CHAPTER TWO

A MODERN SUCCESS STORY
(or, Frege to the Rescue)

Dear Senior Censor,

In a desultory conversation on a point connected with the dinner at our high table you incidentally remarked to me that lobster sauce 'though a necessary adjunct to turbot, was not entirely wholesome!' It is entirely unwholesome. I never ask for it without reluctance: I never take a second spoonful without a feeling of apprehension on the subject of a possible nightmare. This naturally brings me on to the subject of Mathematics . . .

Lewis Carroll

For recent times have seen the development of the calculus of logic, as it is called, or mathematical logic, a theory that has gone far beyond Aristotelian logic. It has been developed by mathematicians; professional philosophers have taken very little interest in it, presumably because they found it too mathematical. On the other hand, most mathematicians, too, have taken very little interest in it, because they found it too philosophical.

Thoralf Skolem

We know that mathematicians care no more for logic than logicians for mathematics. The two eyes of exact science are mathematics and logic: the mathematical sect puts out the logical eye, the logical sect puts out the mathematical eye, each believing that it can see better with one eye than with two.

De Morgan

Frege

The logicians' conceit is due to their supposing that the ideas could only be learned or rendered clear by their method, their studies and their labours. They therefore interpreted a language in which they are weak *and of which their knowledge is imperfect into another in which they are also weak and their knowledge is imperfect. This sort of translation they made into an art, and then declared that they have to do only with words, not with ideas.*

Abu Hayyan

Mathematicians are a species of Frenchmen: if you say something to them they translate it into their own language and presto! it is something entirely different.

Goethe

Generally speaking, the twentieth century has seen a fairly clear division of philosophy into two quite different branches. One, analytic philosophy, has been pursued mostly in English-speaking countries. It is (again, speaking quite generally) primarily interested in the investigation of relatively narrow problems, especially in epistemology and metaphysics, has had a fairly high regard for the natural sciences, and has tended to formulate its questions as concerning language. Most importantly, it has taken formal logic, in one guise or another, to be an essential tool in its investigations. The other branch of philosophy encompasses a much broader range of philosophical programmes, most of which have been pursued by philosophers on the European continent (thus it is often called "continental" philosophy). Continental philosophers have generally abjured recourse to the results of the natural sciences, and have tended to blur distinctions between philosophy and such disciplines as history, psychology, sociology, political theory, anthropology, literary theory, and so on. In particular, they have generally had little regard for logic—especially formal logic. But the two very closely related fields of philosophy of logic and philosophy of language have come to dominate the work of analytic philosophers. These fields might well be said to be the core of twentieth-century analytic philosophy. To be sure, questions concerning the nature of logic and language are as old as Plato and Aristotle, but their modern versions go back only about a century or so. Indeed, many of the key questions in these areas were formulated by one man. Ironically, he was a German and a logician, and his ideas grew out of his attempts to provide a rational grounding for mathematics.

Beginning in 1879, Gottlob Frege, a then relatively obscure mathematician at the University of Jena, developed a conception of logic and an adequate algorithm for it that were radically different from any earlier theory. What Frege started was a revolution in logic unmatched by any other change in logic since Aristotle.[1] The result of that revolution was the theory of logic that has virtually become canonical in most philosophical circles ever since—mathematical logic. From our perspective of more than a century later it is easy to look at Frege's revolution as representing a quick, clean break with the logic that had gone before (viz., that of the algebraists), but the fact is that there was a substantial period of transition (and even

confusion[2]). Actually, the first attacks on the traditional logic of the algebraists came from nonlogician philosophers (especially idealists and pragmatists). Of course, the algebraists themselves often tarnished syllogistic's image by rejecting parts of it or by defending it inadequately.

To understand Frege's revolution and the mathematical logic it initiated, one must first understand something of what mathematical thinking was like during the period from the development of the Leibniz-Newton calculus to the mid-nineteenth century. From the time of Euclid to the nineteenth century, mathematicians, with few exceptions, looked at geometry as providing the foundation for the entire field of mathematics. To be sure, after the development of the calculus, and of analytic methods in general, there was a renewed interest in showing that mathematical reasoning was at least guided by physical observations. But even then, the model for mathematical proof was the derivation of a geometric theorem from Euclidean axioms. Physical science (especially astronomy, physics, cartography) merely supplied mathematicians with motivations and confirmations for their purely formal proofs. However, by the end of the eighteenth century, efforts to make mathematics, particularly the calculus, logically rigorous were becoming strained. An ever-expanding array of new functions, complex numbers, and even negative numbers were being formally manipulated by mathematicians who had little idea how one might construct convincing proofs of theorems in which these were involved. Most mathematicians during that period chose to ignore the problem of logical rigour and simply plunged ahead. As early as 1743, D'Alembert noted that "Up to the present . . . more concern has been given to enlarging the building than to illuminating the entrance, to raising it higher than to giving proper strength to the foundations" (quoted in Kline, 1972: 619). Common sense, intuition, and conformity with nature were sufficient for eighteenth-century mathematicians. Their work, admittedly, was not modelled on Euclidean proof, but no one doubted that it *could* be. Copernicus, Kepler, Descartes, Newton, Leibniz, and their followers all were certain that mathematical truths were in harmony with, even reflections of, God's design of nature. There were exceptions, of course, the most famous being Laplace, who replied to Napoleon's remark that the mathematician's work contained no reference to God by saying that he had no need of such a hypothesis. But for most mathematicians, logical proof could wait.

In spite of the confidence that mathematicians of the seventeenth and eighteenth centuries had in the possibility of grounding the truths of mathematics, a confidence due to the perceived objectivity and self-evidence of Euclidean geometry, little of that confidence survived the nineteenth century. The development of non-Euclidean geometries shattered the comforting old myth that Euclid could ultimately provide the foundations for all mathematics. Gauss, Bolyai, Lobatchevsky, and Riemann gradually convinced mathematicians that Euclidean axioms were merely empirical, based on everyday—but therefore limited—observations of mundane space. Any number of alternative geometries could be formulated that would be equally self-consistent and might very well describe space more generally. Eventually, the foundations of geometry, analysis, and, finally, set theory were all seriously challenged during the nineteenth century. By the middle of the century, the door was open to any number of formal systems unencumbered by the demands of self-evidence or empirical evidence. Thus Abel could charge, "There are very few theorems in advanced analysis which have been demonstrated in a logically tenable manner" (quoted in Kline, 1972: 47).

Still, the hope that mathematics had a rigorous foundation was not completely lost. The search for mathematical rigour led to the investigation of logic as providing a sufficiently strong foundation for mathematics. But this led, in turn, to a search for rigour in logic itself, since traditional logic was seen as inadequate in too many ways. Thus mathematicians like Boole and De Morgan directed their efforts to algebracizing logic and thereby attaching it to mathematics (and separating it from philosophy and psychology, the perceived sources of its old weakness). Later, Hilbert and the so-called formalists sought to ground geometry on arithmetic and logic, which were purely formal systems bound only by the demand for consistency and designed to formally manipulate symbols. But by far the most influential and carefully articulated attempt to give a solid grounding to mathematics was that initiated by Frege. Working quite independently of his contemporaries, he axiomatized logic in his *Begriffschrift* (1879), and then in the *Grundlagen* (1884) and the two volumes of the *Grundgesetze* (1893-1903) he showed that mathematics is an extension of logic. This programme was eventually called "logicism." As it happened, however, Frege's revolutionary work was virtually ignored for more than a decade. It was rescued from oblivion only at the turn of the century, when Bertrand Russell gave the programme of logicism and the new mathematical logic the prominent, international voice it needed.

Frege's logic was revolutionary in several ways. Traditional logicians, with the exception of Leibniz (who forged a connection between logic and algebra), saw no special relationship between logic and mathematics. The nineteenth-century algebraists' logical renaissance was partly due to their insistence that logic was merely a branch of mathematics. Frege went further still: arithmetic is the foundation of mathematics and arithmetic *is* logic. For Frege and his followers, all mathematics is merely an extension of logic. As a consequence of this view, Frege's logic was revolutionary in a second way. Logic was no longer seen as a special concern for either philosophers or psychologists. It does not describe reality, the knowable, or how one reasons. What it does is account for mathematical reckoning. Moreover, it has no close relationship with natural language, which, unlike mathematical language, is plagued by lack of clarity, ambiguity, vagueness, and inconsistency. The language of logic is,

first and foremost, the underlying rational structure of mathematical language. Later logicians would go beyond Frege by holding that even natural language is, in a deep and hidden way, ultimately structured by this logic as well.

The "look" of Frege's logic was also revolutionary. Earlier mathematizing logicians, such as Leibniz, Boole, and De Morgan, were content to use mathematical notation in their constructions of symbolic algorithms for logic. Frege, in order to avoid confusing mathematical expressions with underlying logical notations, chose an algorithm expressed in nonmathematical symbols. Though his own symbolism was not adopted by others, alternative nonmathematical notations were. As an ironic consequence, mathematical logic (often called "symbolic logic") is expressed in a symbolic vocabulary quite foreign to most mathematicians.

But the most important way in which Frege's logic was revolutionary was in its theory of logical syntax—its account of sentential unity. Frege abandoned the traditional analysis of statements into subjects and predicates, substituting an analysis in terms of the mathematician's functions (i.e., function expressions) and arguments.[3] In his theory, complex statements are built up from less complex statements. The least complex of all are "atomic," consisting of a single function and an appropriate number of arguments. The distinction between a function and an argument is initially made in grammatical terms:[4] arguments are grammatically complete, "saturated," contain no gaps; functions are incomplete, "unsaturated," contain one or more gaps. Frege recognized that two complete expressions alone could not form a sentence; nor could two incomplete expressions (again, two axe-heads do not make an axe—nor do two axe-handles). In the first case especially, there must be a "binding agent." As he said, "An object—e.g., the number 2—cannot logically adhere to another object—e.g., Julius Caesar—without a binding agent [or cement: *Bindemittle*]. And this binding agent cannot be an object but must rather be unsaturated."[5] For him, the "binding agent" is not a third expression (a logical copula). It is just an *incomplete* expression, with sufficient gaps to accommodate the complete expressions. Logical sentential unity is, according to Frege's account, not the result of expressions being connected by a connector but of one or more gapless expressions filling the gaps in a gapped expression (linkage as completion rather than connection).

This account is supplemented by a semantic theory. Arguments are names that refer to objects; functions are not names and refer to concepts. No object is ever a concept; no concept is ever an object. No argument can be used as a function; no function can name an object—that is, be used as a name/argument.[6] Consider the simple sentence 'Socrates is wise'. The function expression here is '. . . is wise', which contains a single gap (argument place), and the argument is 'Socrates'. 'Socrates' names an object (viz., Socrates), while the function refers to the concept of wisdom (which, in fact, is not being named by '. . . is wise'). Functions do not name, but they do *apply* to objects (viz., those objects which "fall under" them—i.e., those objects of which they are true). In 'Socrates is wise', 'Socrates' names an object and '. . . is wise' applies to that object.

In effect, according to this theory, singular terms (proper names, personal pronouns, definite descriptions) are names; general terms (verbs, adjectives, etc.) are function expressions. Relational terms are also function expressions. In 'Socrates taught Plato', the function is '. . . taught . . .', an expression with two argument places, two gaps. The logically simplest kinds of statements, atomic statements, consist of a single function expression (still often called a "predicate") and arguments (names) filling each gap. Thus, atomic statements are always singular. Notice that in certain ways Frege's theory of logical syntax (at least for simple, atomic statements) is similar to Plato's binary theory. Each sees the nonformal terms of language to be divided into two exclusive sets: for Plato, nouns and verbs; for Frege, singular terms (names) and general terms (predicates).[7] In each case, logically simple statements contain no formal, logical expressions; they have zero degree of syntactical complexity. Complex ("molecular") statements are built up from less complex ones by the use of "higher" functions. For example, pairs of statements can be combined by "truth-functional" expressions to form such compounds as conjunctions, disjunctions, and conditionals. The gaps of such expressions, like those of general terms, must be filled by names. Grammatically, the only expressions that can fill the gaps of a truth-function (e.g., '. . . or . . .') and result in a statement are statements themselves. For Frege, then, statements must be names, singular terms. Consequently, they are seen as naming objects. According to Frege, there are two possible objects that a statement can name: the True and the False. True statements name the former; false statements name the latter. Truth-functional expressions (now often called "statement connectives" or "propositional connectives") are formal terms that operate on one (in the case of negation) or two (in all other cases) statements to form logically more complex statements. This recursiveness guarantees that there is no limit to the degree of complexity that a statement can have. The logic of statements whose complexity is due only to the use of truth-functions is called "truth-functional logic," or "statement logic," or "propositional logic." Statement logic, according to Frege's theory, is taken to be *basic* logic.

Complexity can be achieved in another way. Consider again 'Socrates taught Plato'. Suppose now that we replace each of the proper names here with a variable name, one which might name any object. Thus we might have 'x taught y'. This is not itself a statement. The 'x' and 'y' here (called "individual variables") are like the personal pronouns of a natural language (cf., 'She taught him'). And, as with pronouns, they can be used sensibly only with an antecedent expression that determines their reference. In the case of individual variables, their antecedents are quantifiers and are said to "bind" their subsequent individual variables. The

expression 'x taught y' (called an "open formula") might be thought of as a template for a statement. It can be turned into a genuine statement by binding each individual variable by a quantifier. There are two kinds of quantifiers, and, for our sample case, two variables to be quantified and two possible sequences of binding ('x' can be bound first or 'y' can be bound first). So there are eight possible ways of turning 'x taught y' into a statement. The two quantifier expressions are 'something, call it v, is such that . . .' and 'each thing, call it v, is such that . . .', where 'v' is an individual variable such as 'x' or 'y' and the gap is filled by an open formula. In the present case, the eight possible statements are:

1. Something, call it x, is such that something, call it y, is such that x taught y.
2. Something, call it y, is such that something, call it x, is such that x taught y.
3. Each thing, call it x, is such that each thing, call it y, is such that x taught y.
4. Each thing, call it y, is such that each thing, call it x, is such that x taught y.
5. Something, call it x, is such that each thing, call it y, is such that x taught y.
6. Something, call it y, is such that each thing, call it x, is such that x taught y.
7. Each thing, call it x, is such that something, call it y, is such that x taught y.
8. Each thing, call it y, is such that something, call it x, is such that x taught y.

Frege is justifiably credited with being the first logician to give an adequate account of the logical syntax of such "multiply general sentences," sentences with more than one quantifier expression. According to Michael Dummett, His success here was due to his recognition of the fact that the logic of such sentences could be revealed by examining the history of their construction.

> Thus we begin with a sentence such as 'Peter envies John'. From this we form a one-place predicate 'Peter envies y' by removing the proper name 'John'—the letter 'y' here serving merely to indicate where the gap occurs that is left by the removal of the proper name. This predicate can then be combined with a sign of generality 'somebody' to yield the sentence 'Peter envies somebody'. The resulting sentence may now be subjected to the same process: by removing the proper name 'Peter' we obtain the predicate 'x envies somebody' and this may then be combined with the sign of generality 'everybody' to yield the sentence 'everybody envies somebody'. (Dummett, 1973: 10)

Notice that beginning with the sentence 'Peter envies John' the choice of which proper name to replace by a variable name first (and, in so choosing, the choice of which "sign of generality," quantifier expression, to use) will determine a variety of different multiply general sentences (e.g., 'somebody envies everybody', or 'everybody envies everybody', etc.).

Such "quantified" statements are genuine statements; thus, they can fill gaps in truth-functional expressions. (This is one of the reasons that, on a Fregean analysis, the logic of truth-functions is more basic than the logic of quantified statements.) Notice that quantifiers are higher functions. A system of logic that incorporates quantifiers as well as truth-functions is now called "quantificational logic," "the functional calculus," or "predicate calculus."

One kind of statement seems to defy the neat syntactical theory worked out by Frege. Consider 'Shakespeare is Bacon'. In this sentence, 'Shakespeare' and 'Bacon' are clearly proper names and operating as arguments. But where is the function expression? In rejecting the traditional subject-predicate analysis of statements, Frege had banned the copular (i.e., qualifier) 'is' from logic.[8] For example, in 'Shakespeare is British' the 'is' plays no logical role. But if 'is' is ignored in 'Shakespeare is Bacon', there is no term to play the role of predicate (function expression). Frege's solution was to introduce the so-called "*is* of identity." The 'is' of 'Shakespeare is Bacon' (contrary to appearances) is not at all like the 'is' of 'Shakespeare is British'. In the case of 'Shakespeare is Bacon' the 'is' is merely short for 'is identical to' (or 'is the same as'), and 'is identical to' is, without question, a function expression, incomplete, unsaturated, gapped (doubly). To ignore the logical distinction between the 'is' of identity and the 'is' of predication (i.e., the copular 'is', the old qualifier) would be to treat a name, say 'Bacon', as a predicate, as incomplete (and, as a consequence, to treat an object, say Bacon, as a concept). But, again, arguments (names, singular terms) and functions (predicates, general terms) are mutually exclusive from a Fregean logical point of view. A predicate logic that incorporates the identity function is called "predicate calculus with identity."

Aristotle (and Plato) and all of his followers for centuries tended to see formal logic as applying to inferences made in the medium of a natural language (Greek, Latin, German, English, etc.). Frege, by contrast, had little regard for the logical powers of natural language. He wrote, for example,

> If our language were logically more perfect, we would perhaps have no further need of logic, or we might read it off from the language. But we are far from being in such a position. Work in logic just is, to a large extent, a struggle with the logical defects of language, and yet language remains for us an indispensable tool. Only after our logical work has been completed shall we possess a more perfect instrument. (Frege, 1979: 252)[9]

For Frege, mathematics represented the paradigm case of rationality. His goal was not to build a system of logic for natural language; rather, he sought to construct a system of logic adequate to the needs of mathematics. The rigour that mathematics seemed to have lost in the nineteenth century was to be recovered by founding mathematics (especially arithmetic) on logic. The fact that the logical system he built required a theory of syntax remote from the forms of natural-language statements was of no concern to him.

From Aristotle to the algebraists, nearly all logicians (and grammarians) recognized that both statements and terms have opposites. Thus, for the traditional logician, every statement that affirms a given predicate of a given subject corresponds to one that denies the same predicate of that subject. Such pairs of statements are contradictories. Every statement has exactly one contradictory. The denial is often said to be the negation of the affirmation. Terms were also taken to have opposites. Let 'T' be a term and 'nonT' be its negation. For every statement that affirms 'T' of a given subject there is one that affirms 'nonT' of that subject. Such pairs of statements are (logical) contraries. Each term has exactly one negation, one logical contrary. Term negation and statement negation were taken by traditional logicians to be clearly distinct, and logical contrariety between a pair of statements was accounted for by term contrariety. The logical contrary and the contradictory of a given statement were not to be equated. The former always entails the latter, but the latter does not always entail the former. So, to summarize the standard traditional view, statement negation, contradiction, is the result of denying the predicate. Statement contrariety is the result of negating the predicate-term. Thus, any statement can be negated and any term can be negated. There are two logically distinct modes of negation.[10]

For Frege and his followers, there is but one kind of logical negation. In the *Begriffschrift* one finds only one sign (the short vertical stroke) for this: it is the sign for statement negation. It is applicable only to an entire statement and results in the contradictory of that statement. Frege recognized that grammatical negation occurs in various places within natural language sentences. But in his essay "Negation" (Geach and Black, 1952) he reminds the reader (as he did so often throughout his writings) that such "languages are unreliable on logical questions" and goes on to warn of the "pitfalls laid by language" (126). So, even though a grammatical sign of negation may appear to apply to a term or part of a sentence, in such cases the logical fact is "that we do [thereby] negate the content of the whole sentence" (131). According to Frege, there is no term negation—there is just sentential negation. However, as it happened, he did consider the possibility of a second mode of negation.

Frege took pains to distinguish between what he called the "content of a thought" and the "assertion of its truth." Suppose I say to you, 'Plato taught Aristotle'. In doing so in the appropriate context (with the appropriate tone of voice, etc.), I am claiming, at least implicitly, that what I say is true. We can think of what-I-say as the content of my thought (the object expressed by my sentence). We can think of my implicit claim as the assertion of its truth. Some things one says are not asserted (as truths), such as questions, commands, and promises. Logically more important are sentences that are truth-functional parts of compounds, such as disjuncts or conditional antecedents. Suppose I say to you, 'If Plato taught Aristotle, then Plato knew syllogistic'. Here, I assert the entire conditional, but I do not assert its antecedent. With this distinction in mind Frege raised the possibility that in addition to sentential negation—the kind of negation that applies to the content of an entire statement and results in its contradictory—there could be a kind of negation that is the opposite corollary of assertion, a second way of "judging a thought."

> Are there two different ways of judging, of which one is used for the affirmative, and the other for the negative, answer to a question? Or is judging the same act in both cases? Does negation go along with judging? Or is negation part of the thought that underlies the act of judging? (129)

Frege's answer is that there is only one mode of judgment—affirmation (assertion): "it is a nuisance to distinguish between two ways of negating" (128).[11] The result of having just one kind of negation, said Frege, is an "economy of logical primitives" (130, cf. 48, 149ff).

One of the prices paid for this economy of logical primitives is the loss of term negation, and thus, it seems, the notion of a statement having a logical contrary (as distinct from its contradictory). But, in fact, contrariety is preserved now in terms of sentential rather than term negation. Consider the two sentences 'All logicians are rational' and 'All logicians are irrational (nonrational)'. The logical contrariety between these two is accounted for by the traditional logician in terms of the logical contrariety between a term ('rational') and its negation ('irrational'). The logical contrariety between a term and its negation is primitive, and it is the basis for defining the logical contrariety between certain pairs of sentences. In contrast, for the Fregean logician, the only primitive negation is sentential. So the term negation in 'All logicians are irrational' is merely grammatical. Logically, the sentence is construed as a negation of another sentence. But what is that other sentence? It cannot be 'Not every logician is rational', since this is the contradictory of 'Every logician is rational' (= 'All logicians are rational'), not its logical contrary. In such cases, the quantifier expression is taken as

a function whose argument is an entire sentence. That sentence is itself a compound (truth-function) of two sentences. The second of these is negated. Our two sentences, then, are construed as follows.

> Each thing, call it x, is such that: if x is a logician then x is rational.
> and
> Each thing, call it x, is such that: if x is a logician then it is not the case that x is rational.

These are clearly not contradictories. It would not be possible for both to name the True—but they could both name the False. They are contraries (neither of which contains a negated term). So Frege, like traditional logicians, could preserve the contradictory/contrary distinction. And he could do so while economizing on the number of logical primitives (viz., forsaking the use of term negation). Nonetheless, in part 2 of this essay we will find reasons for judging this to be a false economy.

Traditional logicians had little hope of building a truly unified logic, a single theory of logic (with an adequate algorithm for logical reckoning) that would accommodate inferences involving categoricals as well as those involving singular terms, compound sentences, and relationals. Leibniz had hope, and important insights, as we have seen. He recognized that unity could be achieved in part by reparsing all statements (including singulars, compounds, and relationals) as categoricals. But Boole and his followers saw that a logic of terms and a logic of sentences could be unified in the sense that a single algorithm could be used for either Primary or Secondary logic, as he called them. In other words, the logical syntax of categoricals and the logical syntax of compounds are isomorphic. De Morgan and Peirce then tried to fit relationals to this logic. Frege achieved a very high degree of unity for logic. In his criticism of Boole and the algebraists he pointed out that genuine unity is not achieved simply by letting term and sentential logic "run alongside one another, so that one is like a mirror image of the other, but for that very reason stands in no organic relation to it" (Geach and Black, 1952: 14). Frege reduced categorical sentences to compound sentences (17) because he took judgment (statement-making) to be prior to conception (term use).[12] He was able to effect this reduction, and thus unify heretofore separate parts of logic, by replacing the subject/predicate distinction with the function/argument distinction. Every statement is a function of one or more arguments. The main function of compounds is truth-functional; relationals are low-level functions ("first-order predicates") on more than one argument. All arguments (names) are singular. In the next chapter, we will see that this last thesis is what Sommers has called the "Fregean Dogma."

I will say more about Frege in the remainder of this chapter. But before concluding these introductory remarks it would be useful to remind the reader, given the post-Fregean developments in philosophical logic, that

Frege was a genuine, serious Realist. He believed that reality consists of objects and concepts, and that among those objects are not only material objects but Thoughts, functions, numbers, and the True and the False. This kind of Platonism has been vigorously rejected by most of the now nominalist heirs of Frege's logic.

As we have seen more than once, syllogistic has constantly been challenged by inferences involving three kinds of statements: singulars, compounds, and relationals. Solutions to these problems (such as those offered by Leibniz or De Morgan) were generally inadequate or unknown to late-nineteenth-century logicians. Frege's revolutionary logic offered a single system, the predicate calculus with identity, which could easily analyse all three kinds of statements. Syllogistic logic could then be incorporated into the new logic as just a small part of it—the part dealing with singly quantified statements containing a single nonrelational predicate. What Frege achieved was a system of formal logic that was far more powerful (in terms both of expressive power—the ability to formulate a broad range of kinds of statements—and of inference power—the ability to account for inferences in a perspicuous manner). After Frege, it was hard for any logician to look back to an earlier system of logic as having more than historical value.

Bradley and Ramsey Raise Some Doubts

Logic sometimes creates monsters.

Poincaré

British empiricists in the nineteenth century, especially John Stuart Mill, had challenged the objectivity of mathematics, and of reasoning in general, by holding that in any judgment the constituent ideas can do nothing more than refer to subjectively held ideas in the mind of the one who makes the judgment. Moreover, the laws of logic were seen as mere generalizations of the natural associations among such ideas. Inference is not the active application of our rational faculty to propositions or judgments; rather, it is merely the passive recognition of the mind's passage from one idea to another in accordance with the laws of association of ideas. Logic is a matter of psychology and nothing more. This sort of view was known as *psychologism*. According to Frege, it was the view that posed the greatest danger to the possibility of objective, rationally grounded mathematics. In his view, the acceptance of psychologism in mathematics would render all mathematical truths subjective and relative. Mathematics would be pointless. In many ways, Frege's entire corpus in logic can be seen as a rebuttal of Mill's position. One of his most basic challenges was the argument that judgments (propositions) could not be analysed into ideas (terms). Propositions are more basic than terms (the "priority principle"),

so that one can inquire into the meaning of a term only in the context of a proposition (the "contextual principle"). Propositions are not, then, built up out of terms, as traditional logicians had believed. Nevertheless, propositions can be analysed (into functions and arguments, as we have seen).

Logicians following Frege were eager to exhibit the powers of the new logic, now unencumbered by ties to either old syllogistic or more recent empiricism. They were particularly proud of the fact that they had successfully incorporated relationals into a unified system of formal logic. But celebration was barred almost from the beginning by a paradox usually attributed to Frege's British contemporary, F.H. Bradley. In his *Appearance and Reality* (1893), Bradley says,

> Relation presupposes quality, and quality relation. Each can be something neither together with, nor apart from the other; and the vicious circle in which they turn is not the truth about reality . . . . (21) But how the relation can stand to the qualities is . . . unintelligible. If it is nothing to the qualities, then they are not related at all . . . . But if it is to be something to them, then clearly we now shall require a *new* connecting relation . . . And, being something itself, if it does not itself bear a relation to the terms, in what intelligible way will it succeed in being anything to them? . . . we are forced to go on finding new relations without end. The links are united by a link. (27-8) . . . If you take the connexion as a solid thing, you have got to show, and you cannot show, how the other solids are joined to it. And, if you take it as a kind of medius or unsubstantial atmosphere, it is a connexion no longer. (28)

Suppose I tell you, 'Alvin is to the left of Calvin'. How can we account for the truth of (the statement I make by appropriately using) this sentence? In one sense, the only things involved here are two people, Alvin and Calvin. If the truth of my statement is due to some thing (or things) having a (nonrelational, monadic) property, what could that property be? An inspection of Alvin and Calvin reveals no such property to account for the truth under consideration. So why not simply say that the truth here is due to a relation (of being to the left of) holding between the two things, Alvin and Calvin? Accordingly, one could say that it is true that Alvin is to the left of Calvin because Alvin and Calvin are related by the relation of being to the left of. But this would introduce a third thing (the relation) in addition to Alvin and Calvin. The question would then immediately arise: Are, say, Alvin and that relation related? And if they are, then it must be because of some other relation holding between them, and so on ad infinitum. One must not assume that Bradley's Paradox necessarily leads to a rejection of relationals in favour of categoricals, for the latter face the same sort of barrier (cf. Vander Veer, 1970: 40n). If it is true that A is B, then it must be that either A and B are identical or that they are not. If they are identical, then the statement is an empty tautology. If they are not, then there must be some connection between them (e.g., predication, exemplification, instantiation, etc.). But such a connection is a relation—and we have seen how relations lead to paradox. As Bradley says, "We wander among puzzles" (16).

Bradley's Paradox raised a serious question about the possibility of giving a logical analysis of any statement, relational or otherwise.[13] Such a question is important for any Fregean logician since, from that perspective, while statements are not built up from constituents they are analysable into them. One way to avoid Bradley's Paradox is to deny that all the elements of a statement are on a logical par. What holds the elements together is never an additional element. Rather, the elements are united by virtue of one element's being logically fit for the others. This was Frege's (and, in a way, Plato's) solution. And how did Bradley respond to his own paradox? He argued that a judgment (proposition) has a unity that cannot be analysed. Judgment cannot be the mere union of ideas (and thus a proposition cannot be the mere union of terms). But a judgment cannot be a relation (or connection) of ideas because, the relation would itself be an idea, which in turn would stand in need of a relation to connect it to the other ideas of the judgment, and so on. So propositional analysis is impossible. Bradley actually had little regard for formal logic (which, after all, relies on such analysis), claiming that while it may be useful and worth preserving (he had already written *Principles of Logic* [1883]), it was metaphysically groundless. As an Absolute Idealist, he held that ultimately all statements are about the same thing—the Real. The unanalysed (indeed, unanalysable) statement expresses a single, unified idea, which is attributed to the Real. Relational thinking is merely a distortion of Reality. However, for finite beings, such as ourselves, to think in terms of "internal" relations, ones grounded in the natures of the relata, is not quite as distorting as to think in terms of "external" relations, ones not so grounded (e.g., being to the left of) (1883, appendix B).

Bradley's Paradox challenged logical orthodoxy while adhering to the standard traditional notion that singular and general terms are logically different. By the 1920s, even this was called into question. The logistic thesis advanced by Frege and Russell sought to secure the moorings of mathematics with the aid of logic (including set theory). In advancing their thesis, both Frege and Russell advanced as well a realist theory of universals. This version of realism holds that in addition to the existence of individual things (particulars), such as the Eiffel Tower, the moon, Socrates, my cat, and so on, there exist universals as well. Universals, unlike particulars, are not limited by space and time. A universal can occur in many places at a given time; examples are redness, blueness, wisdom, sphericality, and humanity. The moon can be in only one place at this time, but sphericality is where the moon is now and also where the sun is now,

where Mars is now, where by old baseball is now, and so on. Particulars and universals are fundamentally different. Support for this view is often gleaned from grammar or logic. The grammarian's distinction between substantives, on the one hand, and adjectives and verbs, on the other, or the logician's distinction between subjects and predicates, is seen to be a revelation of the ontological distinction between particulars and universals. Even while rejecting the subject/predicate distinction for logic, Frege sought to preserve the particular/universal distinction in the guise of his distinction between objects and concepts. Again, objects are the referents of logically complete expressions (names); concepts are the referents of incomplete expressions (function expressions, predicates). Traditional logicians had taken particulars to be the referents of subjects, universals to be the referents of predicates. Frege preserved the ontological distinction while denying the traditional one by simply replacing the latter with his complete/incomplete (name/function) distinction. And that distinction is, in effect, a distinction between singular and general terms.[14] Russell (e.g., 1956) followed Frege here (but not everywhere, as we shall see).

In 1925, young Frank Ramsey challenged both traditional logicians and his contemporaries by arguing that the theory of universals is a "great muddle" and that there are no solid grounds for any of the asymmetries thus far alleged.[15] Both traditional logicians (represented for Ramsey by W.E. Johnson [1921]) and Fregeans (represented by Russell) based the asymmetry of subjects (substantives, names, singular terms) and predicates (adjectives or verbs, functions, general terms) on the assumption that while the latter kinds of expressions can occupy either position in a proposition, the former can occupy only subject (or argument) positions—singular terms cannot be predicated. But, according to Ramsey,

> Both the disputed theories make an important assumption which to my mind, has only to be questioned to be doubted. They assume a fundamental antithesis between subject and predicate, that if a proposition consists of two terms copulated, these two terms must be functioning in different ways, one as subject, the other as predicate. (1925: 404)

Consider the two sentences 'Socrates is wise' and 'Wisdom is a characteristic of Socrates'. These may be different sentences, but they "assert the same fact and express the same proposition . . . they have the same meaning" (404). "Here," Ramsey concluded, "there is no essential distinction between the subject of a proposition and its predicate, and no fundamental classification of objects can be based on such a distinction" (404). This is Ramsey's Symmetry Thesis.

Russell had warned philosophers that the traditional notion that all propositions are subject-predicate in logical form ignores relationals. But, said Ramsey, "Nearly all philosophers, including Mr. Russell himself, have been misled by language in a far more fundamental way than that . . . the whole theory of particulars and universals is due to mistaking for a fundamental characteristic of reality, what is merely a characteristic of language" (1925: 405). One can avoid being misled by paying attention to "atomic" (noncompound) singular propositions (e.g., 'Socrates is wise'). Ramsey cited three possible theories to account for such propositions. The traditional theory, as found in Johnson, holds that the two terms here are linked by the copula, which is a "characterizing tie." The Fregean view, as represented by Russell, holds that the general term is incomplete, gapped, in such a way that it is completed, filled, by the singular term, which is already complete in itself. The third theory is Wittgenstein's. According to Ramsey's account of this theory, there is neither a copula nor a privileged constituent (viz., an expression in need of completion). Atomic propositions depict atomic facts, and the objects that make up atomic facts simply "hang together like the links of a chain." Ramsey said (408) that it is important to look only at the second theory. This theory recognizes the need for a verb in each atomic proposition (and verbs are incomplete). However, for Ramsey, the difficulty here is that *all* objects (Socrates and wisdom alike) are incomplete. What he seems to have had in mind is that an object is incomplete if it somehow depends upon other objects—and no object can occur in an atomic fact without depending upon some other object(s). Consequently, both 'Socrates' and 'wise' are incomplete as well.

If all this is so, one naturally wonders how logicians could have insisted on singular/general (subject/predicate, name/function) asymmetry in their accounts of propositional unity.

> But what is this difference between individuals and functions due to? Again, simply to the fact that certain things do not interest the mathematician. Anyone who is interested not only in classes of things, but also in their qualities, would want to distinguish from among the others, those functions which were names . . . . So were it not for the mathematician's biassed interest he would invent a symbolism which was completely symmetrical as regards individuals and qualities; and it becomes clear that there is no sense in the words individual and quality; all we are talking about is two different types of objects, such that two objects, one of each type, would be sole constituents of an atomic fact. (Ramsey, 1925: 415-16)

But surely, it might be contended, there must still be a tie or relation (of characterizing) between these two "objects." Ramsey's response was, simply, "As regards the tie, I cannot understand what sort of thing it could be" (416). Consequently, Ramsey's own position regarding the problem of propositional unity is agnostic: "The truth is that we know and can know

nothing whatever about the forms of atomic propositions . . . and there is no way of deciding any such question. We cannot even tell that there are not atomic facts consisting of two terms of the same type" (417).

Ramsey's Symmetry Thesis (that in a simple, nonrelational atomic proposition either term may be taken as the logical subject, or argument, and the other as the logical predicate, or function) implies that there is a logical symmetry between singular and general terms. Fregean logicians assume that in such propositions the subject (argument) must be singular—the Fregean Dogma—while the predicate (function) must be a general term (recall Dummett's account of how multiply general sentences are built up from such singular atomic sentences). Those who wish to maintain the logical asymmetry of subjects and predicates have had to counter Ramsey by arguing for the logical asymmetry of singular and general terms. The literature surrounding this "Asymmetry Thesis" has grown large in recent years. Many prominent philosophers and logicians, including especially Geach and Strawson,[16] have offered support for asymmetry. While many arguments have been advanced, most generally go something like this: 'Subjects and predicates are logically asymmetric because singular terms and general terms are logically asymmetric. Singular and general terms are asymmetric because there are logical features which hold of the latter but not of the former.' Two such features are commonly cited: (i) General terms can be negated; singular terms cannot. Moreover, even if singular terms could be negated (none denies that grammatically, if not logically, this is possible), the negation of the predicate (general term) would result in the negation of the entire sentence; the negation of the subject (singular term) does not result in the negation of the entire sentence. (ii) General terms can be compounded (conjoined or disjoined); singular terms cannot. Moreover, even if singular terms could be compounded (again, none denies the grammatical, if not logical, possibility of this), the compounding of the predicate results in a compound sentence; the compounding of the subject does not.[17] In part two of this essay we will see why the Asymmetry Thesis, along with the Fregean Dogma, should be rejected. My challenge, then, will be to make logical sense of negated and compounded singular terms.

Russell and Wittgenstein

> *Even in these semi-sophisticated times, we fall for the myth of the verb.*
> *J.L. Austin*

> *They've a temper, some of them—particularly verbs: they're the proudest—adjectives you can do anything with, but not verbs—*
> *Lewis Carroll*

While it was, ironically, Russell who in 1901 revealed to Frege a serious paradox (or "contradiction," as Russell called it) at the heart of the *Grundgesetze,*[18] Russell was the most articulate and best-known advocate of the new logic (and the logicist theory). He was convinced some time before the turn of the century that traditional logic was fundamentally mistaken. His first target was the logician whom many contemporary logicians regard as the first precursor of modern mathematical logic, Leibniz. Russell's main criticism of Leibniz's logic (Russell, 1937) in particular, and pre-Fregean logic in general, was that it was dependent upon a false account of logical syntax (viz., the view that all propositions are logically analysable into subjects and predicates). Indeed, for Russell, this wrong logical view led to a wrong metaphysics of substances and attributes.

But, while Russell's own logic was Fregean (or Peano-Fregean), his "philosophical" logic was non-Fregean in many ways. For both Frege and Russell the logic of functions and arguments was meant to serve (when supplemented by set theory) as the foundation of mathematics by virtue of its equivalence with arithmetic. Both viewed natural language as logically flawed. In his reply to Max Black, Russell wrote, "We ought [not], in our attempts at serious thinking, to be content with ordinary language, with its ambiguities and its abominable syntax. I remain convinced that obstinate addiction to ordinary language in our private thoughts is one of the main obstacles to progress in philosophy" (in Schilpp, 1944: 694). If anything, Russell had an even lower opinion of ordinary language than did Frege. Unlike Frege, however, Russell seems to have believed that natural language does in some sense have a logic. Natural-language expressions can be translated into expressions of the "logically perfect" language of mathematical logic (the syntax or grammar of which is presented in *Principia Mathematica* [Russell and Whitehead, 1910-13] and the vocabulary of which consists of "logically proper names" and simple predicates [Russell, 1918-19]). The logical form of a natural-language sentence is hidden by its prima facie grammatical form—thus the need for such a translation, for the logical form of a sentence in the logically perfect language is the same as its apparent, surface grammatical form.

Russell came almost to identify logic with philosophy. And, unlike Frege, he took the study of logic and grammar to be a valuable tool for attacking philosophical problems.[19] Thus, for example, where Frege might account for the term 'ghost' by arguing that its referent was merely a mental image, Russell would analyse the term into a complex expression in his ideal language.[20] Finally, while both Frege and Russell could account for logical constants (which, in the long run, determine logical form) in no other way than simply to list them, they offered divergent accounts of propositional unity. As we saw above, Frege took the unity of a proposition to be the result of incomplete expressions being completed by complete expressions. The matter is purely one of logical syntax. Russell accepted this account. Yet it may not be too unfair to say that he often took his directions concerning logical syntax from his prior semantic theses (cf. Sainsbury,

1979). For example, Russell held that the meaning of a complex expression was uniquely determined by the meaning of its component expressions (Frege's compositionality thesis). This, then, guided his account of logical form. The logical form of a sentence must exhibit the meaning of that sentence (by constituting a translation of the sentence into a sentence of the logically perfect language). Russell rejected the Fregean notion that an argument of a function (i.e., a name) could be a complex expression. Names cannot be complex, nor can they be empty. Frege took both ordinary proper names, such as 'Plato' and 'Kant', and definite descriptions, such as 'the teacher of Aristotle' and 'the man who broke the bank', to be eligible as arguments in first-order predicate expressions. Russell rejected Frege's sense/reference (or, for Russell, meaning/denotation) distinction. His semantics (and his epistemology[21]) prevented him from allowing either kind of expression to play such a logical role (be a "logically proper name," as he called it). According to Russell (1912), the denotation of a logically proper name must be a simple, existing object of acquaintance (such as a sense datum). As it turns out, ordinary proper names are merely abbreviations of definite descriptions. Indeed, as he says in *Principia Mathematica*, "In what *we* have in mind when we say 'Socrates is human' there is an apparent variable" (Russell and Whitehead, 1910-13: 50). In other words, we may *say* 'Socrates is human', but what we "have in mind" (presumably the logical form) is a sentence/formula containing not the ordinary proper name 'Socrates' in a denoting role but a variable (say, 'he', or 'it', or 'x'). The contrary view—that definite descriptions are names—was the source of the Meinongian theory, which Russell (1904) had rejected on ontological grounds. Russell (1905) argued, then, that ordinary proper names could be analysed as disguised definite descriptions. And definite descriptions could, in turn, be analysed in terms of indefinite descriptions and identity. Finally, indefinite descriptions could be analysed in terms of predicates (functions) and individual variable arguments bound by existential quantifiers (this last eliminates the need for a *predicate* of existence in the ideal language). Suppose the ordinary proper name 'Socrates' can be replaced by the definite description 'the teacher of Plato'. A sentence such as 'Socrates is wise' would then be analysed, according to Russell's lights, as 'There exists at least one thing that taught Plato and each thing that taught Plato is identical to it and it is wise' (a full logical analysis, of course, would then analyse away the name 'Plato'). The "apparent variable," which we have in mind whenever we use such ordinary names, is simply the existentially quantified variable (expressed by 'it' above) of the logical analysis.

One of Russell's reasons for denying Frege's amalgamation of proper names and definite descriptions was a matter of logical syntax. For Russell, names, unlike descriptions, do not exhibit scope ambiguity (cf. Sainsbury, 1979: 66ff). Consider the following two sentences.

(1) Socrates is wise.
(2) The man who broke the bank was French.

Russell's claim was that any attempt to deny (1) would result in its (sentential) negation, i.e.,

(1.1) Not: Socrates is wise.
(= It is not the case that Socrates is wise.)

'Socrates is not wise' and 'Socrates is unwise' would both be rendered as (1.1). Things are supposedly quite different for the denial of (2). This is because of what Russell (1918-19) called the "possibility of double denial." Sentence (2) can be denied in (at least) two nonequivalent ways:

(2.1) Not: the man who broke the bank was French.
(2.2) The man who broke the bank was not French.

Given Russell's analysis of definite descriptions, a sentence of the form 'The A is B' will be false whenever there exists exactly one A and it is not B, or there is more than one A, or there is no A. So (2.1) and (2.2) are not equivalent, say, when either more than one man broke the bank or no man broke the bank. In either of those cases (2.1) will be true but (2.2) will be false, and this difference is due solely to the difference in relative scopes of the descriptive expression ('the . . .') and the negative expression ('not'). In (2.1) the description lies within the scope of 'not'; in (2.2) 'not' lies within the scope of the description.

To be absolutely fair to Russell, even 'Socrates' in (1) and (1.1) is not really a logically proper name. A sentence with such a name would be

(3) This is red.

Mathematical logicians like Russell would translate this into the ideal language as

(3.1) Rx

—that is, the function R on the argument x. And no matter how we might deny (3) in natural language (e.g., 'This is nonred', 'This isn't red', 'This is not red', 'This is other than red', 'It is not the case that this is red', 'It is false that this is red', etc.) (3.1) can be denied in just one way—by the application of sentential negation:

(3.2) ~(Rx)

In part two I will offer reasons for rejecting this way of distinguishing names and descriptions, as well as the contemporary way with negation.

A central element of Russell's attack on traditional logic's subject-predicate analysis of propositions (as represented by Leibniz) was his insistence that such an analysis must turn a blind eye to relations.[22] Russell's own view was that if relations were reducible to nonrelational properties (or classes), then relational propositions would reduce to categoricals (as Leibniz seems to have thought). But, according to Russell, such a reduction of relational propositions is not possible.[23] And, indeed, Frege and Russell's formal language admits first-order functions with either one or more gaps (places for singular term arguments, i.e., relata expressions). The key to understanding Russell's rejection of Leibniz's view of relationals lies in Russell's Fregean assumption that all syntactical complexity is to be accounted for in terms of sentential complexity. The terms (function and argument[s], or predicate and name[s]) of an *atomic* proposition must always be simple—having no proper parts that are meaningful (i.e., that denote). Russell also assumed that, given that logical syntax is a guide to metaphysics, Leibniz's reduction of relationals to categoricals reflected his rejection of real relations (a view Russell found incompatible with the rest of Leibniz's monadic metaphysics [cf. Ishiguro, 1972-76]).

The Leibnizian theory that relational propositions can be logically reparsed in terms of subjects and predicates was only one of two accounts of relationals that Russell wanted to reject. The other theory was Bradley's.[24] According to Russell, where the "monadistic" (i.e., Leibnizian) theory attempts to reduce a relational proposition, 'aRb', to a conjunction (of some sort) of subject-predicate propositions by replacing the relational expression by a pair of monadic predicates (i.e., 'aR_1 & R_2 b'), the "monistic" (Bradley's) theory analyses such a relational proposition as a subject-predicate proposition with a monadic predicate but a compound subject (i.e., '$(ab)R_3$') (Russell, 1903: 221). For example, 'Paris loves Helen' is analysed by the first theory as 'Paris loves and by that very fact Helen is loved'. It is analysed by the second theory as 'Paris and Helen are lovers', where 'Paris and Helen' denotes a whole composed of Paris and Helen and 'are lovers' is a property of that whole.[25] Among Russell's reasons for rejecting the monistic theory was his observation that when the relation is asymmetric (as in 'David is the father of Solomon'), the proffered analysis would fail to preserve the inequivalence between the relational proposition and its converse. For example, 'David is the father of Solomon' is not equivalent to 'Solomon is the father of David', but, given that there is no difference between the whole composed of Solomon and David and the whole composed of David and Solomon, the analyses of the two propositions would be equivalent.

When it came to Bradley's Paradox concerning relations, Russell insisted that the error that generated the paradox was the failure to recognize that certain elements of a proposition (viz., concepts—specifically, in this case, relational concepts as denoted by relational expressions) can occur in propositions without thereby being subjects of those propositions.[26] The result of this failure is not only paradox, but inability to account for propositional unity. If each expression of a given sentence is viewed as simply naming a subject (thing, object), then the sentence will be nothing more than a list of names—not a unit. Russell directly addressed the problem of sentential, or propositional, unity in the *Principles of Mathematics* in 1903.[27] The result of Russell's struggle is a theory of logical syntax that was different from Frege's in important ways simply because it resulted from Russell's attempt to solve the problem of sentential unity raised by his understanding of the source of Bradley's Paradox. In a way, this was never really a problem for Frege.

In contrasting his new logic with that of Boole, Frege emphasized that he, unlike Boole, gave pride of place to the entire sentence as a unit, arriving only by analysis at the elements of the sentence. Boole had begun with terms and used them to build up sentences. For Frege, then, sentences are, by their very logical nature, unified (by being units!).[28] Russell accepted Frege's notions of functions and arguments, but, unlike Frege, he saw sentences as built up from such expressions (thereby abjuring Frege's priority principle). Russell held that sentences express propositions, the true objects of logical investigation. A (Russellian) proposition is a combination of "terms." A (Russellian) term is any object of thought, a unit, individual, entity; whatever can be mentioned (Russell, 1903: 43). Terms are either "things" (Fregean objects) or concepts. Concepts are expressed by "propositional functions" (or predicates). A propositional function is a proposition whose subjects (things) have been removed, leaving gaps. Singly gapped functions are class concepts; multiply gapped functions are relations. Relations are denoted by verbs; class concepts, by adjectives. Using "verb" in a sense wide enough to encompass all propositional function expressions, Russell claimed that, unlike terms that denote things (i.e., singular terms, names), a verb has a "twofold" nature. It can be used "as actual verb and as verbal name" (Russell, 1903: 49). Subjects (e.g., 'Socrates') are singular. They cannot be used as verbs because they do not have "that curious twofold use which is involved in *human* and *humanity*" (Russell, 1903: 45). 'Human' and 'humanity' are grammatically distinct but logically identical. Their grammatical differences merely reflect the two uses: as a verb ('human', as in 'Socrates is human') or as a verbal noun ('humanity', as in 'Humanity is the curse of this earth').

A verb used as a verbal noun denotes a thing. A verb used as a verb denotes a complex object—class or relation. The verb used as a verb is the source of propositional unity for Russell.

> A proposition in fact is essentially a unity, and when analysis has destroyed the unity, no enumeration of constituents will restore the proposition. The verb when used as a verb, embodies the unity of the proposition, and is thus distinguishable from the verb considered as a term [verbal noun], though I do not know how to give a clear account of the precise nature of the distinction. (Russell, 1903: 49-50)

Russell said this with an eye on Bradley's Paradox. Consider again 'Paris loves Helen'. The proposition expressed here consists of three terms: Paris, Helen, and the relation of loving. But when considering these three terms, we are simply considering three things, three objects. The relating expression is merely a parataxis, not a unit. To unite them into a single propositional unit would require further relations, *ad infinitum*. To avoid this regress, Russell held that the verb ('loves'), unlike other kinds of expressions, has a twofold nature. It can be used "as a term" (thus naming an object, as 'humanity' does), but it can also be "used as a verb." When so used it does not denote another thing constituting the proposition; rather, it is what actually binds those other constituents into a unified whole—a proposition.

As we said, Russell had one eye here on Bradley. His other eye was on Frege, who, as we have seen, in effect avoids the problem of sentential unity by means of his priority principle and his absolute and unqualified distinction between functions (which are incomplete) and names (which are complete). In "On Concept and Object" (in Geach and Black, 1952), he is clear about the price he must, and is willing to, pay for this; he had to admit that, for example, the concept of a horse is not a concept. This *prima facie* paradoxical claim is due, according to Frege, to a logical defect of language in general. "I mention an object, when what I intend is a concept" (54). One might very well wish to mention the concept of a horse, but to do so one must use an expression that is complete (i.e., a name) rather than incomplete (a function or concept expression). One is defeated by language. By insisting on the inviolate distinction between functions and objects (between incomplete predicates—relational and general terms—and complete names), Frege refused to allow any expression the twofold nature of Russell's verbs.

In effect, Frege embraced a radical dualism in logic (like Plato's dualism of *onoma* and *rhema*). Consequently, he was saddled with the 'horse' paradox, but he was able to avoid the problem of sentential unity. In contrast, Russell admitted a single category of "terms," some of which were allowed a double use. In effect, he embraced (at least in 1903) a kind of Aristotelian (of the *Analytics*) monism in logic. Expressions are logically of one sort. Yet Aristotle had a solution to the problem of sentential unity. Russell's attempted solution was far from satisfactory. Why did Russell fail? What he lacked that Aristotle did not was a sentence-unifying expression—a logical copula. All of Russell's talk about verbs used as verbs embodying the unity of a proposition turned out to be an expression of hope in place of argument. Aristotle had the logical copula and Russell did not, having followed Frege in abandoning it.

Recall that Ramsey had contrasted three theories concerning the logical structure of "atomic" propositions. These were the traditional (copular) theory, the Fregean (gap filling) theory, and Wittgenstein's (chain link) theory. However, in examining the view of logical form that Wittgenstein offered (especially in the *Tractatus* [1961]), it is absolutely imperative to keep in mind his distinction between what can be said and what can be shown (as found at remark 4.1212). In laying out his theory of logical form Wittgenstein kept in mind two principles—one from Frege, the other from Russell. Wittgenstein took from Russell the notion that the form of a proposition is determined by its predicate (verb); from Frege, he took the idea that what can be predicated can never be a subject. Subjects, for Frege and Wittgenstein, are names. But Wittgenstein meant by a "simple sign" (3.202) a name—not an ordinary name, but something like Russell's "logically proper" name. Names denote, or name, "objects." Predicates—general terms and verbs—denote concepts. Names *say* what they name; predicates do not. There is no "twofold" use of verbs, as in Russell of 1903. Thus predicates cannot name, or be used to say, what they denote. They can only *show* this by their use as predicates.

It looks as if Wittgenstein held the Fregean view that a proposition consists of names and a predicate, where names and predicates are fundamentally different. But this is hard to reconcile with his "picture theory" of meaning, according to which an atomic proposition ("propositional sign") pictures an atomic fact (2.141). It does this because the fact is a combination of objects (2.01, 2.0272) and the proposition is a concatenation of names of those objects (4.22), names in "immediate combination" (4.221). Moreover, the names are combined in a definite way (2.14, 3.14) corresponding to the configuration of objects in the state of affairs depicted (3.21). As well, since every proposition has a definite sense, there must be simple names (of simple objects) (3.23). Names have sense only in the context of a proposition—Frege's priority principle (3.3, 3.314). So, objects are always in the context of a fact. And the world is nothing more that the totality of such facts (1.1).

Given the picture theory, a proposition is logically analysable as a combination of names (not names and predicates). And this is surely the view Ramsey was attributing to Wittgenstein when he claimed that for Wittgenstein the elements of a proposition are united like links in a chain (2.03, 3.14, 3.141, 3.142). In a chain, each link is like every other; they combine with one another not by means of an intermediary but by their very nature. Names, too, according to this view, are like one another (while names and predicates are not). Names simply link with one another in concatenation.[29] Clearly, the kinds of propositions Wittgenstein had in mind were quite different from ordinary sentences (3.325, 4.002, 4.1213). Indeed,

he admitted that it is not obvious that ordinary sentences have the kind of *logical* form that he attributed to them (4.002). This is especially so for relational sentences. Concerning such sentences, he said (3.1432), "We must not say, "The complex sign '*aRb*' says '*a* stands in relation *R* to *b*' "; but we must say, "*That* '*a*' stands in a certain relation to '*b*' says *that aRb*." More specifically, we must not say this in a language of "adequate notation" (6.122; also 3.325, 5.533, 5.534). In other words, in a logically adequate, or correct, notation—a *Begriffschrift* (4.1273)—a notation in which sentences consist only of simple names naming simple objects—what would be expressed in, say, English by a sentence of the form 'aRb' would be expressed by a sentence in which only 'a' and 'b' appear. Such a sentence would, in turn, depict a state of affairs consisting only of the two objects *a* and *b*, standing in some relation to one another. The fact that *a* stands in relation *R* to *b* (if it is a fact—an existing state of affairs) is pictured by a sentence (of the logically correct language) in which the two names 'a' and 'b' are in *some* relation. The relation between 'a' and 'b' need not be the same as the relation between *a* and *b* (3.1431) (cf. Copi, 1958). In a logically adequate language, there are no predicates, no relational terms, no Russellian verbs. In one stroke, Wittgenstein has accommodated Frege's demand that concepts are not objects (by not allowing predicates to be used as names—indeed, by allowing only names to occur in logical notation) and answered Bradley (by allowing relations among objects to be represented not by relational terms but by relations among names). As a consequence of this theory, Wittgenstein was committed in the *Tractatus* (1961) to the view that all logically adequate propositions are relational, in that each consists only of names, which, themselves being objects, stand in relations, which, in turn, represent relations among the objects so named (2.0121).

To be absolutely fair to Wittgenstein of the *Tractatus*, it must be noted that sentences of a logically correct language do have, in addition to names, logical constants as elements. However, these expressions, unlike names, are "not representatives" (4.0312, 4.441).[30]

Negation was the logical operation that most interested Wittgenstein (as it had Frege). Still, his remarks concerning negation hardly constitute a satisfactory theory. In the *Tractatus,* Wittgenstein claims that simple facts (the ultimate constituents of the world) are independent of one another (1.2, 1.21, 2.061). Because of this mutual independence, there can be no logical relations (implication, equivalence, incompatibility, etc.) between any two such facts, nor, presumably, between any two atomic propositions depicting such a pair of facts: "From the existence or non-existence of one state of affairs it is impossible to infer the existence or non-existence of another" (2.062). Suppose I know that the state of affairs depicted by a proposition of the form '~p' exists (i.e., is a fact). From this, I can infer the nonexistence of the state depicted by 'p' (5.512). Such inference would be legitimate, according to Wittgenstein, because at least one of these ('~p') is not atomic. But what of the seemingly obvious inference from 'This is red' to 'This is not blue' (cf. Ramsey, 1923: 18)? This inference depends upon the incompatibility of 'This is red' and 'This is blue' (given a common denotation for each token of 'this' here). As a Fregean, of course, Wittgenstein had no access to any kind of negation other than sentential (4.0641, 5.1241, 5.2341) to account for this. Consequently, he had to deny that any logical relation could hold between the two atomic propositions expressed by 'This is red' and 'This is blue'. Indeed, it appears that in the *Tractatus* view there can be no negative atomic facts at all.[31]

Before leaving Wittgenstein, we must take note of a fascinating turn that he took in the *Tractatus* away from the logical path followed by virtually all other Fregeans. In "Sense and Reference," Frege had made a very clear distinction between the 'is' of predication and the 'is' of identity. A proposition such as 'Shakespeare is Bacon' must be analysed as a relation between two objects (viz., the relation of identity between Shakespeare and Bacon), rather than as a function on a single object (viz., the function of *being Bacon* applied to Shakespeare). This is because in the latter analysis a complete expression, a name ('Bacon'), would be used as an incomplete expression, a predicate. Wittgenstein did not offer an alternative to Frege's account of identity—he simply rejected the idea of identity altogether: "The identity sign, therefore, is not an essential constituent of conceptual notation" (5.533). In a logically adequate notation one cannot even express a proposition such as 'a is identical to b' (5.534): "Identity of object I express by identity of sign, and not by using a sign for identity" (5.53); "It is self-evident that identity is not a relation between objects" (5.5301). Either a and b are two things or they are not. "Roughly speaking, to say of *two* things that they are identical is nonsense, and to say of one thing that it is identical to itself is to say nothing at all" (5.5303). It is safe to say that very few mathematical logicians have felt enough logical security, as Wittgenstein did, to throw out the oar of identity. Indeed, as we will see, some have even augmented it.

Strawson, Geach, and Quine

> *I am the most readily disposed person to do justice to the moderns, yet I find that they have carried reform too far, among other things, by confusing natural things with artificial things.*
>
> *Leibniz*

As we shall see in the next chapter, Frege's revolution in logic has only recently begun to face serious challenges (both from those within the citadel of contemporary mathematical logic and from the few traditionalists beyond the moat). Still, now near the end of the twentieth century, the logic derived from Frege is well entrenched and powerful. Its hegemony is still intact. The sources of its pre-eminence are not in doubt. Modern mathematical logic was created by Frege and was soon pushed into a place of prominence

in philosophy by Russell, Whitehead, and Wittgenstein. In comparison with what had thus far been offered by traditional logic, the new logic clearly deserved its rapid rise to power. It provided an algorithm for the analysis of mathematical reasoning far more perspicuous and effective than anything before. Even after the 1920s and 1930s, when the new logic's ability to generate mathematics (and the logicist programme in general) was forcibly abandoned, it continued to be logical orthodoxy—especially among analytic philosophers.

Three of these philosophers, P.F. Strawson, Peter Geach, and W.V. Quine, are individually representative of certain aspects of late-twentieth-century orthodoxy in philosophical logic. Of the three, Strawson is the least committed to orthodox mathematical logic. By the middle of the twentieth century, Fregeans, having generally abandoned logicism, had become more eager to claim that mathematical logic is actually the hidden, underlying logic of everyday discourse. This claim was not new, of course. Russell, for example, had held such a view early in the century. Indeed, Russell's Theory of Descriptions was generally regarded not only as a paradigm example of philosophical analysis, but as a way the new logic could be used to reveal the hidden, underlying logical forms of ordinary language expressions.

Strawson first gained fame in 1950 with his "On Referring" (1950a), in which he challenged Russell's theory of descriptions. Shortly after, in *Introduction to Logical Theory* (1952), he extended his challenge to the standard logic in general. Strawson represented a brand of analytic philosophy called "Oxford analysis," or "ordinary language analysis." This kind of analysis was inspired primarily by Wittgenstein's work in the 1930s and 1940s (esp. 1953), when Wittgenstein abandoned the view that mathematical logic was the proper tool for language analysis (and that mathematico-scientific language was the proper language to be analysed). Ordinary-language analysts generally hold that the philosopher's central task is to solve (or at least dissolve) philosophical problems by showing how they are generated by a misuse or misunderstanding of expressions of ordinary language as they are used in everyday circumstances. Ordinary-language philosophers such as Strawson take mathematical logic to be in order as far as it goes, but they deny that it could shed much light on the workings of ordinary, nonscientific, nonmathematical language. In *Introduction to Logical Theory,* Strawson was at some pains to show that modern formal logic is insufficiently close to natural, ordinary language (e.g., 193-94). The logic of ordinary language is less elegant and less systematic than the artificial language of formal logic (232). In fact, "expressions of everyday speech . . . have no exact and systematic logic" (57).

In spite of Strawson's early denial of a logic of ordinary language, he has spent much of the past forty-five years examining what can only be described as the logical form of simple statements made in the medium of ordinary language.[32] This concern is due in large measure to the fact that

Strawson is one of the few contemporary philosophers who takes Ramsey's challenge to subject/predicate asymmetry seriously. Strawson is a staunch and persistent defender of the asymmetry thesis, and it is in this defence that one can discern how little he has actually moved away from the now entrenched view of logical syntax.

As we have seen, Russell was willing to abandon (or at least temper) the radical asymmetry between names and function expressions (predicates) that Frege had advocated (an asymmetry essential in accounting for "the sharp distinction between concept and object" [1979: 177]). Ramsey went further and overtly challenged *any* such asymmetry. For Frege, predicates are never names (concepts are never objects). Since objects are, by definition, what are referred to by names (including definite descriptions), the concept of a horse is not a concept. Frege's distinction was primarily logico-linguistic-namely, the distinction between expressions which are complete, saturated (names) and those which are not (predicates). Strawson's asymmetry is neither as radical as Frege's nor primarily logico-linguistic. According to Strawson, the basic, logically simplest sentences consist of two expressions: a subject and a predicate. But his distinction is really one between the ways in which things are "introduced into discourse" (1957: 441). Things are either particulars (individuals) or universals (characteristics and kinds of individuals), and the distinction between particulars and universals is basic and ontological. Both particulars and universals can be introduced into discourse by expressions. Particulars are introduced by subjects (singular terms, such as names); universals are introduced by predicates (general terms). The differences between subjects and predicates are important because they reflect important differences between particulars and universals. Thus individuals, unlike universals, "cannot have instances" (1953-54: 31). For example, while man is a universal having as instances John, Peter, and Ralph, John himself is an individual, and nothing is an instance of John, according to Strawson. An individual is what can be counted as one. Consequently, "anything whatever is an individual" (1957: 442). Even universals can be counted as one and thus be individuals, which means that they can be introduced into discourse via subjects (i.e., by the use of abstract names). Indeed, for Strawson, we can ignore the distinction between abstract nouns and general terms (e.g., 'wisdom' and 'wise') (1987: 404). Individuals can never be introduced via general terms—that is, singular terms can never be predicated (1957: 446). Strawson, like an orthodox Fregean, presupposes that the singular/general distinction is reflected in (indeed, is identical to) the subject/predicate distinction, which in turn reflects the individual/universal distinction.[33] The ontological distinction between individuals and universals is reflected in the semantic distinction between singular and general terms. Whether one takes the logico-grammatical distinction between subjects and predicates to be reflective of such ontological and semantic distinctions depends upon how much of logical syntax one wants to be determined by ontology and

semantics. In 1959, 1961, and 1974, Strawson continued his attempt to defend this notion that logic "must reflect fundamental features of our thought about the world. And at the core of logic lie the structures here in question, the 'basic combination' (as Quine once called it) of predication" (Strawson, 1974: 4; cf. 113).

Strawson's most fully articulated defences of the asymmetry thesis are found in his "The Asymmetry of Subjects and Predicates" (1970) and *Subjects and Predicates in Logic and Grammar* (1974). According to Strawson subjects and predicates are logically asymmetric in at least two ways: regarding negation and regarding composition. With regard to negation, the argument is that general terms (the only kind that can play the role of predicate) "come in incompatibility groups" but singular terms do not (1970: 102-3; 1974: 19). Consequently, one can negate a subject-predicate sentence simply by negating its predicate, but one cannot negate a sentence by negating its subject (1970: 98). In fact, subjects (singular terms) cannot be negated at all. The negation of a general term, say 'red', can be thought of as the disjunction of all the terms in its incompatibility group ('blue', 'green', 'pink', etc.). So the negation of 'red' ('nonred') is 'blue or green or pink or . . .'. The terms 'red' and 'nonred', and, generally, any nonsingular term and its negation, are logically incompatible, and it is this incompatibility that accounts for the incompatibility of sentences (1970: 104-5). But there is no term incompatible with a singular term. The asymmetry of subjects and predicates is due to a more fundamental ontological asymmetry, which "seem[s] to be obvious and (nearly) as fundamental as anything in philosophy can be" (1970: 102). As we have seen, subjects introduce particulars (individuals) into discourse, while predicates introduce universals (characteristics or kinds of individuals). It is the asymmetry of individuals and universals that is fundamental and obvious for Strawson. The negatibility of predicates is due to the fact that general terms come in incompatibility groups, which, in turn, is due to the fact that universals come in "incompatibility ranges" (1970: 102). The terms 'red' and 'nonred' are incompatible because the characteristic of red is incompatible with the characteristics of blue, green, pink, and so on. Singular terms have no negations because individuals do not come in incompatibility ranges. Nothing is incompatible with, say, Socrates. Now, the reason nothing (i.e., no individual) can be incompatible with Socrates is that any such individual would have to have all the properties Socrates lacks and lack all the properties he has. But any such purported individual would then have to possess incompatible properties, *per impossibile* (see 1970: 111n, final paragraph). For example, a nonSocrates would have to be both French and Chinese, both over seven feet tall and under three feet tall.

For Strawson, the fundamental ontological asymmetries between individuals and universals "explain and vindicate" (1970: 104) the logical asymmetries between subjects and predicates. The second of these asymmetries is that regarding composition. Strawson's claim here is that while predicates can be compositionally compounded (i.e., conjoined or disjoined) to form new predicates, "there are no such things as compound (conjunctive or disjunctive) subjects" (1970: 100; see also 1974: 4-9). Compound subjects have no place in logic; they are "pseudo-logical-subject terms" (1970: 101). Again, the difference between subjects and predicates (singular and general terms) is "explained and vindicated" by a fundamental ontological asymmetry. Universals (characteristics and kinds) can be conjoined or disjoined to one another to form new universals. We even have specific terms for some important such compounds (e.g., 'bachelor' for the conjunction of the characteristics male, adult, and unmarried). In contrast, there are no individuals that are composed of other individuals. Like negative individuals, such purported compound individuals would be impossible. Consider, says Strawson (1970: 111n), Tom and William. Suppose there is an individual who is the conjunction of Tom and William, call him 'Tolliam'. Tolliam must have all and only those properties shared by Tom and William. Suppose Tom has property P and William is nonP. It follows that Tolliam, *per impossibile*, has neither P nor nonP. Suppose there is an individual who is the disjunction of Tom and William, call him 'Tilliam'. Tilliam must have all and only those properties that either Tom or William have. Again, suppose that Tom is P and William is nonP. *Per impossibile*, Tilliam must be both P and nonP.

In summary, for Strawson, subjects and predicates are logically asymmetric because subjects (qua singular terms), unlike predicates (qua general terms), cannot be negated or composed, because subjects introduce individuals into discourse, while predicates introduce universals. No individual is incompatible with another individual, nor are any individuals composites of individuals. In contrast, universals come in incompatibility ranges and can be composites of other universals. We shall leave Strawson for now, but certain points concerning his view must be kept in mind: (i) he assumes that "basic" sentences must be singular (in other words, the Fregean Dogma); (ii) he believes basic sentences are, from the point of view of logical syntax, concatenations of two expressions, each having a unique semantic function; (iii) he assumes that singular terms can never be predicated; (iv) he believes that the negation of a singular term must be another singular term; and (v) he believes the composition of two or more singular terms must be a singular term.[34]

As we saw in chapter one, Peter Geach has little love for traditional logic, a logic conceived in a brief Aristotelian Eden but lost in *Prior Analytics*. According to Geach, "Traditional 'Aristotelian' logic is full of mistakes and confusions" (1950: 461). It is "the miserable mutilated torso that passed for the whole body of logic from about 1550 to 1847" (1969a: 77). While most contemporary logicians have simply turned their backs on traditional logic, confident that the path first marked by Frege leads to logical perfection, Geach has taken up the challenge of showing just how the old logic is

inadequate, misleading, or mistaken. Though he views the logic of Aristotle from a solidly Fregean point of view, and thus with a fair degree of hostility,[35] his critique offers the kind of challenges that friends of traditional logic must meet if they are to have any hope of offering a viable alternative to the standard mathematical logic.

Geach's Fregean outlook is not the result of a radical, unquestioning allegiance to Frege and his logic; it is tempered by some important departures from Frege.[36] One of these is Geach's conviction that logicians must deal with natural (rather than artificial or mathematical) language. Throughout his work during the second half of the twentieth century, Geach has consistently urged logicians to take the new logic as the best (or even the only) tool for analysing natural language. Classical Fregeans ignored natural language or insisted that whatever can be viewed as logical in natural language can be seen as such only once expressions in that language have been *translated* into the formulae of the standard predicate calculus. The irony here is that Geach counsels the application to natural language of a logical tool designed on the understanding (by Frege) that natural language has no logic. Frege held that only *after* one has clarified the artificial, logically constructed language will one be in a position to apply it to natural languages. Geach feels ready now. A second difference between Geach's logical views and orthodox Fregeanism is his rejection of the contextualist thesis (that only in the context of a sentence can a word be said to have meaning): "The view put forward by Frege and Wittgenstein, that it is only in the context of a sentence that a name stands for something, seems to be certainly wrong" (1950: 462). This departure from strict Fregean doctrine is the result of Geach's own theory of logical syntax, which is intended to extend Frege's theory. Accordingly, it insists on "an absolute" (1950: 464), a "fundamental distinction between names and predicates" (1950: 474, 476), and "predicables" (1962: 34), for it is only by insisting on such an inviolable distinction that Ramsey's challenge can properly be met (1950: 474).

A predicable, for Geach, is a template expression that is turned into a sentence by filling its blanks with names. Predicables are incomplete; names are complete. A predicable actually used in a sentence (i.e., one having its blanks appropriately filled) is a predicate. Logically simple sentences are subject-predicate in form. Predicables have no logical role to play outside of their roles as predicates. In contrast, names can play the role of naming even outside of the context of a sentence (hence Geach's rejection of the contextualist thesis). Used in a sentence, a name plays the role of subject. But it can also play the role of simply naming, "to acknowledge the presence of the thing named" (1962: 26). "An act of naming is of course not an assertion . . . it does, however, 'express a complete thought' " (1950: 462). For Frege, a thought (actually, "Thought") is the sense of a sentence—never of a (nonsentential) name. At any rate, Geach's contention that names and predicables are fundamentally distinct *is* Fregean. Names can never be logical predicates, and a "predicate can never be used as a name" (1950: 463). Predicables can never be names because they never have a "complete sense" (1962: 32). Any shifting about of logical positions (names to predicates; predicables to subjects) would require a shifting of sense. "It is logically impossible for a term to shift about between subject and predicate positions without undergoing a change of sense as well as a change of role" (1972a: 48). It is the licence for terms to play roles in either subjects or predicates without any semantic modifications that Geach finds most disgusting in syllogistic. Geach, like Plato and Frege, has conceived of terms (names and predicables) as logically heterogeneous (as sentence parts, i.e., subjects and predicates, undeniably are). Each kind of term has its own kind of sense—a sense that fits it to one logical role only (either subject or predicate). Moreover, he clearly takes naming as a logically primitive kind of sense, for he defines predicables as the results of removing names from sentences (1962: 22-25).

An important consequence of Geach's theory of logical syntax (as of Frege's) is that copulae have no place in logic, for "no link is needed to join subject and predicate; the incomplete sense of the predicate is completed when the subject is inserted in the empty place" (1950: 464). Failure to see this would lead to Bradley's Paradox. Geach points out that even "Aristotle had little interest in the copula" (1950: 465; 1962: 34). But what he has in mind here is the Abelardian copula (the qualifier) rather than the Aristotelian logical copula (consisting of the quantifier-qualifier complex).

In his criticism of Strawson's theory of logical syntax, Geach (1980) accuses Strawson of preserving "two bits of old logical lore" (179): (i) taking contrariety as logically more primitive than contradictoriness, and (ii) analysing (atomic) sentences as subject-copula-predicate rather than subject-predicate. His first charge is directed at Strawson's notion of "ranges of incompatibility." Geach argues that the idea of contradictoriness (between certain pairs of sentences) is a precondition for having the idea of incompatibility (between certain pairs of properties). (In part two I will offer reasons for holding neither idea to be a prerequisite for the other.) Concerning the second charge, Geach says,

> On the Fregean view, a predicative expression needs no glue or bond to make it adhere to a subject. At some point we must have an expression that simply adheres to others without needing some further linguistic device to make it adhere; why not ascribe this character to predicables themselves, rather than postulate a copula? *Standum est in primo*! If there were a genuine problem how subject and predicate *can* adhere together, then only by mere fiat could we avoid raising the problem how the copula *can* adhere to both. (1980: 182)

Thus, for Geach, Bradley's Paradox is blocked only by Frege's syntax of complete-incomplete expressions or by fiat. Nonetheless, Strawson is no innocent here. He claims that the copula "indicates the mode of combination" (in van Straaten, 1980: 293) of the individual object specified by the subject with the concept, quality, or activity specified by the predicate, though he makes clear that the role of logical copulation and the role of predication are *both* played by the predicate. But, of course, Strawson's defence is ineffective. His analysis of the logical roles to be played in a sentence *is* tripartite. And while it makes sense to bind pairs of terms by a copula (as did Aristotle) or a quantified term with another term by a copula (as did Abelard) or subjects with predicates (sans copulae, as did Frege), short of taking subjects as nothing but subject-terms (as Frege and Geach do) and predicates as nothing but predicate-terms (which nobody does), binding subjects with predicates by means of a copula makes no sense.

As I have already noted, Geach, like Strawson, has been a staunch defender of the Asymmetry Thesis (e.g., 1962, 1969a, 1972a, 1975). So, for him, sentences can be conjoined, disjoined, and negated, and predicates can be conjoined, disjoined, and negated. In the latter cases, the result is a logical conjunction, disjunction, or negation of the sentences embedding the predicables. But names cannot be conjoined, disjoined, or negated.[37] One of the ways in which Geach uses the contrast between the negatibility of predicables (and sentences) and nonnegatibility of names is to enforce a distinction between genuine logical names and quantified general terms (cf. 1972b). Since he allows that names may be either proper or common, one might conclude that a noun phrase such as 'some man' is merely a complex name. But this would be a mistake, according to Geach. For him, as for Frege, a sentence is logically negated only by negating its *main* function expression (predicate). The latter may be a verb (as in 'Socrates walks') or a quantifier (as in 'Every man walks').[38] Geach holds that for each of these sentences negation is achieved in a manner that would not result in the negation of the other sentence if applied to it, because each sentence involves a different main function expression. 'Socrates walks' can be negated by negating '. . . walks', its predicate. But the negation of '. . . walks' in 'Every man walks' results only in that sentence's contrary. The main function expression of 'Every man walks' is 'Every' (a "second-order predicate"). It attaches to the first-order function, '. . . walks', to yield 'Every . . . walks', which, in turn, is completed by the (general, common) name, 'man'.

One of the features of traditional logic that has most exercised Geach is its doctrine of distribution. In chapter one, we saw how Geach has attacked Aristotle's position (as set out in *Prior Analytics*) that the two terms of a categorical are interchangeable (i.e., are logically homogeneous). Geach compared Aristotle's rejection of the Platonic binary theory (endorsed in *De Interpretatione*) in favour of the ternary analysis of the

Analytics to Adam's Fall. "Aristotle's going over to the two-term theory was a disaster" (1972a: 47). Yet this "was only the beginning of a long degradation" (1972a: 51). The two-term theory was coupled with the view that since the subject-term could be taken as a name, the predicate-term could be viewed as a name as well. The two-name theory construed categoricals as pairs of names joined by a copula (now taken as a sign of identity). It was this theory, says Geach, that dominated the Middle Ages.[39] Finally, the two-name theory was coupled with the idea that what a general term names is the class of objects that it denotes. "By this slide the rake's progress of logic . . . reaches its last and most degraded phase: the *two-class* theory of categoricals" (1972a: 53). The evils of the two-class theory are legion, according to Geach's reading of logic's history. Among them is the perverted notion that quantifier expressions somehow contribute to the reference of the terms to which they are attached, what Frege contemptuously called "quantificatious thinking." A second abomination was the doctrine of distribution.

The Scholastics' doctrine of distribution was a result of their theory (or theories) of supposition—their semantics. According to the traditional account, since subjects are always syntactically complex, consisting of a quantifier and a subject-term, the semantic roles of subjects and subject-terms are different. The former *refer* while the latter *denote*. Subjects-terms are said to be distributed just in case their denotations and the references of the subjects in which they are embedded coincide; otherwise, they are undistributed. The doctrine was extended to singular subjects (taking them, usually, to be distributed), and then expanded into the well-known theory involving necessary conditions for syllogistic validity. As a Fregean, Geach rejects any claim that subjects might be syntactically complex. As a consequence, he rejects any distinction between reference and denotation, and with it the notion of distribution (cf. 1956; 1962, ch. 2; 1976). For Geach, reference/denotation is the role played by names. And any time a term is used to make reference it is in logical order to ask about what is being referred to.

> Even if we knew what 'referring' was, how could we say that 'some man' refers to some man? The question at once arises: Who can be the man referred to? . . . which men will the subject refer to if a predication of this sort is false? No way suggests itself for specifying which men from among all men would then be referred to; so are we to say, when 'Some men are P' is false, all men without exception are referred to—and 'men' is distributed? (1962: 6-7)

Geach would reject the view that quantified terms (traditional subjects) refer at all. Thus 'some S' and 'every S' do not refer (i.e., do not denote, do not name). Much of the plausibility of his stand derives from the fact that he insists on including 'no S' among such expressions: "When 'nothing' or 'no

man' stands as a grammatical subject, it is ridiculous to ask what it refers to" (1962: 12). But, of course, 'no' is no (logical) quantifier. One who, like Geach, takes all logical subjects to be names is right to require an answer to the question of what is being referred to, or named, by a given subject. Friends of distribution (among whom I will count myself) must reject not only the Fregean Dogma (that all logical subjects are singular) but the weaker Geachian claim that all logical subjects are names.[40] At any rate, Geach has not been content to reject just the semantic grounds of distribution.[41] He has also sought to overturn the doctrine as it relates to syllogistic validity (e.g., 1962, ch. 1; 1972a, sections 2.1 and 2.2). The distribution of predicate-terms especially disturbs Geach. "If a predicate-term 'P' can indeed be understood to refer now to any and every P, and now only to some P, then it seems natural to mark this fact by attaching quantifiers to the predicate as well as the subject" (1962: 18). This is just what Hamilton tried, as Geach goes on to note, a century and a half before. But the fact is that no one defending distribution would say that predicate-terms, or even predicates, refer. The doctrine holds only that a sentence in which the predicate-term is distributed implies a sentence in which that term is now a universally quantified term (and thus *part* of a referring expression). Even if we could make sense of the distribution of both subjects and predicates, does the doctrine actually work as applied to syllogisms? According to Geach, the syllogistic rules of distribution are inconsistent. Thus, the inversion of an A categorical to an O (viz., 'Every S is P' to 'Some nonS is not P') has, contrary to the rules of distribution, a predicate term distributed in the conclusion but not in the premise. Suffice it to say for now that what needs to be doubted here is not distribution but inversion. A Keynesian sort of solution (seeing the inversion here as an enthymeme) is to be recommended.[42] Needless to say, we will return to Geach from time to time throughout the second part of this essay.

A century after the initiation of the Fregean revolution in logic, no philosopher better represents the pervasive use of that logic as a *philosophical* tool than W.V. Quine. Through the period of accretions, changes, and challenges from the 1930s to the 1990s Quine has remained the heir to and defender of Frege and Russell's logicism. His contributions to logic, philosophy of language, epistemology, and metaphysics have been enormous and influential. Geach spoke for many when he wrote recently, "My intellectual debt to Quine is immeasurable" (in Lewis, 1991: 252).

From his simplification in "New Foundations for Mathematical Logic" (Quine, 1937) of Russell and Whitehead's system to his most recent work in logic, Quine has been consistent in his advocacy of a small number of core theses. Among these are the following: (1) The proper logical task with respect to natural language is regimentation, the building of an artificial language free of the ambiguities, vagueness, and other qualities of natural language, and adequate for scientific discourse. The logic of natural language can be revealed only through a translation into the regimented language that eliminates the plaguing "quirks of usage" (1960a: 158). The resulting constructed language—the standard first-order predicate calculus with identity—"is a paragon of clarity, elegance, and efficiency" (1970: 85). (2) In constructing an artificial logical language, the logician must seek to display the logical grammar of expressions (viz., sentences) as perspicuously as possible. Such forms must aim to reveal the truth-conditions of their sentences.

> Logic chases truth up the tree of grammar. . . The grammar that we logicians are tendentiously calling standard is a grammar designed with no other thought than to facilitate the tracing of truth conditions. And a very good thought this is. (1970: 35-36)

> Revision of grammar is an important part of the logician's activity. . . For the latter-day logician, logical regimentation of grammar is standard procedure. . . . what we call logical form is what grammatical form becomes when grammar is revised so as to make for efficient general methods of exploring the interdependence of sentences in respect of their truth values. (1980: 20-21)

(3) The distinction between statements true by virtue of their logical form alone (analytic statements) and those true by virtue of the way things are in fact (synthetic statements) is, at least, difficult to draw. An adequate understanding of the former would require an understanding of all members of a family of terms, including 'necessary', 'synonymous', 'meaning'. But none of the terms in this circle can be explicated clearly and independently. As a result, Quine has steadily maintained a conservative attitude toward all attempts to extend the standard system of first-order predicate logic to include modalities, propositional attitudes, and any kind of intensional object (such as propositions and meanings). (4) Logic is not ontologically neutral. While it cannot reveal what there is, it can reveal what a person or theory "says there is" (1960a: 253; see also 1953: 15). According to Quine, what is (i.e., what is assumed, presupposed, taken, counted to be) is just what a speaker (or theory) admits as a possible referent of his or her referring expressions, and in Quine's "canonical" regimented language these are bound individual variables—personal pronouns. As he so memorably put it, "To be is to be the value of a variable" (1953: 15; other, more accurate but less striking, versions are 1943: 25; 1953: 13, 14; 1960a: 242, 243).

Quine's commitment to theses such as these has determined his ideas concerning logical syntax. In the canonical language all sentences are either atomic or functions of atomic sentences. Atomic sentences are formed by the "basic combination":

> The basic combination in which general and singular terms find their contrasting roles is that of *predication*. . . . Predication joins a general term and a singular term to form a sentence that is true or false according as the general term is true or false of the object, if any, to which the singular term refers. (1960a: 96; see also 1970: 28)

This is a succinct but rich statement of Quine's central thesis concerning logical syntax and logical semantics. Predication is the joining of a singular and a general term. Natural-language singular terms (what he calls "ordinary" or "definite," as opposed to "indefinite" or "dummy," singular terms [1960a: 112-14]) are expressions such as names, definite descriptions, personal pronouns, and demonstratives (dummy singulars are quantified terms). The role of (definite) singular terms is reference. Since all such terms can be paraphrased in terms of just personal pronouns, existential quantifiers, and general terms, and since (by Quine's thesis 4 above) ontological commitment is revealed by those terms used to make objective reference, the singular terms of logically regimented sentences (i.e., individual variables) carry the entire burden of reference. Consequently, no other kind of expression, particularly general terms, can refer—the Fregean Dogma. In the basic combination, the singular term is referential. The general term is predicative. Indeed, it is only by this difference of "role that general and singular terms are properly distinguished" (1960a: 96).[43] The predicative role is the role of being *true of* what is referred to by the singular term (the subject). Only general terms can play the predicative role in a sentence. Like Frege, Strawson, and Geach, Quine subscribes to the Asymmetry Thesis. Singulars can never play the predicative role; general terms can never play the referential role.[44]

Sentences in the basic combination have no formatives. Sentential unity appears to be merely the result of the primitive relation of predication, which binds singulars to general terms when these are brought sufficiently close to one another. Most importantly, no logical copula is required to effect this binding. The grammatical copula (e.g., 'is') in some natural-language approximations of the basic combination is logically inert. Such expressions are nothing more than devices for converting "a general term from adjectival or substantival forms to verbal form for predicative position" (1960a: 97). Note that while general terms may have the *grammatical* forms of adjectives, substantives, or verbs, they are *logically* verbs, "for predication the verb may even be looked on as the fundamental form" (1960a: 96). Relative terms, likewise, are general terms that may be substantives (plus prepositions), such as 'brother of'; adjectives (plus prepositions or conjunctions), such as 'part of', 'bigger than', 'same as'; transitive verbs, such as 'loves', 'killed'; or lone prepositions, such as 'in', 'under', 'like' (1960a:105-6). Fundamentally, then, a basic combination is a predication of a singular term and a verb.[45] The following represent ever closer approximations to logical purity.

a. Socrates is a philosopher.
b. Socrates is philosophic.
c. Socrates philosophizes.

According to Quine, indefinite singular terms can be translated into the canonical notation via quantifiers and the individual variable that they bind. But, as mentioned above, definite singulars can be given this same sort of translation as well. Names can be paraphrased in terms of identity (thus 'Socrates' becomes 'is identical to Socrates'), which, in turn, can be paraphrased in terms of a general term (verb) designed to be true of just the thing named (thus: 'Socratizes') (1960a: 178ff). This procedure for translating names is generally called "Pegasizing", since it is meant to be applied primarily to names that fail to refer (e.g., 'Pegasus'). In such cases, it permits the closing of truth-value gaps so that sentences like 'Pegasus runs' (= 'Something both Pegasizes and runs') are false. This way with names may seem artificial, but "All in all, who shall say whether English is more radically modified by a canonical notation in which names consort with the singular pronouns and indefinite singular terms or by one in which they consort with the general terms (1960a: 181).

Singular definite descriptions, such as 'the queen of England' or 'the present king of France', are regimented according to Russell's theory (thus: 'exactly one thing is queen of England', 'exactly one thing is presently king of France'), again allowing truth-gap closure for cases such as the second example. Class, attribute, and relation abstractions (e.g., 'the class of red things', 'sanity', and 'superiority', respectively) are paraphrased as definite descriptions and then eliminated by Russell's theory in favour of bound variables and predicates.

> Thus evidently nothing stands in the way of our making a clean sweep of singular terms altogether, with the sole exception of the variables themselves. (1960a: 185) . . . That variables alone remain as singular terms may be seen as testifying to the primacy of the pronoun. (186)

Quine's emphasis on the variable, in both logic and ontology, cannot be exaggerated. On one side it is the logical analogue of the natural-language pronoun;[46] on the other it is the expression par excellence for reference to bare particulars (Locke's "unclothed substances," Wittgenstein's "colourless objects"). As Quine says, "The pronoun is the tenable linguistic counterpart of the untenable old metaphysical notion of a bare particular" (1980: 165) and "The variable is the legitimate latter-day embodiment of the incoherent old idea of a bare particular" (1981a: 25).

Quine does not make it clear just what is "untenable" or "incoherent" in the idea of a bare particular—perhaps just that it is an old idea, for he appears to like it well enough in its "latter-day embodiment." At any rate, as he sees it, singular terms in natural language tend to work at two tasks simultaneously, though the two tasks—identification and reference—can be separated. In a regimented language, the identificatory work is assigned to general terms, leaving variables to do the purely referential work. Thus, what such variables refer to must be bare, unidentified, but merely enumerated. These bare particulars constitute the range of values, the domain of discourse, for the bound variables of a regimented sentence, "but the idea of [such sentences] being about certain things and not others seems dispensable" (Quine's reply to Strawson in Davidson and Hintikka, 1969: 321). Quine's emphasis on the variable notwithstanding, we will soon see that he has found ways of eliminating variables altogether from a regimented langauge. But first, a few comments concerning Quine's notion of identity are in order.

Modern mathematical logic has made much of Frege's distinction between the 'is' of predication and the 'is' of identity. The distinction is seen as logically required, since otherwise some singular terms, when following 'is', would have to be construed as predicates. Thus, in 'Tully is Cicero' either 'is' is not the dispensable 'is' of predication or 'Cicero' is not singular. The 'is' of identity is seen as indicating a binary relation and is appropriately flanked by a pair of singulars. The identity statement is true if and only if the two singular terms are co-referential. Notice that Quine's Pegasizing procedure is a device not for predicating singular terms (in the case of names), but merely for converting such singulars to general terms (which *can* be predicated). Now, according to Quine, "Identity evidently invites confusion between sign and object" (1960a: 117). Oddly enough, he thinks that Aristotle was not subject to this widespread confusion (116n). This is odd because Aristotle and Quine hold quite different views of logical syntax—and thus of the logical form of identity statements. Quine sees his position to be similar to Aristotle's here only because he fails to see that Aristotle, unlike him, is willing to admit singular terms after the qualifier. Aristotle would take 'is' to be always predicational. Whether a term is used referentially or predicationally is, for Aristotle, independent of whether it is singular or general—it is just a matter of the term's position, or syntax. In contrast, Quine takes the role of 'is' to depend on whether it is followed by a singular or general term.[47] We saw that Leibniz was able to exploit this Aristotelian view in attempting to incorporate singular sentences into the categorical mould. In part two we shall see how Sommers has exploited it even further.[48]

For Quine, all referential expressions—singular terms—can be paraphrased in terms of pronouns and appropriate predicates and formatives. Translated into the language of the canonical notation, these pronouns become individual variables. In effect, descriptive phrases, demonstratives, and names have been eliminated in favour of variables. Yet, as noted above, Quine allows for the possibility of eliminating even these. Given the enormous logical weight placed on variables, the natural question is: Why should they be eliminated, and how? The elimination of any device that does essential work in a given system must be accompanied by the substitution for that device of another device (or devices) that does the same job. From a theoretical point of view, the contemplation of such a substitution will reveal just what the work of the original device has been. The intention is to instil an appropriate level of understanding and appreciation of the old device. This is the motive for eliminating variables. But how can this be done? One of the jobs assigned to variables in the standard predicate calculus is that of indexing. Consider, for example, the sentence 'Every boy kissed every girl'. The canonical translation of this is usually '(x)(y)(Bx & Gy .⊃ Kxy)'. Note that the (individual) variables here show which reference is bound by which quantifier and keep track of which is the subject and which the object of the relational ('kissed'). If one could reduce all polyadic (relational) predicates to monadic (nonrelational) predicates, such indexing—and thus the use of such variables—might be eliminated (see 1980a: 23-24). In a series of studies spread out over several years Quine developed a formal language, the Predicate Functor Algebra, "a drastic alternative to standard logical grammar" (1970: 30), which makes no place for individual variables.[49] As it turns out, the Predicate Functor Algebra, which makes use only of predicates and functors on them, has no need to distinguish between the singular and general terms of its lexicon. Singular terms—names, descriptions, demonstratives, and even pronouns—have been eliminated. Unanalysed sentences (as in the logic of truth-functions) have been taken as zero-place predicates. The result is a logic that is, in effect, equivalent to the first-order predicate calculus with identity. But, unlike the calculus, the algebra is a *term logic*, for, when all the terms of the lexicon are general terms, predicate terms, 'term' will do. Aristotle's syllogistic was a term logic. The universal characteristic sought by Leibniz was a term logic. The algebras of Boole, De Morgan, Jevons, and Venn were term logics. Quine's algebra of predicate functors demonstrates via its equivalence to the standard calculus that even the predicate calculus is a term logic.[50] In what remains of this essay, I will examine and develop a system of logic due to Fred Sommers. My contention is that it is a term logic par excellence .

Notes for Chapter 2

[1] Indeed, Kneale and Kneale (1962: 511) call 1879 "the most important date in the history of [logic']," and Quine (preface to Clark 1952: v) says that "1879 did indeed usher in a renaissance [in logic]."

[2] Some of this can be seen in the account of this transition period by one writing in the midst of it: Shearman (1906).

[3] For example, see Frege (1879: 7, 12; 1892: 54; 1979: 141).

[4] Prominent Fregeans, such as Geach and Dummett, disagree about the priority of the grammatical or the ontological distinction in Frege. For a brief discussion of this debate, see Sommers (1982: 36-37). See also Wetzel (1990). For Frege's saturated/unsaturated distinction, see especially Frege (1891: 31); see also Frege (1979: 177, 187).

[5] Quoted in Furth (1968: 16). See also Frege (1892: 54-55; 1979: 177).

[6] See especially Frege (1891, 1892b). See also Furth (1968).

[7] See Englebretsen (1986a).

[8] For example, in Frege (1892a). See also Mendelsohn (1987) and Englebretsen (1990a). For an attempt to "repair" Frege by reintroducing the copula (as a predicate formative) see Wiggins (1984).

[9] Also see 6f, 12ff, 142, 188, 190, 266, 269ff.

[10] For more on this contrast see, for example, Horn (1989); Sommers (1982, esp. ch. 13); Englebretsen (1981a); and Sanford (1966).

[11] See Frege (1979: 185; also 198, 253).

[12] See 46. This is the basis of Frege's *contextual thesis* (only in the context of a sentence can a work be said to have meaning).

[13] See Bradley (1914, 1935); Hunter (1985).

[14] See Englebretsen (1986a, 1986c).

[15] Ramsey (1925). See Russell's response (1931).

[16] Geach (1962, ch. 2; 1969a; 1972a, esp. sections 3.5, 3.6, 8.1, 8.2; 1975); Strawson (1952, esp. 170ff; 1957; 1959, esp. part 2; 1970; 1974). See also Heintz (1984) (and the response by Linsky and King-Farlow [1984]). For criticism of the Asymmetry Thesis see, for example, Grimm (1966); Hale (1979, along with a brief comment by Geach, 146); Nemirow (1979); Clark (1983); and Bradley (1986). I have addressed the topic in Englebretsen (1985b, 1985c, 1987b, 1990d).

[17] Traditionalists such as Johnson had held that general, but not singular, terms could be compounded. In response, Ramsey argued that even general terms could not be compounded (see 1925: 405-07, 411).

[18] See Russell's letter to Frege in van Heijenoort (1967a).

[19] This is especially so in Russell's *Principles of Mathematics* (1903) and "The Philosophy of Logical Atomism" (1918-19).

[20] This point is discussed by Kaplan (1972: 239-41).

[21] In addition to Sainsbury (1979), see White (1979).

[22] Russell (1945: 13) claimed that Leibniz had actually made an "awkward discovery," relational logic (a logic that admits irreducible relationals), but that he chose to suppress his discovery, taking relations to be merely ideal.

[23] See Korner (1979: 176ff). In fact, the reduction of relations to classes was achieved. See Wiener (1967).

[24] See especially Russell (1903, ch. 26). Russell's critique is examined in Sprigge (1979).

[25] Of course, as we saw above, Bradley went on to hold that both relational and nonrelational judgments are distortions of Reality. Non-relational judgments are simply less distorting.

[26] In fact, Russell had held that Bradley's Paradox was genuine but that, being "logically quite harmless," it posed no real threat to logic. See Russell (1903: 100).

[27] The following remarks rely heavily on the excellent analyses of the problem in Palmer (1988) and Linsky (1992) (the brief appendix to the latter work is especially recommended).

[28] See "Boole's Logical Calculus and the Concept-Script" in Frege (1979). Sluga (1987) calls Frege's insistence on giving priority to the sentence over its constituents the "priority principle."

[29] As Linsky has said (1992: 26-7), "*All* of the constituents of the proposition in the *Tractatus* are incomplete." Names achieve propositional unity by completing each other.

[30] But in a sense, claimed Wittgenstein, all of the logical constants occur in any atomic sentence; there is, in effect, just one such constant, which is what atomic sentences have in common—that is, their logical form (1961: 5.47). He had in mind "mutual rejection," the Sheffer stroke (5.1311, 6.001).

[31] See remark 5.5151. Ramsey (1923: 17) made the same point. Wittgenstein (1929) tried to accommodate Ramsey's criticism concerning the incompatibility of 'This is red' and 'This is blue', claiming that such propositions are not contradictory but may "exclude" one another (35-36).

[32] Since a statement is always a truth-claim, Strawson has also devoted much attention to the concept of truth (see, for example, 1950b, 1954, 1964, 1965).

[33] For an extended discussion see Englebretsen (1986a).

[34] For a more critical appraisal of Strawson concerning asymmetry see Englebretsen (1985b, 1987b). See also Strawson's reply to Geach in van Straaten (1980: 293-94).

[35] Geach's "History of the Corruptions of Logic" (in Geach, 1972) is especially hostile. See Englebretsen (1981c) for a response.

[36] See Hintikka (1991) for a fuller survey of Geach's hidden non-Fregean views. In his reply to Hintikka, Geach denies any departure from orthodox Fregean theory.

[37] But Geach (1950: 463) does suggest that while the negation of a name is not a name, it isn't nonsense either. And in *Logic Matters* (1962a, ch. 7) he comes close to my own notion of composed names. For an alternative view of compound predicates see Stalnaker (1977).

[38] For a more extensive discussion see Englebretsen (1985a).

[39] Though Geach has not been reluctant to charge Scholastic logic in general with the sin of the two-name theory, he has exempted his favourite Scholastic, Aquinas. See especially Geach (1962: 22-46; 1969b: 42-64). Veatch (1974: 416-22) responds to Geach's views of Aquinas on logical syntax.

[40] Recent discussions of distribution are: Toms (1965), Makinson (1969), Richards (1971), Williamson (1971), Sommers (1975), Katz and Martinich (1976), Friedman (1978ba), Rearden (1984), Englebretsen (1985d), Wilson (1987), Peterson (1995).

[41] But see Geach (1950: 475), where his "total" and "partial" identity sound suspiciously like "distributed" and "undistributed."

[42] See Englebretsen (1979: 115-17).

[43] For more on this topic see Englebretsen (1982a, 1986a, 1986c). See also Strawson (1969), along with Quine's reply in the same volume.

[44] This last is meant to protect us from Platonism.

[45] Recall Russell's notion of the verb "used as a verb," which embodies the unity of the proposition.

[46] While many contemporary linguists have taken to heart this Quinean attitude toward the pronoun-variable parallel, not all have. See, for example, Wasow (1975) and Higgenbotham (1980).

[47] For more, see Englebretsen (1985b).

[48] In a sense, of course, Quine is willing to dispense with the identity relation altogether—at least, where the lexicon of the language at hand has a finite number of predicates. Given such a language, indiscernibility can do all the work of identity. For a brief statement of this see Quine (1981a: 27-28).

[49] See Quine (1936a, 1936b, 1937b, 1959, 1960b, 1971a, 1971b, 1972, 1981c, 1981d). Quine has often acknowledged his debt to the work of Schönfinkel (1924) and Curry (see esp. Curry and Feys, 1958).

[50] Though much of my knowledge of the Predicate Functor Algebra comes from a reading of Quine's work cited above, I owe most of my understanding and appreciation of the theoretical import of the algebra (particularly its status as a version of term logic) to the

work of Aris Noah. No one should consider the study of Quine's algebra complete without having attended to Noah (1980, 1982, 1987).

Part II