a proposition is best construed as a quantified term (subject) concatenated with a qualified term (predicate). That is why he said, in "Of the Art of Combination," that " 'is' is not properly a copula, but part of the predicate." Indeed, for Leibniz, 'is' is a qualifier. More generally, for Leibniz, a universal language would have a rational grammar. And the logical forms dictated by such a grammar would be revealed in natural-language sentences by particles—the copula being the foremost among these.[30]

Before going on to look more closely at Leibniz, we cannot leave Hobbes without remarking on Geach's criticism of him. Geach accuses Hobbes of holding the "two-name theory" of predication (recall that, according to Geach's reading of history, the two-name theory followed the two-term theory and led to the two-class theory): "Hobbes, who held the two-name theory of predication, held also that the copula was superfluous; but we might very well object that on the contrary it is necessary, because a pair of names is not a proposition but the beginning of a list, and a redundant list at that if the two names do name the same thing" (1962: 35). It must be pointed out in response that (a) Hobbes did *not* hold the copula to be superfluous, for, as we saw above, he explicitly claimed that every proposition consists of two copulated names and that in analysing a proposition the copula is one of the three things to be considered; (b) Geach has misunderstood Hobbes's use of the word 'name'. Geach seems to think

that Hobbes's use of 'name' is similar to his own, which construes a name as a referential expression (paradigmatically a proper name or a personal pronoun, but never a quantified expression). In Hobbes's Latin, the word usually translated as 'name' is *nomen*, but it could just as easily and accurately be translated as 'term', 'expression', 'linguistic item'. Furthermore, Hobbes's talk of such expressions "naming" cannot be understood simply as referring (as Geach and most moderns would have it). Rather, Hobbes held that when used in a given proposition a name (term) denotes an individual or individuals in virtue of its signification. Names do not signify thoughts, ideas, and so on. Names are significant by virtue of their uses in acts of signifying the thoughts or beliefs of speakers when intending to communicate to an audience.[31] Hobbes may not have been much of a mathematician or a logician, but he was a semanticist of the first rank.

Leibniz, by contrast, was a mathematician and logician of the first rank. The idea that reasoning was a form of calculating, of adding and subtracting ideas, and the idea that statements are copulations of pairs of terms, may have been Hobbes's. However, only a logician of Leibniz's talents could develop these into a fully articulated theory of logical reckoning. Scholasticism hung on long enough in the German universities of the seventeenth century to touch Leibniz. His initial views of logic were clearly those of the later Schoolmen, yet he was soon surrounded by the anti-Scholasticism of the Humanist logicians—and was not completely uninfluenced by it. The Humanists had tended to dismiss logic in favour of a search for method (i.e., a means for discovering the truth). Having read Ramus, Descartes, and Hobbes, Leibniz, too, saw the search for method as a proper philosophical goal. What separated Leibniz from the others was his conviction that logic, far from being an impediment to the search for method, *is* a method for discovering truth. From his youth, Leibniz was convinced that it was possible to devise a symbolic calculus, the terms of which could be manipulated mechanically, according to simple laws, to yield truths. The terms of such a device would be supplied by a *characteristica universalis*, or alphabet of human thought. Moreover, Leibniz was always convinced that, given such an alphabet, all terms could be seen explicitly to be either simple or combinations of such simples. If an encyclopedia of established knowledge could be gathered, this, along with the universal language in which to express such knowledge and a logical algorithm for manipulating mechanically, according to logical laws, the terms of such a language, would place within the grasp of the human mind all possible knowledge. The task envisaged here was monumentally ambitious—and impossible for Leibniz to complete—but his enthusiasm for it never waned. He spent much of his life attempting to build a viable logical algorithm, and he made numerous attempts to enlist the aid of learned societies and other researchers in his project.

Leibniz's fertile mind produced a steady stream of insights into the nature of logic and logical algorithms. Of particular interest are those concerning logical syntax, of course, and his theses concerning the extensions and modifications of syllogistic. Russell made popular the charge that the failures of Leibniz's logical programme were due to his uncritical insistence on viewing all statements as subject-predicate in logical form. The fact is that, while Leibniz did insist on a subject-predicate analysis of statements, this attitude was well thought out, and the failures of his logic are due only to his inability to devise an appropriate system of notation for his logical algorithm. Indeed, Leibniz's insistence on a subject-predicate analysis of statements (i.e., a Scholastic analysis) may have been suggested to him by his reading of Scholastic logicians and his respect for Aristotle, but his conviction that it was the correct analysis was the result of his attempts to modify and extend syllogistic so that it could be used as a general logic.

We have seen that traditional syllogistic was unable to give an adequate account of inferences involving three kinds of statements: singulars, relationals ("oblique sentences"), and compounds ("hypothetical sentences"). Leibniz was convinced that the best course was to augment syllogistic so that it could be used to analyse inferences involving these kinds of statements. Since syllogistic, as he saw it, depends upon a categorical (subject-predicate) analysis of logical syntax, he saw his task to be to discover how to construe singular, relational, and compound sentences as categoricals.[32]

For many years Leibniz's view of singulars was a typically Scholastic one. Singular terms in subject position were taken to have implicit universal quantity. Singular predicate-terms were simply treated like any other predicate-term.[33] At one point, he did seem to suggest, however, that singular subjects could be construed as having an implicit particular quantity (Leibniz, 1966: 65). But he finally offered a considered view of the issue in a brief study written late in his career, "A Note on Some Logical Difficulties":

> Some logical difficulties worth solution have occurred to me. How is it that opposition is valid in the case of singular propositions—e.g. 'The Apostle Peter is a soldier' and 'The Apostle Peter is not a soldier'—since elsewhere a universal affirmative and a particular negative are opposed? Should we say that a singular proposition is equivalent to a particular and to a universal proposition? Yes, we should. So also when it is objected that a singular proposition is equivalent to a particular, since the conclusion in the third figure must be particular, and can nevertheless be singular; e.g. 'Every writer is a man, some writer is the Apostle Peter, therefore the Apostle Peter is a man'. I reply that here also the conclusion is really particular, and it is as if we had drawn the conclusion 'Some Apostle Peter is a man'. For 'some Apostle Peter' and 'every Apostle Peter' coincide, since the term is singular. (1966: 115)

What Leibniz was claiming, in other words, was that for singular subjects we can arbitrarily take their implicit quantity to be either universal or particular, since the two "coincide." They coincide in the following sense. The reference of any subject expression (quantified term) is the result of the joint semantic work performed by both the quantifier and the denotation of the term. A universally quantified term makes reference to the entire denotation of the term (the Scholastics called this distributed supposition). Thus, 'every composer', when used in a statement, makes reference to all individuals denoted by the term 'composer': Mozart, Beethoven, Brahms, and so on. A particularly quantified term makes reference to an indeterminate part of the denotation of the term (for the Scholastics, such a term had undistributed supposition). Thus, 'some composer', when used in a statement, makes reference to some indeterminate part (though perhaps all) of the denotation of the term 'composer'—that is, to one or another of the individuals denoted by 'composer'. For example, while 'Every composer is a musician' uses a quantified term referring to Mozart and Beethoven and Brahms and . . . , the statement 'Some composer is a musician' uses a quantified term referring to either Mozart or Beethoven or Brahms or . . . (with 'or' inclusive). Now, suppose that the term involved is not general but singular, such as 'Bach'. The denotation of 'Bach' is just Bach. So the universal quantification of 'Bach' yields an expression that can be used to refer to the entire denotation of 'Bach', in other words, Bach. And the particular quantification of 'Bach' yields an expression that can be used to refer to a part of the denotation of 'Bach', which, since it has but one part, is, again, just Bach. In summary, when the subject-term of a statement is singular we can arbitrarily take it to have either an implicit universal or an implicit particular quantity, since in either case the very same reference is made. In other words, as Leibniz said, the two "coincide." This idea, or one very close to it, has been advocated by others more recently, but as far as I know it belongs originally to Leibniz. Later we will see that Sommers has made much use of it under the title "the wild quantity thesis."[34] Finally, note in passing that in the quoted remarks above, Leibniz mentions without hesitation a sentence using a singular as predicate ('some writer is the Apostle Peter'). In the second part of this essay, we will see that Leibniz's wild quantity thesis, coupled with the admissibility of singulars as predicate-terms, gives an advantage to syllogistic logic not enjoyed by mathematical logic (which must augment its calculus with an appended "theory of identity").

Modern logicians, following Russell in his criticism of Leibniz's logic, generally have been most exercised by Leibniz's attempt to construe relational statements as categoricals. The problem with relationals is that they seem to have too many subjects. 'Some man is a lover' is clearly categorical, having one subject and one predicate. But 'Some man loves some woman' has two subjects, two quantified terms, so how could such a sentence possibly be construed categorically? Leibniz's solution was to take such sentences to be at bottom conjunctions of categoricals. Thus, 'Some man loves some woman' would be parsed initially as 'Some man loves and some woman is loved'. But, of course, this leaves doubt about whether the loving man is the one who loves the beloved woman. So such a sentence is finally parsed as 'Some man loves and *eo ipso* some woman is loved'. In other words, 'A is R to B' would be analysed as 'A is an R'er and by that very fact B is R'ed'. Consider 'David is the father of Solomon'. A Leibnizian analysis renders this as 'David is a father and by that very fact Solomon is a son (i.e., is fathered)'. What Leibniz was aiming at here was an analysis that would construe relational terms such as 'loves' and 'father of' as being simultaneously predicated of two subjects, to be "Janus-faced," as Sommers has called it (1983: 188), turning each face toward a different subject.[35] Russell quoted Leibniz concerning this very point:

> I do not believe that you will admit an accident that is in two subjects at the same time. My judgment about relations is that paternity in David is one thing, sonship in Solomon another, but the relation common to both is merely a mental thing whose basis is the modifications of the individuals. (Russell, 1900: 206)

It cannot be said that Leibniz was very successful in his attempt to give a categorical reading of relationals and thereby to incorporate inferences involving them into syllogistic. What he did do was introduce the notion that relational sentences should be analysed as triples (or n-tuples) of terms, such that appropriate pairs of those terms can be taken as forming (by the presence of, possibly implicit, quantifiers and qualifiers) categorical phrases. Moreover, he saw that inferences involving such statements would be governed by the same rule of mediate inference that governs categorical syllogisms, namely, the *dictum de omni et nullo*.[36] It was in "A Specimen of a Demonstrated Inference from the Direct to the Oblique" that Leibniz proposed a method for demonstrating the validity of inferences in which the terms of the premise(s) are all nominative but some of the terms of the conclusion are non-nominative (as is the object-term of a relational expression). Such arguments became of interest to Leibniz after reading Joachim Jungius's *Logica Hamburgensis*. One of Jungius's examples was: Omnis circulus est figura. Ergo, quicumque circulum describit figuram describit. Leibniz analysed a similar example: Painting is an art, therefore he who learns painting learns an art. In this case the term 'art' in the conclusion is not in the nominative case. Since his version of the *dictum de omni et nullo* (in terms of subjects and predicates) requires that 'art' be nominative in the conclusion, Leibniz supposed an equivalence between any expression of the form 'thing that is X' (with 'X' nominative) and 'X'

(where 'X' is oblique). His proof then proceeds, in effect, as follows:

1. Painting is an art.	premise
2. He who learns painting learns painting.	assumption
3. He who learns painting learns a thing that is painting.	his equivalence
4. He who learns painting learns a thing that is an art.	the dictum
5. He who learns painting learns an art.	his equivalence

Leibniz was on the right track here, but his system was still unsound. There are invalid inferences that it could prove, such as, 'Painting is an art, therefore he who learns an art learns painting.' What is required is a formulation of the dictum in terms of term distribution. Leibniz, like most traditional logicians, always took distribution in terms of quantity and quality. To break the hold of this dogma the logician must let go of the analysis of propositions into subjects and predicates. In part 2 we will see that Leibniz's failure here was due not to his inability to recognize that relational terms are always polyadic, never monadic,[37] but to his inability to recognize the Aristotelian ternary form of relational predicates and to his failure to devise a suitable system of notation flexible enough to symbolize statements containing such terms.

Thus far, we have seen that Leibniz was successful in his attempt to incorporate inferences involving singular statements into his syllogistic by construing singulars as categoricals. He was less successful in his attempt to do the same for relationals. His third task was to find a way of analysing compound sentences ("hypotheticals") as categoricals. He realized that by doing so the logic of compounds (what is now called propositional logic) would be then viewed as simply a part of the more general logic of categoricals—syllogistic. In "General Inquiries about the Analysis of Concepts and of Truths" he wrote, "If, as I hope, I can conceive all propositions as terms, and hypotheticals as categoricals, and if I can treat all propositions universally, this promises a wonderful ease in my symbolism and analysis of concepts, and will be a discovery of the greatest importance" (1966: 66). While Leibniz was certain that entire propositions could be conceived of as terms, he was less than clear about just how this could be revealed logically. At any rate, he was clear about how to "treat all propositions universally." What he meant was that any proposition could be viewed as claiming that the concept expressed by the subject *contains* the concept expressed by the predicate. A proposition will be taken to be true whenever its claim holds—that is, whenever the subject concept does contain the predicate concept. If containment is the relation claimed to hold between a subject and a predicate, then, given that entire propositions can be taken to be terms, and that hypotheticals are to be taken as categoricals, the claimed relation between an antecedent and a consequent must be containment as well. And indeed this is just what Leibniz says: "Une proposition catégorique est vraie quand le prédicat est contenu dans le sujet; une proposition hypothétique est vraie quand le conséquent est contenu dans l'antécédent" (quoted in Couturat, 1966: 423). Whether or not one accepts Leibniz's containment account of truth, the fact is that if there is to be a general logic incorporating both principles of inference for categorical statements and principles of inference for compound statements, it must be one that, as he saw, first takes entire statements to be terms and then bases its account of inference on a theory of logical syntax that sees terms as logically homogeneous (i.e., a non-binary theory of logical syntax).[38]

Like De Morgan, Leibniz has become famous for a law—Leibniz's Law.[39] That the law can even be found in the corpus of Leibniz's work is questionable. At any rate, it is usually stated in the form of a biconditional or a conjunction of conditionals concerning the connection between identity and indiscernibility: two things are identical if they are indiscernible and only if they are indiscernible. Usually, indiscernibility is defined as follows: Two things are indiscernible if and only if any predicate true/false of one is true/false of the other. Sommers (1982: 127-30) is among those who have denied that Leibniz formulated Leibniz's Law. Moreover, he has argued that what Leibniz did in fact formulate was an altogether different law, from which, when properly understood, Leibniz's Law can be derived. This other law is the Principle of Substitutability, and there is no doubt about its author. Leibniz states the principle as follows (quoted in Couturat, 1966: 259): "*Eadem sunt quorum unum in alterius locum substitui potest, salva veritate*" (Two things are the same that can be substituted for one another everywhere, without destroying truth). It is unclear here whether Leibniz is talking about objects or terms, but in a later work he makes it explicit. He is talking about the necessary and sufficient conditions for a pair of terms to be interchangeable: "Those terms are 'the same' or 'coincident' of which either can be substituted for the other wherever we please without loss of truth—for example, 'triangle' and 'trilateral' " (Leibniz 1966: 131).

Earlier I mentioned that traditional logic, unlike modern mathematical logic, has no need for a special "theory of identity" (and I will say much more about this in later chapters). Suffice it to say for now that modern logicians take identity to be a relation that holds between an object and itself. Leibniz nowhere talks of such a relation. His Principle of Substitutability governs the terms of a sentence: it states the conditions in which a pair of terms are replaceable for one another in a given sentence without altering the truth-value of that sentence. It is only when seen in this way that the principle is regarded as an integral part of Leibniz's general program for logic. For it is merely a special case of an even more general logical principle, one considered by Leibniz to be the most important principle governing logical inference—the *dictum de omni et nullo*. Look again at Leibniz's formulation of the dictum: "To be a predicate in a universal affirmative proposition is the same as to be capable of being

substituted without loss of truth for the subject in every other affirmative proposition where the subject plays the part of predicate" (1966: 88). In effect, the dictum is a very general principle of substitutability. It lays down the conditions governing when one term can be substituted *salva veritate* for another in a given sentence. Notice that there are pairs of terms for which the dictum holds but the principle does not. In a Barbara syllogism, for example, the major term can be substituted for the middle term in the minor premise to yield the conclusion. But no law permits the substitution of the middle term for the major term—they are not mutually interchangeable. Many pairs of terms are such that one can be substituted for the other in a given sentence but the other cannot be substituted for the first. Other pairs of terms are such that *either* can be substituted for the other in a given sentence. When is this so? According to the dictum, whenever each is truly affirmed of the universalization of the other. In other words, 'A' and 'B' are intersubstitutable for one another in a given sentence *salva veritate* whenever 'every A is B' and 'every B is A' are both true. The principle of substitutability is merely a special case of the dictum.

What, now, of Leibniz's Law? Recall that the law states, in effect, that two things are identical if and only if they are such that whatever predicate is true of one is true of the other. This law can now be derived from the Leibniz's principle and his wild quantity thesis. Let 'a' and 'b' be two singular terms. To say that a predicate, 'P', is true of 'a' (given the wild quantity thesis) is to say 'every/some a is P', which, by the dictum, means that 'P' is substitutable *salva veritate* for 'a' in any sentence in which 'a' is affirmed. Now, let 'P' be 'b'. To say, then, that 'b' is true of 'a' is to say that 'b' can be substituted for 'a' in a given sentence *salva veritate*. The same holds for 'a'. So to say that a and b are identical is to say that 'a' and 'b' are intersubstitutable in a given sentence *salva veritate*. To say that Tully is identical to Cicero would be to say that every Tully is Cicero and every Cicero is Tully—'Cicero' and 'Tully' are intersubstitutable.

Before leaving our discussion of Leibniz's insights, it would be instructive to say something more about the wild quantity thesis. It is possible to show that the wild quantity thesis can be derived from two other Leibnizian theses: the Conceptual Containment thesis and the Completeness thesis.[40] As we saw above, the first thesis holds that a statement is true if and only if the concept expressed, or "signified," by the subject-term contains the concept expressed by the predicate-term. The clearest mark of containment is the copula 'every . . . is'. Thus, in 'Every man is rational' the concept signified by 'man' is claimed to contain the concept signified by 'rational'. The statement (made by the appropriate use of this sentence) is true just in case the claim is upheld. For Leibniz, every concept is either simple or complex. A complex concept is the result of a combination by addition or subtraction of less complex concepts, and any concept can be said to contain itself. Particular propositions are governed by the thesis only by taking them to be elliptical. Thus, Leibniz held that in a particular proposition "something is added" to the subject-term. For example, 'Some man is musical' is true just in the case when the concept signified by 'musical' is contained in the concept signified by 'man' with an added term, 'musical', so that the real significant of the subject-term is the one signified by 'musical man'. Negative propositions simply deny their corresponding affirmations, and thus are taken as falsity rather than truth claims, or, equivalently, as truth claims depending on conceptual exclusion rather than containment.

The Completeness thesis[41] claims that the concept of an individual substance (Leibnizian monad) is complete. The content of an individual is a combination of concepts. That complex concept is complete in the sense that for any pair of logically incompatible concepts (e.g., rational/nonrational, massive/massless, red/nonred) exactly one is contained in it. This thesis, along with the Conceptual Containment thesis, can be used to establish Leibniz's wild quantity thesis. Let 'X' name an individual (i.e., 'X' is a proper-name singular term). The wild quantity thesis holds that every X is Y if and only if some X is Y. Since subalternation, a rule accepted by Leibniz, guarantees that every X is Y only if some X is Y, it is sufficient to show that every X is Y if some X is Y. Suppose it is asserted that some X is Y. According to the Containment thesis, this is true just in case 'some XZ is Y' is true (where Z is the concept added for particular statements and where every Z is Y and Z is contained in X). Next, suppose it is asserted that every X is Y. This is true, according to the Containment thesis, just in the case when the concept signified by 'X' contains the concept signified by 'Y'. Since X is an individual, according to the Completeness thesis, the concept of X is complete. No X could be a nonY. The concept of X contains the concept of Y. And, since X is an individual, its concept contains one of every other pair of incompatible concepts. No concept could be added to the concept of an individual that is not either already contained in it or inconsistent with it. So the concept of Z is already contained in that of X or it is inconsistent with it. If it is already contained in the concept of X, then 'Some X is Y' (='some XZ is Y') entails 'Every X is Y'. If it is not already contained in the concept of X, then 'Some X is Y' is a contradiction. So, if 'Some X is Y' is true, then 'Every X is Y' is true. The logical quantity of singular subjects is wild.

Logicians of our own century are fond of pointing to Leibniz as one of the great precursors of modern mathematical logic (doomed to failure only by his excessive respect for tradition and the categorical analysis of statements). That Leibniz saw that mathematical techniques could be adapted to the needs of a logical algorithm, and that he viewed the establishment of a single general logic, fit for the analysis of all kinds of formal inferences, as a goal worth striving for, are indeed reasons for returning to his ideas and for claiming him as a source of inspiration. But the fact remains that Leibniz is best viewed not as the first mathematical logician but as the last (and best) Scholastic.

Nineteenth-Century Algebraists

For whenever you think the two premises, you think and put together the conclusion.

Aristotle

In effect, we judge and reason with words, just as we calculate with numerals; and languages are for ordinary people what algebra is for geometricians.

Condillac

L'expression simple sera algébrique ou elle ne sera pas.

Saussure

British philosophers from the Renaissance to the mid-nineteenth century were generally unsympathetic or critical toward the enterprise of formal logic. The great empiricists, especially Locke, were particularly harsh in stating their sentiments against logic. Ironically, however, it was in England, Scotland, and Ireland that the great revival (and the subsequent burst of creative activity) concerning formal logic began. The central figure in this revival was George Boole, but the stage for Boole's work had been set by a number of predecessors in logic, mathematics, and philosophy. As well, many of Boole's ideas were worked out in detail or set in proper order by those who followed his lead.

These "algebraic" logicians—Boole, Hamilton, De Morgan, Jevons, Venn, Peirce, Schroeder, J.N. Keynes, W.E. Johnson—dominated logic in the nineteenth century until its final few years, when Peano, Frege, and Russell revolutionized the subject completely. From the time of Leibniz to the mid-nineteenth century the basic logic text in English schools had been Henry Aldrich's *Artis Logicae Compendium.* First published in 1691, it was a collection of aids, mnemonic verses, and so on, for traditional logic. Aldrich's text was replaced (finally) in English schools by Richard Whately's *Elements of Logic,* which was published in 1826 and the standard text known to the British algebraists. In the 1830s Sir William Hamilton published a series of papers on logic that helped to initiate the renewed activities. In particular, Hamilton's work helped to establish the view (contrary to Kant's claim) that traditional syllogistic logic was far from complete, and that as yet unimagined alterations and additions could be made to the old logic. Mill's publication of *A System of Logic* in 1843 was, in many ways, a negative instigator of the renewed interest in formal logic. Mill's logic was an attempt to carry out the older empiricist (and Hobbsean) tradition, which the algebraists generally abandoned. For these logicians, there was no question of going back to what they saw as Aristotelian syllogistic (as found in Whately). They were ready to reject the humanist (or, later, empiricist) notion that a logic that is not a tool of discovery is worthless (as represented by Mill). And they would never consider going back to Scholasticism (as summarized in Aldrich); for one thing, they now knew far too much science and mathematics.

The algebraists were convinced that it was possible to build an algebraic system for the manipulation of properly symbolized inferences in ordinary language. The most successful of these was, of course, Boole's. We will look briefly at his work below. But for my own purposes the work of De Morgan is far more interesting. We will look closely at some of De Morgan's original logical insights.

George Boole built his famous logic as an example of how one could generalize algebra, which itself was a generalization of arithmetic. Like Leibniz, Boole saw certain analogies between numerical addition and multiplication and the logical conjunction and disjunction of terms. The idea that algebraic formulae could be used to express logical relationships followed naturally. Boole presented his logic in *Mathematical Analysis of Logic,* published in 1847, and (more fully) in *An Investigation of the Laws of Thought on which are Founded the Mathematical Theories of Logic and Probabilities,* usually referred to as *The Laws of Thought*, in 1854. Here, Boole's algebra governs a language whose variable expressions ("elective symbols") are general enough that they can be interpreted in at least three ways. Such a term could be interpreted as a class name, a proposition, or a degree of probability. The first interpretation gives a generalization of the traditional logic of terms, now seen as a logic of classes. Boole's idea was that, once formalized, the drawing of syllogistic conclusions was simply a matter of mechanical manipulation of symbols according to the laws of his algebra.

It is important to note that Boole, like Hamilton, De Morgan, and the other algebraic logicians, assumed that any algebra of logic must be equational. In other words, the general conviction was that the propositions of logic should be formulated as equations. Initially, this was taken to mean that the copula must be represented as a relation of equivalence,[42] and that very idea led to the suggestion (made popular, if not originated, by Hamilton) that the predicate as well as the subject of a categorical proposition must be logically quantified. Boole, however, took the equivalence in question to hold between a specified class and either the universal class or an indefinite nonempty class.

Boole symbolized the empty class by '0', the universal class (i.e., the class of entities constituting the "universe of discourse", a concept originally introduced into logic by De Morgan) by '1'. The intersection (conjunction) of a pair of classes was indicated by juxtaposition. Thus, if 'x' stands for the class of men and 'y' stands for the class of unmarried adults, then 'xy' stands for the intersection of these two classes—namely, the class of bachelors. The union (disjunction) of two classes was indicated by the interposition of a '+'. The complement of a class was indicated by the subtraction of that class from the universe. For example, the complement of the class of men (namely, nonmen), where 'x' stands for the

class of men, is symbolized as '1-x'. An E categorical, say 'No x is y', would be formulated as the claim that the intersection of x and y is empty, equivalent to the empty set: xy=0. An A categorical, 'Every x is y', claims that the intersection of x and the complement of y is empty: x(1-y)=0. I and O categoricals could be taken as the negations of E and O (thus yielding inequations), or as claims that the intersections in question are equivalent to some indefinite nonempty class, which Boole symbolized by 'v'. So, I would be symbolized as: xy=v; O as: x(1-y)=v. Syllogistic inference was taken as the reduction of a pair of equations representing the premises to a single equation representing the conclusion, from which the middle term was then eliminated by means of the algebraic rules. These rules, by the way, tended to be those normally found in numerical algebra, with one important exception. Boole realized that his algebra could be interpreted in yet a fourth way, as an algebra of 1 and 0. In such a restricted algebra one finds the special rule: $x^2=x$. This allows the algebra to reflect the important fact about classes that any class is equivalent to its intersection with itself.

Boole called the interpretation of variables as class names the "primary" interpretation. By interpreting his terms as propositions (the "secondary" interpretation), Boole showed that his logic was also a propositional logic, a logic of compound statements. On such an interpretation '1' was read as 'all cases', '0' was read as 'no case', and a propositional variable, 'x', was read as 'cases when x is true'. For example, a conditional, 'If x then y', was parsed as 'No case is a case when x is true and y is false': x(1-y)=0. In general, we have the following parallels:

Formula	**Term reading**	**Propositional reading**
x(1-y)=0	Every x is y	If x then y
xy=0	No x is y	If x then not y
xy=v	Some x is y	x and y
x(1-y)=v	Some x is not y	x and not y

Boole noted "the close and remarkable analogy which [the theory of Secondary Propositions] bears with the theory of Primary Propositions." He added, "It will appear, that the formal laws to which the operations of the mind are subject, are identical in expression in both cases. The mathematical processes which are founded on those laws are, therefore, identical also" (1854: 171). Notice that Boole did not claim that the logic of terms (classes), the algebra given the primary interpretation, was in any way more basic than the logic of propositions, the algebra given the secondary interpretation. Neither logic was taken as the foundation for the other. But the issue was not settled. Until the take-over by mathematical logic at the turn of the century, the question of which logic, term or propositional, was foundational for the other remained a topic of much dispute among algebraic logicians. By the first decade of the new century, the mathematical logicians and the algebraic logicians, such as MacColl, had established what is today generally held as the "default" position: the logic of propositions is foundational for the logic of terms. While Boole, De Morgan, Pierce, and others were apparently neutral with respect to this position, I will not be. In the second half of this work I will reverse the now commonly held view. My claim will be that the logic of propositions is an important but small branch of the logic of terms, so that the logic of terms is foundational.[43]

The great contribution made by Boole to formal logic was his production of a thoroughly general formal system. While those who followed him rejected various details of his logic, his vision of a fully formalized language, subject only to specified general rules, constituting an algorithm for modelling natural language inference, was shared by all of them. The traditional logic was never changed by Boole. He simply supplied it with a symbolic algorithm, which happened to admit alternative interpretations. The most ambitious attempt radically to change traditional syllogistic, without abandoning it, belonged to Boole's sponsor, friend, and fellow mathematician, Augustus De Morgan.

The nineteenth-century algebraic logicians introduced three important ideas into logical investigations: the idea that there was at least a formal connection between the logic of terms and the logic of compound statements, the notion of a universe of discourse, and the realization that relational statements cannot be ignored by any adequate logic of terms. Boole was responsible for clarifying the first idea; De Morgan introduced the other two. De Morgan's work on logic extended from the early 1830s to the mid-1860s. His most original and important ideas are found in his series *On the Syllogism* (1846-62).

It was well recognized by the early nineteenth century that traditional syllogistic could not easily model mathematical proofs (e.g., Euclid's proofs). One reason for this was the increasingly common view that mathematical statements are relational (and thus apparently not categorical). As a mathematician, De Morgan was committed to the view that mathematics is an instrument of sound reasoning. But the apparent relational character of mathematical statements means that either (i) mathematical reasoning is not syllogistic, or (ii) mathematical statements are in fact (and contrary to appearances) categorical. If the latter is the case, then the relational expressions of such statements must be construed as copulae. Initially De Morgan took the mathematician's *is* and *is equal to* as copulae, trying to reduce relationals to categoricals. Eventually, however, he concluded that in effect these two copulae are actually abstract relational terms. In *On the Syllogism, II,* he wrote that an abstract copula is "a formal mode of joining two terms which carries no meaning, and obeys no law except such as is barely necessary to make the forms of inference follow" (1966: 51). This view soon led him to the more radical notion that all logic is simply the study of relations. Any relation (not just *is* and *is equal to*) can

be a copula. Moreover, only the *formal* features of a relation are important for logic. These formal features, for De Morgan, included especially transitivity and symmetry. In other words, De Morgan considered copulae/relations purely abstractly. Indeed, the attempt to look at logic as abstractly as possible (to achieve generality) was a hallmark of all of De Morgan's work in that area. In his *Formal Logic* of 1847, he wrote, "In the form of the proposition, the copula is made as abstract as the terms: or is considered as obeying only those conditions which are necessary to inference" (1966: ix).

For De Morgan, then, inference in mathematics depended on the transitivity of the two copula. But to see this is to recognize—as De Morgan eventually did—that the copulae are themselves relations. Indeed, other mathematical relations (e.g., 'is greater than', 'is less than') are transitive and could easily be construed as copulae as well.[44] Since he took mathematical reasoning to be the discovery of new relations on the basis of known relations, it was natural for De Morgan to conclude eventually that (a) logic in general is the study of relations, and (b) any relation can be viewed as a logical copula.

It was in *On the Syllogism, II,* that De Morgan began his attempt to build an algebra of logic. In doing so, he emphasized the important similarities between logical reasoning and mathematical reasoning. He held that all kinds of opposition (e.g., universal/particular, affirmation/negation, black/nonblack) are formally equivalent: "Every pair of opposite relations is indistinguishable from every other pair, in the instruments of operation which are required" (1966: 23). Thus, the mathematical +/- opposition could be used in an attempt to find an algebraic algorithm for reasoning involving any kind of relational opposition. "I think it reasonably probable that the advance of symbolic logic will lead to a calculus of opposite relations, for mere inference, as general as that of + and - in algebra" (1966: 26). Moreover, the process of algebraic elimination could then serve to model logical inference in general: "Speaking instrumentally, what is called *elimination* in algebra is called *inference* in logic" (1966: 27). Note that in a valid syllogistic inference the middle term *is* eliminated. In building symbolic logic, De Morgan did not, unfortunately, take his own hint and make use of + and - to symbolize any opposition. Instead, he introduced his "spicular" notation. In this notational scheme, parentheses are used to indicate the quantity (distribution) of a term, and negation is indicated by a dot or the use of a lowercase term letter. Distribution is indicated by a parenthesis facing the term; otherwise, the parenthesis faces away from the term. Thus the four standard categoricals would be formulated as follows. A: S))P, E: S).(P [or: S))p], I: S()P, O: S(.(P [or: S()p]. The system was cumbersome and far from perspicuous.

A system of inference that takes any relation as a logical copula must account for inferences in which the premises do not share a common copula/relation. In such cases, there will be a question of which copula/

relation is to be found in the conclusion. De Morgan's response was to make use of the notion of the "composition of relations" (1966: 55ff, 231, 253). An example of an inference whose two premises make use of different copulae/relations (a "bicopular syllogism") is

John can persuade Thomas
Thomas can command William
So, John can control William

Here, the two relations 'can persuade' and 'can command' are composed to yield the relation 'can control'. The relation 'can control' is seen as equivalent to 'can persuade what can command'. Logic is simply the study of such inferences—that is, the Logic of Relations; syllogistic is the part of logic that examines inferences in which the premise copulae/relations happen not to differ from one another.

In *On the Syllogism, II,* De Morgan offered his famous challenge to traditional logicians. "I gave a challenge in my work on formal logic to deduce syllogistically from 'every man is an animal' that 'every head of a man is the head of an animal' " (1966: 29). He went on there to claim that this is not a syllogism but "the substitution, in a compound phrase, of the name of the genus for that of the species." What he meant was that, given the assumption 'every head of a man is the head of a man', one can substitute the name of the genus ('animal') for the name of the species ('man'), where the species-genus relation is given by the explicit premise, in at least one of its occurrences in that assumption. Substituting 'animal' for 'man' in its second occurrence in 'every head of a man is the head of a man' yields the conclusion 'every head of a man is the head of an animal'. De Morgan saw such "oblique inferences" as applications of the *dictum de omni et nullo,* and he took the *dictum* as a rule of substitution. For, just as in algebra "we know that in $x>y$ our right of substitution is, that we may for x write an equal or a greater, for y an equal or a less. In $x>y$, y is used after the manner of a universal term in logic, x after the manner of a particular" (1966: 28).[45] Consider the following argument.

Every person who loves some human is happy.
Every American is human.
So every person who loves some American is happy.

Here the *dictum* requires 'human' to be universal in the first premise. But, as we will see later, this is merely the result of a confusion between the quantity of a term and its "distribution value." For traditional syllogisms, with simple terms, these are the same. But this is not so for complex cases, such as those containing relational terms. In the example above, 'human' is particularly quantified, but it has universal distribution. Once my own algorithm, making use of De Morgan's suggested +/- notation, is developed

below it should be clear that the charge that "in the absence of some other criteria of particular and universal use, the *dictum* cannot be applied" (Merrill, 1990: 88) is best answered simply by providing just such a criterion. In general, De Morgan made the mistake of formulating the *dictum* in terms of species/genus. The fact is that often the universal premise is not a species/genus proposition at all. Thus in *Formal Logic* he wrote that "when X)Y, the relation of X to Y is that of species to genus" (1926: 75).

For De Morgan, all statements are relational (rather than categorical) in that they always relate (bring together, connect) pairs of terms, a view that at least reminds us of Aristotle's ternary analysis. While he was on the right track in taking relationals into a logic of terms, he was clearly mistaken in seeing relational terms as copulae rather than as (material) terms in their own right. In his recent study of De Morgan's logic of relations Merrill argues, "It is one thing to extend logic to include relations; it is quite another thing to think of subject-predicate propositions as relational propositions" (1990: 107). This suggests the stronger claim that it is also wrong to think of relational propositions as subject-predicate. I will reject that suggestion below.

De Morgan, and the algebraists in general, was a "monist" in the sense that he held that there is just one general logical form for all categoricals and relationals. For most, this general logical form was categorical; for De Morgan, it was relational. Later logicians, especially Peirce and Frege, were "pluralists" in that they insisted that there are as many general logical forms as there are numbers of argument places of functional expressions (viz., 0 for unanalysed propositions, 1 for simple monadic predicates, 2 for binary relations, 3 for triadic, etc.). But De Morgan did not see such expressions as functions. Instead, he saw each proposition as consisting of a pair of terms bound together into a syntactic unit by use of a connecting element, a copula. All relational terms, in spite of the fact that they are *prima facie* material expressions, are copulae (thus formatives as well). But in at least one place in *On the Syllogism, II,* De Morgan suggests that it is possible to analyse a relational proposition as a categorical. Using "=" as the logical copula, he wrote, "The algebraic equation $y=\phi x$ has the copula =, relatively to y *and* ϕx: but relatively to y and x the copula is $=\phi$. This is precisely the distinction between 'John can persuade Thomas' and 'John is {one who can persuade Thomas.}' " (1966: 56). Aside from the confusion due to using equality as the standard logical copula, this alternative analysis wants only a way to analyse expressions such as 'one who can persuade' as genuine nonformal terms connected by a logical copula to the next term. The result would be a logical syntax that takes, as Aristotle suggested, all propositions as pairs of terms connected by a logical copula. We will see how to achieve such a theory in the second part of this essay.

Critics argued that the algebraic logicians, by making reasoning a kind of computation (as Hobbes had seen it), subordinated logic to mathematics, particularly algebra. This charge may be appropriate for Boole, for example, who made important use of algebraic and geometric techniques in his system. But while De Morgan did want somehow to unite logic and mathematics, he did not actually make much use of mathematical techniques in his logic. He saw that traditional syllogistic was too weak to handle certain kinds of inferences (viz., those involving relationals) and developed (or tried to develop) the logic of relations as a general logic fit for the analysis of a very broad range of inferences. He saw syllogistic as being a proper part of that logic. What he did historically, in effect, was to usher in a period (along with Boole, Peirce, etc.) during which a number of attempts were made either to extend or to replace syllogistic, with the aim of obtaining the kind of universal logic envisaged by Leibniz, one fit especially for the analysis of proofs in mathematics. By the end of this period, logicians were convinced that syllogistic logic could no longer be considered seriously as anything more than a respectable but powerless alternative to a new, comprehensive, powerful "symbolic logic." Unfortunately, syllogistic logic was presumed identical to term logic. While De Morgan and other algebraists might have been willing to demote, or even reject, syllogistic logic, they were nonetheless *term logicians*. For better or worse, term logic was tarred with the anti-syllogistic brush, and symbolic logic (eventually, mathematical logic) soon replaced the old logic and quickly used its new success to bar the door to any term logic—even a nonsyllogistic one such as De Morgan had envisaged.

Notes for Chapter One

[1] Geach (1972a: 44-61).

[2] We will encounter this view (called "verbism") in chapter 2. For now, see Englebretsen (1982a, 1985a, and 1986a).

[3] For Russell's attempt to reject Aristotle, see Russell (1945: 195-202).

[4] I have offered a slightly different survey of Aristotle's logic in Englebretsen (1981). See, as well, Lukasiewicz (1957) and Lear (1980). Corcoran and Scanlan (1982) is an ideal place to start. See also the essays in Corcoran (1974a).

[5] Geach claims on p. 53 of *Logic Matters* (1972) that Aristotle still held in *Prior Analytics* that a statement could consist of just a pair of unlinked terms, that such expressions as 'applies to' do "not supply a link between 'A' and 'B' " but were "meant only to give a sentence a lecturer can pronounce."

[6] Cf. Englebretsen (1989, 1990b).

[7] For a very thorough discussion of negation in all of its aspects see Horn (1989). See also Sommers (1982) and Englebretsen (1981b).

[8] For more on squares of opposition see Sommers (1982, ch. 14); Sommers (1970); Englebretsen (1984a, 1984b).

[9] For discussion concerning the Aristotelian notion of valid syllogism see Barnes (1969); Frede (1974); and Hadgopoulos (1979).

[10] For an alternative view see Corcoran (1974b).

[11] Cf. Englebretsen (1980a); Barnes (1983).

[12] For a survey of Aristotle's attempt here see Englebretsen (1982b). See also Thom (1977).

[13] See especially Ross (1949: 289); Lukasiewicz (1957: 6-7); Bird (1964: 90); Patzig (1968: 5-7). A response to all of these claims is found in Englebretsen (1980b).

[14] For a brief discussion of Scholastic semantics and its relation to modern logic see Henry (1972: 47-55). See as well A. de Libera (1982); Spade (1982); Nuchelman (1982).

[15] See, especially, Abelard's *Dialectica* (1956).

[16] *Philosophica Disciplina* in Lafleur (1988: 282). I owe this reference and the next, as well as other valuable information here, to Graeme Hunter.

[17] See C. Bazan's "Les questions disputées principalement dans les facultés de théologie" in Bazan et al. (1985, esp. 40). Also see Angelelli (1970).

[18] See, for example, Kneale and Kneale (1962: 232-3).

[19] For a nice summary of this, see Kneale and Kneale (1962: 274-97). The classic extended discussion is Moody (1953). See also Durr (1951).

[20] A nice account of how Ockham might have used his supposition theory to account for oblique inferences is given in Sanchez (1987).

[21] See Ashworth (1974).

[22] For two excellent surveys see Jardine (1982, 1988).

[23] See Ashworth (1988).

[24] An excellent extended account of Descartes's views on logic is found in Gaukroger (1989) and in Clarke (1981).

[25] For example, Locke (1924: 346-7) writes, "If syllogisms must be taken for the only proper instrument of reason and means of knowledge, it will follow that before Aristotle there was not one man that did or could know anything by reason; and that, since the invention of syllogisms, there is not one of ten thousand that doth."

[26] A version of what follows has appeared in Englebretsen (1990c).

[27] See especially Arnauld and Lancelot (1975); Arnauld and Nicole (1964).

[28] A similar summary is given in Sommers (1983a).

[29] See the excellent study by Castaneda (1982).

[30] In the papers on grammatical analysis (Parkinson, 1966: 12-6), Leibniz shows how the number of natural-language particles can be reduced to a few, which correspond to logical particles. McRae (1988) has pointed out that in addition to this purely logical interest, Leibniz also had an interest (shared with Locke) in the project of using natural-language particles as indicators of the internal operations of the mind.

[31] An excellent examination of Hobbes's theory is found in Hungerland and Vick's "Hobbes's Theory of Language, Speech, and Reasoning," in Hungerland and Vick (1981a). See also Dascal (1987, ch.1 and 2).

[32] The account below closely follows those given in Englebretsen (1981a, 1982c).

[33] According to Ashworth (1974: 247), "any proposition whose predicate was singular was treated as if it were of a standard form."

[34] In addition to Sommers, two others have held something like the wild quantity thesis. See Czezowski (1955) and Copi (1982). All are discussed in Englebretsen (1986a, 1986b, 1988b).

[35] Compare Leibniz's remarks in Parkinson (1966: 14-5). For a brief defence of Leibniz's way with relationals against Russell's see Angelelli (1967: 19ff).

[36] In "A Specimen of a Demonstrated Inference from the Direct to the Oblique" (in Parkinson, 1966: 88), Leibniz says that the following (i.e., the *dictum*) holds of such inferences: "To be a predicate in a universal affirmative proposition is the same as to be capable of being substituted without loss of truth for the subject in every other affirmative proposition where that subject plays the part of predicate."

[37] Parkinson (1966: xx) seems to be making such a criticism.

[38] For a useful, extended account of Leibniz's attempt to incorporate the logic of compound statements into syllogistic see Castaneda (1976, 1990). See also Ishiguro (1982, 1990).

[39] More on what follows regarding Leibniz's Law is found in Englebretsen (1984c).

[40] This has been shown in Englebretsen (1988b).

[41] See, for example, section 9 of "Discourse on Metaphysics" (in Leibniz, 1989: 41ff) and section 40 of the "Monadology" (in Leibniz, 1965: 154).

[42] As Patzig (1968: 11) says, "I offer here the hesitant conjecture that the tendency which has continually reappeared in the history of logic, not least in more recent times, of conceiving a judgement as an equation, or even as an expression of identity, derives a good deal, if not all, of its force from the purely conventional wording of the schema 'S is P'."

[43] For more on the controversy among late-nineteenth-century logicians, see Shearman (1906).

[44] For a time, De Morgan considered the idea that all relations could be reduced to a single one—identity. For more on this see the excellent discussion in Merrill (1990, ch. 2 and 3).

[45] Merrill (1990) tries to show that De Morgan was unable to defend the use of his substitution dictum for cases where the substitution applies to parts of complex terms. De Morgan had formulated the dictum in terms of 'some' and 'all' as signs of particularity or universality. But where these occur within complex terms, they are sometimes disconsonant with the true distributivity of the terms to which they apply.