[18] See Englebretsen (1986a).

[19] For more on this point see Vendler (1967, ch. 2), Paduceva (1970), and Chastain (1975). A theory similar to Sommers's is found in Heim (1982, ch. 3).

[20] See Englebretsen (1984b, 1985e).

[21] See Sommers (1982, app. A) and Englebretsen (1982d).

[22] A fine account of this is offered in Dipert (1981).

[23] For a similar account of proper names see Lockwood (1971).

[24] See Englebretsen (1972c) for a clarification of Sommers's account of this distinction.

[25] A much more extensive examination of this device is found in Englebretsen (1984b).

[26] Other logicians, including especially Boole and Frege, had held similar views. Frege took every statement to refer to either the True (what makes the statement true) or the False (what makes the statement false).

[27] In ch.14 of Englebretsen (1987), I try to show that while, with respect to any term 'P', a given thing may be either P or nonP, with respect to any constitutive characteristic [p], every domain is either [p] or un[p]. In other words, the polarity of nonsentential terms is reversible but the polarity for sentential terms is not. This fact is the true basis for the contrary/contradictory distinction.

[28] For recent work on this question see, for example, Braine (1978), Evans (1982), Henle (1962), Johnson-Laird (1983), Johnson-Laird and Byrne (1991), Osherson (1975), Wetherick (1989), and Rips (1994).

CHAPTER FOUR

IT ALL ADDS UP

We cannot go back to the prison that would confine all logic to the Aristotelian syllogism, but it is possible to defend (a) something like the view that the form "Every X is Y" is more fundamental than either "For all x, f(x)" or "If p then q" and (b) the traditional ignoring (in inference by subalternation, etc.) of terms that have no application.

A.N. Prior

Plus/Minus

Certainement calculer c'est raisonner, et raisonner c'est calculer . . . . Lorsque je dis que les quantités sont ajoutées ou soustraites, et que conséquemment je les distingue en quantités en plus et en quantités en moins, je ne les confonds pas avec l'opération qui les ajoute ou qui les soustrait; et on voit comment, étant les mêmes en algèbre que dans toutes les langues, il n'y a de différences que dans la manière de s'exprimer: mais quand on nomme quantité positive l'addition d'une quantité, et quantité négative la soustraction d'une quantité, on confond l'expression des quantités avec l'expression de l'opération qui les ajoute ou qui les soustrait, et un pareil langage n'est pas fait pour répandre la lumière. Aussi les quantités négatives ont-elles été un écueil pour tous ceux qui ont entrepris de les expliquer.

Condillac

The concepts of addition and subtraction. The rudiments of logic.

Don De Lillo

In chapter three I offered a brief summary of the many contributions to term logic made over the past several years by Sommers. Such a summary cannot in any measure serve as a substitute for Sommers's own work, but I hope that it will kindle a degree of interest in it. As well, it is meant to show to some extent just how Sommers's logical ideas are actually the latest stage of a very long historical development that did not, contrary to the view of many contemporary logicians, end with Frege or retreat to a few "Colleges of Unreason." My aim in the present chapter is to give a simple, consolidated picture of the logical algorithm for term logic (the plus/minus system envisaged by Hobbes, Leibniz, and De Morgan and built by Sommers). Along the way I will offer a few modest amendments and additions.

According to the Scholastic logicians, as we have seen, the proper concern of logic is at once both speech and thought (*scientia sermocinalis* and *scientia rationalis*). As a science of thought, it aims at an account of what were commonly known as the three "acts of the intellect": (1) understanding/comprehension, (2) composition and division, (3) reasoning/syllogistic. The first deals with the meaning of terms; the second with the formation of sentences from terms; the third with the formation of arguments (syllogisms) from sentences. The nature and order of the three parts was claimed to have been inspired by Aristotle's *Categories, De Interpretatione*, and *Prior Analytics*. In modern terms, it would be fair to describe the content of the three studies as (1) semantics, (2) syntax, (3) deduction. Serious debates have erupted from time to time among post-Fregean logicians, but traditional logicians were in general agreement about this ordering. One cannot understand syllogisms without first accounting for the sentences that constitute their matter. And these, in turn, require a prior account of the terms that constitute sentential matter. Post-Fregeans have been particularly worried about the relative order of semantics and syntax. Some give priority to the former; others to the latter; and still others take semantics and syntax to be interdependent. As we saw in chapter two above, whatever view is taken, there is a sense in which the standard system of mathematical logic now must give pride of place to at least a certain measure of semantics. In particular, the syntactic theory of the standard logic rests on the prior division of the elements of the lexicon into absolutely distinct categories: general terms (predicates) and singular terms (names, pronouns, individual variables). For all the talk of those who would give priority to syntax, the fact remains that such lexical classification must rest on semantic distinctions. Ironically perhaps, in spite of the traditional priority of semantics, pre-Fregean logicians avoided the temptation to allow logical syntax to be determined by semantics. The first act of the intellect was directed toward understanding the meanings of terms. In studying this act, the logician is concerned with accounting for the various semantic roles that any term can play in a sentence. The theory of supposition and comprehension (in all of its many guises) was meant to provide a complete account of the extensional and intensional meanings of *any* (used) term. There was little interest at this stage in classifying *kinds* of terms. All terms—singular, general, relational, sentential, compound, and so on—were given the same semantic treatment and were passed on to the second, syntactic, stage undivided. This traditional way, approaching syntax with an undivided lexicon, was followed by Sommers. I shall follow it as well.

There is a systematic ambiguity of plus and minus expressions in mathematical language. This ambiguity is not only benign, it is a source of great expressive power for the mathematician. Leibniz, De Morgan, and Sommers have suggested that natural language has a logic that, like arithmetic and algebra, makes use of two kinds of basic formal expressions, the signs of opposition. In fact, their common position seems to be that all the expressions of natural language that carry the responsibility for determining logical form are either positive signs or negative signs or signs definable in terms of these. If this is so, it means, among other things, that one could build an artificial formal language that would model natural language by using the mathematicians' opposition signs for all formatives. The result would be an algorithm for natural-language reckoning that would model natural statements as arithmetical, indeed algebraic, formulae and inference as algebraic calculation. There is little doubt that this was Leibniz's goal throughout his logical studies, and Sommers has effectively reached that goal in his own logical work.

It would seem, therefore, that the idea of using signs of opposition to model natural-language formatives is a good one, leading, as it seems to have done, to rich programmes of logical investigation and to viable systems for logical reckoning. One of the consequences of this idea has been, as we saw, great optimism among those who have shared it that a clear and precise account of the nature of logical formatives, and their distinction from nonlogical expressions, can be provided. In a sense, their account is quite simple: logical formatives, unlike other expressions, are oppositional in just the way that plus and minus are oppositional in mathematics. But to appreciate fully this kind of account, we need to look more closely at the oppositional character of formatives, their roles in inferences, and the kind of algorithm that could model those inferences. For if natural language has a logic (something assumed by all traditional logicians but denied by many modern logicians), then it ought to be possible to devise a formal language that models all kinds of statement-making sentences, as well as inference patterns among them. In other words, it ought to be possible for the logician to construct a formal system that closely matches the expressive and inferential powers of a language such as English.

The Simple System

We begin with a simple, abstract formal system consisting of the following: upper-case letters, a plus sign, and a minus sign (as well as any parentheses, brackets, etc., that we need for punctuation). The letters are the system's variables, its lexicon or vocabulary; the plus and minus signs are its formatives. The plus is a binary formative; the minus is a unary formative. The formation rules are:

(i) Every letter is a term.

(ii) If X is a term, so is −X.
(iii) If X and Y are terms, so is X+Y (called a phrase).

Our binary connective, +, is symmetric; thus the terms of a phrase like A+B can be commuted to give us B+A (for convenience, we will usually allow formulae to be their own quotations). As well, + is associative. Thus a term like A+(B+C) is equivalent to (A+B)+C.

So far the system is extremely simple—and weak. We can apply commutative and associative rules to phrases to yield new (and equivalent) terms. But we could not, for example, derive a new phrase from a pair of phrases, neither of which is equivalent to the new phrase. What is needed, of course, is a binary connective that is transitive. Suppose we had such a connective, perhaps *. Then we could formulate a deduction rule such that, for instance, A*C followed from A*B and B*C. As yet, we have no transitive binary connective, but we can define one in terms of our unary minus and binary plus. Notice that if X+Y is a term, so is −(X+Y). A system consisting of just a unary minus and a binary plus is already familiar to us. We can all do arithmetic using just negative and nonnegative numbers and addition. Subtraction is a binary operation defined in terms of negativity and addition. Thus: 3−2 = 3+(−2), where the minus sign in 3−2 is a binary operator defined by the binary plus (addition) and the unary minus (negativity) of 3+(−2). In like manner, we will define a binary minus in terms of our unary minus and binary plus as follows:

D.1 X−Y = df −(−X+Y)

(Compare: 5−4=−(−5+4).) This new binary connective is reflexive and transitive, and, unlike our binary plus, is nonsymmetric.

We now have a binary plus, a binary minus, and a unary minus. As well, we have implicitly defined a unary plus. Thus:

D.2 +X = df −(−X)

And then, as in arithmetic and algebra, we suppress unary plus signs when convenient, taking all unmarked terms as implicitly positive. The addition of these defined formatives simply amounts to a conservative extension of the original system.

The system of four formatives (two binary, two unary) is still relatively simple, but it has far greater expressive powers than does our original system of a binary plus and a unary minus. And the introduction of our transitive binary connective yields increased inference power. Let us say that the expressive power of any formal logic is a function of the extent to which it can formulate natural-language expressions. The greater the number of kinds of natural-language expressions that can be formulated, the greater the expressive power of the system. The inference power of a logic

is a function of the extent to which it can model inferences made in the mode of a natural language. Ideally, the logician wants not only a system exhibiting power (expressive and inferential), but, as well, a system that is simple (relative to alternative logics of comparable power). An additional criterion of adequacy might be naturalness. One logic is more natural than another in the sense that the first formulates natural-language expressions in ways that are closer (syntactically) to the original than does the second. The criterion of naturalness has not been accepted by all logicians (recall Frege's views about this), and even among those who do accept it, naturalness has often been honoured more in word than in deed. We will apply the criterion.

Formulating English

The simple system we have in hand thus far can be used to formulate many natural-language (e.g., English) expressions. Let our variables stand for natural-language terms. In natural languages such as English, terms come in charged (positive/negative) pairs (e.g., 'massive'/'massless', 'wed'/'unwed', 'confidence'/'nonconfidence', 'painful'/'painless'). These charges are clearly reflected by our unary formatives. And, just as a sign of positive charge on terms is rarely explicit in English, our positive unary sign is generally left tacit.

Our binary plus has the formal features of nonreflexivity, symmetry, and nontransitivity. In English there are several formative expressions with just these formal features. The most obvious of these is 'and', as in 'wealthy and happy', where 'and' connects a pair of terms to form a compound term, and in 'It is raining and it is cold', where 'and' connects a pair of sentences to form a compound sentence. Let us place a phrase in angular brackets when it formulates a conjunctive term, and in square brackets when it formulates a conjunctive sentence. We might formulate our two samples here as ⟨W+H⟩ and [r+c], respectively (adopting in the latter case the additional convention of symbolizing logically unanalysed sentences by lower-case versions of our variables).

In the *Analytics,* Aristotle tended to paraphrase categorical sentences by using a single formative expression between pairs of terms. For example, he would write (the Greek version of) 'A belongs to some B' and 'A belongs to every B', rather than 'Some B is A' and 'Every B is A'. The first of these, 'belongs to some', is a binary connective that forms a sentence from a pair of terms. It is the Scholastics' I-connective—as when they formulated the conclusion of Darii as 'SiP'. Such a connective is nonreflexive, symmetric, and nontransitive. So, like 'and', it can be formulated using our binary plus (for we can think of our binary plus as having been created with just those formal features and no others—recall De Morgan's notion of the purely formal copula, =). We could then formulate an I categorical, paraphrased first as 'P belongs to some S', as P+S.

All of the other three categorical forms can be expressed in our formal language. An O categorical makes use of our binary plus and unary minus. 'NonP belongs to some S' would be formulated as $-P+S$. A and E categoricals are the contradictory negations of O and I, respectively. Thus we could formulate 'P belongs to no S' as $-(P+S)$. And, given D.1, this can be expressed in terms of our defined binary minus as $-P-S$, where the second minus is now our defined binary connective and can be read as 'belongs to every'. Similarly, we could formulate 'P belongs to every S' as $-(-P+S)$, which, by D.1 and D.2, is equivalent to $P-S$. The new connective, read here as 'belongs to every', is reflexive, nonsymmetric, and transitive.

The English connective 'if' has the formal features of reflexivity, nonsymmetry, and transitivity. This suggests that we can formulate it using our defined binary minus, and a bit of calculation shows that we can indeed do so. A sentence of the form 'p if q' is the contradictory of '(both) not p and q', which is symbolized as $[-p+q]$; its negation would be $-[-p+q]$. By applying D.1 and D.2, we get $[p-q]$ for 'p if q'. In effect, we have defined 'if' in terms of 'not' and 'and'.

It might be objected at this stage that we are treating 'and' and 'belongs to some' as equivalent expressions (likewise for 'if' and 'belongs to every'). But from a purely formal point of view they *are* equivalent; both 'and' and 'belongs to some' are nonreflexive, symmetric, and nontransitive (and both 'if' and 'belongs to every' are reflexive, nonsymmetric, and transitive). They share the same formal features. (This fact is the one De Morgan sought to express when he tried to reduce, for example, 'only if' to 'is'.) Moreover, the following kinds of equivalences ought to convince one of the formal parallels we have drawn.

> Some A is B and C = B and C belongs to some A = B belongs to some C and A = $B+\langle C+A\rangle = \langle B+C\rangle+A$

Here the associativity of our binary connective reflects the formal equivalence in English between 'and' and 'belongs to some'. Similar equivalences show that 'if' and 'belongs to every' are formally equal.

> Every A is B if C = B if C belongs to every A = B belongs to every C and A = $B-\langle C+A\rangle = \langle B-C\rangle-A$

Relationals

Thus far, we have seen that a system of two unary and two binary formatives can be used to express a wide variety of kinds of natural-language expressions: conjunctive and conditional terms, conjunctive and conditional sentences, and categoricals. But it can be used to express still more.

Consider a sentence like 'Some boy loves every girl'. Its Aristotelian paraphrase is 'Loves every girl belongs to some boy', which we might begin to formulate as (L every G)+B. We are now tempted to

symbolize 'every' by our binary minus since we have already used it for 'belongs to every'. Yet, 'every' and 'belongs to every' appear to be different expressions; are they formally equivalent—do they share the same formal features of reflexivity, nonsymmetry, and transitivity? The answer is yes. One way to see why this is so is found in Leibniz's idea that relational terms are "Janus-faced," facing in two directions at once. Consider 'Paris loves Helen'. Leibniz would analyse this initially as 'Paris loves and, *eo ipso*, Helen is loved'. The relational term 'loves' applies to 'Paris' as subject and to 'Helen' as object. In 'Some boy loves every girl', the expression 'loves every girl' has 'girl' as the object of the relation—that is, 'loves' (with its passive face, 'is loved') is said to belong to every girl (by virtue, *eo ipso*, of 'loves every girl' being said to belong to some boy). Leibniz was correct that relational terms are Janus-faced, but he was wrong to conclude that relational sentences must therefore be conjunctions.

Relative terms, such as 'loves', 'killed', and 'gave . . . to', can be predicated of more than one term at a time. We will indicate this by subscribing to each subject or object of a relation a unique numeral, which, in turn, will be subscribed to the relational term, along with other such numerals, in the appropriate order, to indicate how the relational is to be read. Thus, in formulating 'Some boy loves every girl' we symbolize 'boy' as 'B_1', 'girl' as 'G_2', and 'loves' as 'L_{12}': $(L_{12}-G_2)+B_1$.

Our letter variables can be used to symbolize any kind of simple (noncompound, nonsentential, nonrelational) term. So far, we have used them to formulate general count nouns (e.g., 'logician') and adjectives (e.g., 'happy'). But they can be used as well to symbolize general mass nouns (e.g., 'wine', as in 'Some wine is sour'—i.e., 'Sour belongs to some wine', or, S+W). And, most importantly, they can be used to symbolize singular terms, especially proper names. Thus, to assert 'Socrates is wise' is to say of Socrates that wise (=wisdom) belongs to him, in other words, 'Wise belongs to Socrates': W+S. Notice that we have used our binary plus here. This is justified if a sentence like 'Socrates is wise' can be commuted (since our binary plus is symmetric). And, indeed, 'Socrates is wise' is logically equivalent to 'Some (one who is) wise is Socrates'—that is, 'Socrates belongs to some (one who is) wise', or S+W. But suppose I add 'Some Greek is Socrates'—that is, S+G. Now, from 'Socrates is wise' and 'Some Greek is Socrates' one can intuitively derive 'Some Greek is wise', W+G. Yet this is contrary to our observation that the binary plus is nontransitive. The solution here is to see that a sentence like 'Socrates is wise' ('Wise belongs to some Socrates') is logically equivalent to 'Wise belongs to every Socrates'. In other words, when S is singular we can say $W+S = W-S$. To see how this is so, we will make an important change to our formal system.

Splitting Connectives

We began with a formal system consisting of just a unary minus and a binary plus. Recognition of the formal features of these operators revealed a surprising expressive power, which was then greatly increased by adding to the system defined unary plus and binary minus operators. However, in spite of this greatly increased power of expression, we have yet to achieve very much naturalness. For example, Aristotelian paraphrases of categorical sentences are less than perfectly natural. In English, for example, the expressions formulated so far as binary formatives in fact often (indeed, in some cases, usually) come not as single expressions (e.g., 'belongs to some') but as a pair of expressions—*split* (e.g., 'some . . . is . . .'). What is clearly wanted, then, is a way to split our binary connectives, so that they more closely match their natural-language counterparts, while simultaneously keeping their formal features (symmetry, transitivity, etc.).

Let us begin by looking at some formatives in English for combining pairs of sentences to form compound sentences. For example, we say 'Sue will go to the party only if Ed is not there', or 'Ed will not be there if Sue goes to the party', where the binary connective, or formative, is a single expression occurring between the two constituent subsentences. But we just as readily express the same proposition by splitting the connective: 'If Sue goes to the party then Ed will not be there', with the single connective, '. . . if . . .' (or '. . . only if . . .'), now split into 'if . . . then . . .'. So, in English, at least, we form conditionals with split as well as unsplit binary formatives. The same is true for compounds such as conjunctions and disjunctions. Thus, '. . . and . . .' can be split into 'both . . . and . . .', and '. . . or . . .' can be split into 'either . . . or . . .'. When it comes to compound sentences and phrases, both split and unsplit formatives are common. What now of formatives used to bind terms into categoricals? We have seen that the Aristotelian formatives 'belongs to some' and 'belongs to every' are not always natural. What is natural is split versions of these. We do not ordinarily say 'Happy belongs to some bachelor', but rather 'Some bachelors are happy', where the single binary formative is now split into 'some . . . are . . .'. In the same way, we normally split '. . . belongs to every . . .' into 'every . . . is . . .'. When it comes to relational expressions, however, we have seen that the unsplit version is the norm. Thus we say 'Some boy is kissing every girl', with 'every' as an unsplit binary formative connecting the terms 'kissing' and 'girl' to form the relational expression 'kissing every girl', but the split binary connective 'some . . . is' connecting the terms 'boy' and the relational phrase to form the categorical sentence.

Traditional logicians were of two minds (perhaps appropriately) when it came to the categorical formatives. On the one hand, they developed an algorithm that took such connectives as unsplit. Thus they used the a, e, I, and o signs as symbols for the unsplit term connectives 'belongs to every', 'belongs to no', 'belongs to some', and 'does not belong to some'—that is, 'PaS', 'PeS', 'PiS', and 'PoS', for the four standard categorical forms. On the other hand, they recognized the natural split

versions of such formatives, and, indeed, elaborated various semantic theories based on the analyses of categoricals formed with split connectives. Each half of a split Aristotelian formative can be treated as if it were an independent logical expression. Yet it must always be kept in mind (though this has not always been so in the tradition) that the two parts of any split connective are just that—parts of a whole. They are not two formatives; they are two parts of a single formative. The two parts of these formatives are called, respectively, the quantifier and the qualifier. In what follows, I shall restrict the use of the term *logical copula* (or simply, *copula*) to any unsplit Aristotelian formative. The part of a sentence consisting of a quantifier and a term is the subject; the part consisting of a qualifier and the other term is the predicate. A subject, then, is a quantified term. Thus, from a formal point of view, object expressions in relationals are logical subjects, since they are quantified terms. Predicates are qualified terms. The fact that traditionally qualifiers were often called copulae indicates the longstanding ambivalence over split and unsplit formatives for categoricals.

Quantifiers are of two kinds: particular (expressed in English usually by expressions such as 'some', 'one', 'a(n)', 'at least one') and universal (e.g., 'all', 'every', 'each'). It should be kept in mind that in ordinary uses of English particular quantifiers can often play the logical role of universal quantifiers, though this can be determined by context. Moreover, in the ordinary use of a natural language, contextual clues often allow the explicit use of any quantifier to be suppressed.

Qualifiers are of two kinds as well: affirmative and negative. English examples of affirmative qualifiers are such words as 'is', 'are', 'was', 'were'; negative qualifiers include 'is not', 'isn't', 'ain't', 'was not', 'wasn't'.

Given any pair of terms, it is easy to see that using pairs of a quantifier and a qualifier one could form four possible kinds of sentences (viz., the four classical categorical forms): universal affirmations, universal negations, particular affirmations, and particular negations. Each form is the result, we recall, of splitting the single formative, the copula, which connected the terms to form a categorical. Thus 'P belongs to every S' is now formed as 'Every S is P'; 'P belongs to some S' is now 'Some S is P'; 'P belongs to no S' is now 'No S is P'; 'NonP belongs to some S' is now 'Some S is nonP'. We have symbolized our unsplit formatives (copulae) by + and − signs. How shall we symbolize our formatives now that we have split them into quantifiers and qualifiers?

We are going to continue to use plus and minus signs as our only formative symbols. This means that our original unsplit copulae signs must now be rendered as pairs of signs (quantifiers and qualifiers). In the case of qualifiers, the choice of signs is natural. Affirmative quality can be symbolized by +; negative quality can be symbolized by −. Our choice of signs for the two quantifiers is now determined algebraically by the requirement to preserve commutativity for I categoricals and reflexivity and transitivity for A categoricals. Consider 'Some logicians are fools', which had been symbolized (with an unsplit formative) as F+L. Splitting the connective, and using + for the affirmative qualifier, we have the following first approximation: some L + F. Now, given that such a sentence can be commuted (i.e., its two terms can exchange places to yield a formally equivalent sentence), we have F+L = L+F. So, our particular quantifier must be symbolized in a way that will guarantee some L + F = some F + L. And clearly, only a + will do here: +L+F = +F+L ('Some logicians are fools' = 'Some fools are logicians').

Symbolizing the particular quantifier by + suggests that the universal quantifier be symbolized by −. And, indeed, the proper algebraic equivalence is preserved for transitivity by doing so. Consider 'Every logician is a philosopher, and every philosopher is wise, so every logician is wise'. The transitivity that 'every . . . is' inherits from 'belongs to every', and that guarantees this logical truth, can be preserved only by symbolizing the universal quantifier as a minus. Thus, 'Every logician is wise' is formulated as −L+W. It is easy to see that 'every . . . is' is nonsymmetric.

There is a second way to show the proper symbolism for universal quantifiers. Recall that our original system consisted of a binary plus and a unary minus. We then defined binary minus in terms of our binary plus and unary minus, and we defined a unary plus in terms of our unary minus. Our binary plus and unary minus were *elementary formatives*. Let us take the split version of our binary plus, along with our unary minus, as elementary as well, defining other split formatives in terms of them. Thus an I categorical and an O categorical can be formulated using only elementary formatives: Some S is P (+S+P), Some S is nonP (+S+(−P)). A and E forms are the contradictory negations of O and I forms, respectively. They can be formed by applying the unary minus to the entire sentence. So we can negate our I sentence, 'Some S is P', to give us 'Not: some S is P' (−[+S+P]), an E form. Likewise, we can negate our O sentence, 'Some S is nonP', to give us 'Not: some S is nonP' (−[+S+(−P)]), an A form. These defined forms are not natural, however. What is required is a method that will allow us to distribute these external minus signs into parenthetical expressions. And this is just what we have in arithmetic and algebra (e.g., −(2+3)=−2−3). Consequently, we will adopt

D.3 $\quad -X-Y \ =\text{df}\ -(+X+Y)$

Our E form can now be simplified, using D.3, to give us −S−P. Applying D.3 to our A form yields, first, −S−(−P), which, after then applying D.2, gives us −S+P. In effect, then, we have symbolized the universal quantifier by a minus, defined in terms of our elementary formatives.

Our new formulations for the categoricals closely match their natural-language counterparts. Each consists of a sign for quantity, a subject term, a sign for quality, and a predicate term.

A	E	I	O
Every S is P	No S is P	Some S is P	Some S is nonP
− S + P	− S − P	+ S + P	+ S + −P

It is important to keep in mind that a pair consisting of a quantifier and a qualifier is simply a split version of a logical copula, an unsplit formative.

Just as the formatives connecting pairs of terms for forming sentences have been split, we can split those formatives when they are used to form compound terms and sentences. Thus, just as we split 'belongs to some' to give us 'some . . . is', we can split 'and' to give us 'both . . . and'. And just as we defined 'every . . . is' in terms of 'some . . . is' and our unary minus, we can define 'if . . . then' in terms of 'both . . . and' and minus. So, in summary, we have the following formulations (with which we will take the liberty of using the traditional labels for forms having the same formal features).

A	E	I	O
Every S is P	No S is P	Some S is P	Some S is nonP
− S + P	− S − P	+ S + P	+ S + −p
If p then q	If p then not q	Both p and q	Both p and not q
− p + q	− p − q	+ p + q	+ p + −q

The parallelism between the categorical and compound forms here is striking, and hints at the possibility of a single algorithm for analysing inferences involving either kind of statement. For convenience, I shall continue to refer to the first part of any split connective as a quantifier and the second part as a qualifier. Thus, for example, we will talk of 'if' as a quantifier and 'then' as a qualifier.

So far, we have seen that our formatives can be used to express not only statements formed from simple terms (e.g., the categoricals), but also conjunctive and conditional compound terms and conjunctive and conditional compound statements. But if we are to build a system of logic that is as natural as possible, we cannot ignore disjunctive compounds. Consider a simple disjunctive statement of the form 'Either p or q'. This is the negation of 'Both not p and not q'. So we could initially formulate it as −[+[−p]+[−q]]. This formula, after we have algebraically distributed the external minus sign, yields −[−p]−[−q], or, more briefly, −−p−−q. (Keep in mind that the first and third minuses here constitute a split connective; the second and fourth are unary.) We will use the defined '−− . . . −− . . .' notation as a formulation of 'either . . . or'. For example, 'Some senator is a liberal or a democrat' could be formulated as +S+⟨−−L−−D⟩.

It is important to note that in introducing our splitting procedure we are not *reducing* 'every . . . is' to 'if . . . then' (nor 'some . . . are' to 'both . . . and') or vice versa. We are simply symbolizing each of these by an expression, – . . . + (and + . . . +), that has just those formal features shared by each formative. Compare this to the practice of others who tried to reduce compounds to categoricals. For example, De Morgan wrote, "In the forms of propositions, the copula is made as abstract as the terms: or is considered as obeying only those conditions which are necessary to inference" (1926: ix). Logicians such as Leibniz and De Morgan tried to place *all* of the burden of form on the "copula" (seen as equality, =) rather than on the logical copulae (e.g., Aristotle's or my split formatives).

Wild Quantity

I am now in a position to explain my earlier claim that 'Wise belongs to some Socrates' and 'Wise belongs to every Socrates' are equivalent. These sentences are in their unsplit, Aristotelian formats. Let us split the connectives here to give us 'Some Socrates is wise' and 'Every Socrates is wise'. My claim, then, is that these are, in effect, logically equivalent (the Leibniz-Sommers wild quantity thesis). When the subject-term of a sentence is singular, there is no logical difference between taking the quantifier to be particular and taking it to be universal. (Singular statements are simply particulars that semantically, nonformally, entail their corresponding universals. One might say that their "default" quantity is particular.) In natural language, this logical indifference to quantity for singulars is reflected in the fact that no quantifier at all is attached to singular subjects. Nonetheless, sentences with singular subjects enter into all kinds of logical relations with other sentences, and, as we will see, from a logical point of view they can be thought of as having whichever quantifier we want them to have.

Singular sentences are indifferent to quantity because they happen to share the formal features of both particulars and universals. For example, 'Socrates is wise' is like 'Some philosopher is wise' in that it is commutable. Just as 'Some philosopher is wise' is logically equivalent to 'Some wise (person) is (a) philosopher', 'Socrates is wise' is logically equivalent to 'Some wise (person) is Socrates'. This suggests that singular sentences are implicitly particular in quantity; thus '(Some) Socrates is wise' is, formally, +S+W. But, unlike particulars, singulars are both reflexive and transitive (like universals). Thus, just as 'Every human is human' is tautologous, so is 'Socrates is Socrates' tautologous. Moreover, just as 'Every logician is wise' follows from 'Every logician is a philosopher' and 'Every philosopher is wise', so does 'Socrates is wise' follow from 'Socrates is a philosopher' and 'Every philosopher is wise'. This suggest that singular sentences are implicitly universal in quantity. Thus: '(Every) Socrates is wise', formally: –S+W.

We have seen that singular terms can be subject-terms, which requires that they be logically quantified. We have seen as well that such quantity is not overt in natural language, but that logic requires that such

subjects have (as any logical subject must) some (at least tacit) quantity. And we have seen that there are reasons for taking the quantity of singular subjects to be indifferently either particular or universal. There is one further consideration that should strengthen our resolve to so treat singulars (and that will, in the long run, actually contribute to the simplicity of our system).

We have, so far, made no semantic distinctions among the terms fit for formulation by our symbolic algorithm. Any term, singular or general, mass or count, abstract or concrete, or whatever, can be symbolized by one of our letter variables for terms. And any term can be either quantified or qualified, so that any term can be a logical subject and can also be a logical predicate. In other words, our variables are semantically opaque. One might object at this stage that we have, after all, introduced the semantic singular/general distinction into our discussion. Yet it should be noted that we have recognized this distinction not by a distinction of variables but by a formal distinction, for, while the singular/general distinction is indeed semantic, its only effect on formal inference (i.e., its only logical effect) is syntactic (i.e., due to the indifference to quantity of singular terms when in subject positions). But this still leaves us with the fact that any kind of term can also be qualified, and thus become a logical predicate.

When singular terms are in predicate positions, the subjects of those sentences are usually singular as well. Sentences whose terms are both singular are no different, formally, from other sentences. They consist of a pair of terms connected by a formative. When the formative is split, one part of it is the quantifier and the other is the qualifier. Now, modern mathematical logicians have assumed what was not generally assumed before the late nineteenth century: that all predicate terms must be general (the other side of the Fregean Dogma). This assumption presents logicians with a problem when it comes to accounting for sentences in a natural language. Consider, for example, 'Shakespeare is Bacon'. Ignoring for now the problem of whether the subject is quantified, what is the logical form of the predicate? If predicate terms cannot be singular, then 'is Bacon' cannot be construed as a qualified term. Since 'Bacon' is undeniably a term (even though it is singular), the only option appears to be to deny that the 'is' in such a case is a qualifier. And this is exactly what the modern logician does. He or she claims that 'is' (and such words) is systematically ambiguous. Sometimes it is a genuine qualifier and other times it is itself a general term. It is a qualifier whenever it accompanies a general term, but when it accompanies a singular term, since that term cannot be the predicate term (and there must be a predicate term), 'is' itself must do duty as the predicate term. Thus 'is' in such cases must, contrary to all appearances, be a general term. In these cases, the 'is' is taken to be the " 'is' of identity" and generally read as a contraction of the expression 'is identical to', which is, in turn, without question a general term. 'Shakespeare is Bacon' is taken to be 'Shakespeare is identical to Bacon', and the general term here is appropriately symbolized by '=', which, as in mathematics, indicates the equivalence relation par excellence.

Needless to say, this logic, like traditional logics, makes no assumption about the semantics of predicate terms. Singular terms, like general terms, can be qualified and thus predicated. 'Shakespeare is Bacon' is construed, then, as a pair of terms connected by a formative. The formative in such a case has been split and the quantifier part suppressed. But such an analysis now presents a problem for us. So-called identity sentences are transitive, reflexive, and symmetric, all of which are guaranteed in a formal system that analyses them as pairs of singular terms connected by a general term that is itself an identity (equivalence) relation. How can our system preserve these features for such sentences? Since they have singular subjects, we are free to assign to them whichever quantifier we choose in any given context. By taking our tacit quantity to be particular, we guarantee that sentences like this are commutable (for all I categoricals are commutable—'belongs to some', 'some . . . is' are symmetric). By taking our tacit quantity to be universal, we guarantee (by the reflexivity and transitivity of 'belongs to every', 'every...is') the reflexivity and transitivity of sentences whose terms are both singular. In summary:

Symmetry:	(Some) Shakespeare is Bacon.
	So (some) Bacon is Shakespeare.
Reflexivity:	(Every) Bacon is Bacon.
Transitivity:	(Every) Shakespeare is Bacon.
	(Every) Bacon is Johnson.
	So (every) Shakespeare is Johnson.

We have symbolized the particular quantity by a plus and the universal quantity by a minus. Whenever the logical quantity of a formula is indifferently either particular or universal we will follow Sommers and indicate its quantity by * (e.g., 'Shakespeare is Bacon' becomes *S+B).

Before leaving the topic of symbolization, we can take an additional step toward the naturalization of our symbolic language. Sentences like

$$*S-P$$
$$+S-P$$
$$-S-P$$

(with split connectives) can be read as having the predicate 'isn't P'. But this is a contraction of 'is not P'. Is this 'not' binary or unary? Were we using a language like Latin, our question would be: Is the predicate to be read as 'non est P' or as 'est non P'? In other words, is the predicate to be parsed as $-(+P)$ or as $+(-P)$? As it turns out, the two are logically equivalent. The rule of obversion was the traditional logician's recognition of this equivalence. English, unlike Latin, allows us to suppress even the

appearance of a difference by contracting the 'is' and the 'not' to give 'isn't'. Consider: 'No human is immortal' is equivalent to 'Not a human is immortal', which equals 'Every human isn't (fails to be) immortal', which, in turn, is equivalent to 'Every human is mortal'. Symbolically:

$$-H-(-M) = -[+H+(-M)] = -H-(-M) = -H+M$$

Our system is further simplified and naturalized by two additional notational conventions. Consider the relational sentence 'A man gave a rose to a woman'. Here, the subject is 'a man' and the (complex) relational predicate is 'gave a rose to a woman'. The phrase 'gave a rose to a woman' consists of a logical predicate, the (complex) relational term 'gave a rose', and a subject-term, 'woman'. These are connected by the unsplit binary plus. This relational term is itself composed of a (simple) relational term, 'gave', and a subject-term, 'rose', and they are connected also by the unsplit binary plus. Fully symbolized, the sentence is:

$$+M_1+((+G_{12}+R_2)_{13}+W_3)_1$$

This formula is cluttered with several numerical subscripts. It can be simplified by adopting a pair of conventions. First, the final 1 is unnecessary (just as both subscripts are in $+F_1+B_1$ [= 'Some flowers are blue'] to yield +F+B). Such subscripts merely indicate which pair of terms constitutes a phrase. We will assume that for any well-formed phrase the two terms that constitute it must share at least one numerical subscript, which may be suppressed if unnecessary (as when there are no other subscripts or when one of the terms is relational). So our formula now becomes:

$$+M_1+((+G_{12}+R_2)_{13}+W_3)$$

The subscribed 1 and 3 indicate that the complex relation of having given a rose holds between a man and a woman. Thus, in effect, we are taking all relations to be binary, two-place. However, we can adopt an additional convention of amalgamating the subscripts of complex relations to yield relations of higher degrees. In other words, relational terms nested within relational terms are amalgamated, fusing their subscribed numerals so as to preserve order. Thus:

$$+M_1+((+G_{123}+R_2)+W_3)$$

Now we see 'gave' as a three-place relation. The convention gives us a new formula that is simpler and more natural. One further note before leaving this section: We assume that every well-formed phrase consists of a pair of (possibly complex) terms sharing a common (but sometimes implicit) subscript. We will soon specify a deduction rule that will permit simplification from complex formulae. For example, from our formula above we can derive any of the following:

1. $+M_1+(+G_{123}+R_2)$
2. $+M_1+(+G_{123}+W_3)$
3. $+G_{123}+R_2$
4. $+G_{123}+W_3$
5. $+M_1+G_{123}$

(where apparently extraneous subscripts, such as the 2 and 3 of the fifth formula, are ignored). But we cannot derive:

6. $+M_1+R_2$
7. $+M_1+W_3$

The derivable formulae can be read as:

1.1 A man gave a rose.
2.1 A man gave to a woman.
3.1 Something given was a rose.
4.1 Someone given to was a woman.
5.1 A man gave.

DON and EQ

Our system of split connectives provides us with a formal logic that is relatively natural, simple, and expressively powerful. Its ability to model in a perspicuous manner a wide variety of kinds of inferences will be a measure of its deductive power.

A natural deduction system consists primarily of a small set of rules for deducing the conclusion from the premises of valid arguments. If the arguments can be formulated in a single, abstract notational system, then the deduction system amounts to an algorithm for manipulating the symbolic representations of the premises in order to arrive at the symbolic representation of the conclusion. The notational system of variable letters and plus and minus signs was motivated by our desire to use an algebra-like algorithm for deduction. Indeed, I have already made use of a part of such an algorithm when, for example, I showed the equivalence of phrases with symmetric formatives after commutation of their terms (+S+P = +P+S).

The fundamental principle of this algorithm recalls the ancient law known as the *dictum de omni et nullo*. As we have seen, this has often been claimed (though not without challenge) as the underlying principle of traditional syllogistic reasoning. In effect, the law says that whatever is said of all or none of something is likewise said of what that something is said of. In other words, any term predicated (affirmed or denied) of a universal

subject (quantified term) is predicated in the same manner (affirmatively or negatively) of any subject of which the universally quantified term is predicated (viz., affirmed). Consider, for example, the inference

Every A is B
Every B is C
So every A is C

This inference satisfies the law and is valid. The term C, affirmed of the universally quantified term B (as in the second premise), is affirmed (in the conclusion) of the subject, 'Every A', of which that term, B, was affirmed of (in the first premise).

Consider next the inference

Some boy kissed a girl.
Every girl is a female.
So some boy kissed a female.

Here, what is affirmed of a universally quantified term ('female' in the second premise) is affirmed in the conclusion of the subject of which that quantified term ('girl') has been affirmed ('some boy kissed' in the first premise).

The *dictum de omni et nullo* applies directly to classical valid syllogisms and, as we saw, can be extended to apply to inferences involving relationals. But, as stated, it is hard to see how it can be extended to apply to all kinds of inferences. Nevertheless, a close inspection of the law and my examples reveals what Sommers saw, that the law really amounts to a rule of substitution. It says, in effect, and most generally now, that, given a sentence with a universally quantified term (subject or object) and another sentence in which that term is positively qualified, we can deduce a third sentence that is exactly like the second sentence except that the given term has been replaced by the first sentence minus the given term. In classical syllogisms, the given term is the middle term; the first sentence is the major premise; the second sentence is the minor premise; the second sentence minus the middle term is the minor term; and the first sentence minus the middle term is the major term. In effect, we substitute the major term for the middle term in the minor premise to get the conclusion. Thus we substituted C for B in 'Every A is B' to get the conclusion of our first example above, and 'female' for 'girl' in 'Some boy kissed a girl' to get the conclusion of our second example. This rule, allowing the substitution of one term for another in certain circumstances, always results in the cancellation of a term (viz., the middle term). The cancellation of terms suggests algebraic addition, as when we add 'a+b' and 'c−a' to get 'b+c', where pairs of oppositely charged terms of an addition are cancelled. This is exactly what happens in this logical algorithm. Middle-term pairs are oppositely charged and, so, can be cancelled. We symbolize the two inferences above as

1. $-A+B$
$-B+C$
$-A+C$

Notice here that the middle term, B, is positive in the minor and negative in the major. We cancel these two, in effect, adding the premises to get the sum—the conclusion. For a valid inference, it all adds up.

2. $+B_1+(+K_{12}+G_2)$
$-G+F$
$+B_1+(+K_{12}+F_2)$

In this case, as in 1, the application of the substitution law amounts to adding the premises algebraically (i.e., cancelling middle terms) to get the conclusion as sum.

Consider now a slightly more difficult inference.

No A is B
Some B is C
So some C is not A

Before applying the dictum directly, we would have to commute both the major and minor premises. Once we recognize the law as merely a rule of algebraic addition, however, we merely need to symbolize and add:

3. $-A-B$
$+B+C$
$+C-A$

But in carrying out this addition we see the need for a logical restriction. Notice that we could have added our two premises to get $-A+C$ ('Every A is C'), which does *not* follow from our premises. $+C-A$ and $-A+C$ are algebraically equal, but they are definitely not logically equal. What we require is a further restriction on premise addition to guarantee validity. The equivalence of the conclusion with the algebraic sum of the premises is a necessary but not sufficient condition for validity. A second requirement, which is also a necessary condition for validity, will, conjointly with the other restriction, be a sufficient condition for validity.

Keep in mind that our split binary connectives will be said to consist of two parts: the quantifier and the qualifier, and we will continue to use this terminology even for split binary connectives that combine with phrases and sentences as well as with simple terms. We can now say that our second necessary condition for the validity of any inference is this: the

number of conclusions with particular quantity must be the same as the number of premises with particular quantity. It follows that in the case of logical equivalence the two statements must not only be algebraically equal but must have the same quantity as well. From now on, we will refer to the necessary and sufficient conditions for validity as EQ. E reminds us of the algebraic equivalence condition; Q reminds us of the quantity condition.

> EQ: A conclusion follows validly from a set of premises if and only if (1) the sum of the premises is algebraically equal to the conclusion and (2) the number of conclusions with particular quantity (viz., zero or one) is the same as the number of premises with particular quantity.

The principle EQ amounts to a definition of 'validity' and accounts for classical conversion, obversion, contraposition, and all valid categorical and hypothetical syllogisms. We can use it, in effect, as a decision procedure for determining validity. Thus, with our example 3 inference above we now know that $+C-A$, but not $-A+C$, follows from the premises. Both formulae satisfy the equality condition, but only the former also satisfies the quantity condition.

Rules of Inference

Once an inference has been determined to be valid, what is next required is a *proof* of that validity. As we have said, the fundamental principle of our algorithm for proving validity is the *dictum de omni et nullo*. We will see what role it plays in proving validity shortly.

Let us say that *a proof is a finite sequence of formulae such that the first n (n≥0) formulae in the sequence are the premises, the last formula is the conclusion, and every formula is justified by as least one rule of inference*. In the algorithm offered here, there are two kinds of inference rules. Rules that permit the creation of a new formula on the basis of a single previous formula are Rules of Immediate Inference; rules that permit the creation of a new formula on the basis of a pair of previous formulae are Rules of Mediate Inference. The main rule of mediate inference will be the *dictum de omni et nullo*, also often called the Rule of Syllogism. As is normally the case for systems of proof, each of these rules is nothing more than a simple and obvious pattern of valid inferences. And, as is usual, there is no restriction on the number of rules in this system. But too many rules, while making proofs short, will fail to model perspicuously the ways we ordinarily deduce conclusions from premises; too few rules, while making each step in a proof perspicuous, render proofs too long to be practical. I, like others, seek an optimal number of rules; as we shall see, the number can be kept relatively low because, unlike the standard algorithm now in general use, this one recognizes the common formal features of statements composed of simple terms as well as those composed of subsentences. Any subsentence is a sentence; any sentence is a phrase; any phrase is a (complex) term. This is a term logic. A single set of rules will suffice for reckoning inferences involving all kinds of statements. A further consideration in choosing these rules is my intention to preserve as many of the correct logical insights of traditional logic as possible.

Before presenting the rules of inference, I will define 'tautology' in my system and show how tautologies can occur in proofs. I define a tautology as follows: *any universally quantified formula that is algebraically equal to zero is a tautology*. Note that we could think of a tautology as the conclusion of a valid zero-premise argument. Such an inference satisfies EQ; the algebraic sum of the premises is zero and so is the conclusion, and the number of particular conclusions (0) is the same as the number of particular premises (0). In general, statements of the universal affirmation form −X+X will be tautologies. Obviously, the negation of any tautology will be a contradiction. A contradiction is a particularly quantified formula that is algebraically equal to zero.

I begin with the Rules of Immediate Inference. Such rules allow the creation of a new formula on the basis of a single previous formula in the proof sequence.

Rules of Immediate Inference

Premise (P): Any premise or tautology can be entered as a line in proof. (Tautologies that repeat the corresponding conditional of the inference are excluded. The corresponding conditional of an inference is simply a conditional sentence whose antecedent is the conjunction of the premises and whose consequent is the conclusion.)

Double Negation (DN): Pairs of unary minuses can be added or deleted from a formula (i.e., recalling D.2, −−X=X).

External Negation (EN): An external unary minus can be distributed into or out of any phrase (i.e., recalling D.3, −(±X±Y)=∓X∓Y).

Internal Negation (IN): A negative qualifier can be distributed into or out of any predicate-term (i.e.,±X−(±Y)=±X+(∓Y)).

Notice that we need different rules here, because external negation is term negation while internal negation is actually negative quality. External negation is unary; internal negation is part of a split binary formative. The first minus of −[+S+P] is a (sentential-)term negation. So we have: −[+S+P] = −S−P. But the minus of +S−P is a qualifier. Thus: +S−P = +S+(−P) = +S−(+P). Consequently, we have a rule for distributing term

negation *and* a rule for "amalgamating" a qualifier and the unary formative of the following term—namely:

...+(−...) = ...−..
...−(+...) = ...−...
...+(+...) = ...+...
...−(−...) = ...+...

Commutation (Com): The binary plus (split: +...+) is symmetric (i.e., +X+Y=+Y+X).

Association (Assoc): The binary plus (split: +...+) is associative (i.e., +X+(+Y+Z)=+(+X+Y)+Z).

Contraposition (Contrap): The subject- and predicate-terms of a universal affirmation can be negated and can exchange places (i.e., −X+Y=−(−Y)+(−X)).

Predicate Distribution (PD): A universal subject can be distributed into or out of a conjunctive predicate (i.e., −X + ⟨+Y + Z⟩ = +[−X + Y] + [−X + Z]) and a particular subject can be distributed into or out of a disjunctive predicate (i.e., +X + ⟨−(−Y) − (−Z)⟩ = −−[+X + Y]−−[+X + Z]).

Iteration (It): The conjunction of any term with itself is equivalent to that term (i.e., +X+X=X).

Notice that the traditional rules of conversion are preserved in the rules above. I turn now to the rules of mediate inference.

Rules of Mediate Inference

Dictum de Omni et Nullo (DON): If a term, M, occurs universally quantified in a formula and either M occurs not universally quantified or its logical contrary occurs universally quantified in another formula, deduce a new formula that is exactly like the second except that M has been replaced at least once by the first formula minus its universally quantified M.

Simplification (Simp): Either conjunct can be deduced from a conjunctive formula; from a particularly quantified formula with a conjunctive subject-term, deduce either the statement form of the subject-term or a new statement just like the original but without one of the conjuncts of the subject-term (i.e., from +⟨+X+Y⟩±Z deduce any of the following: +X+Y, +X±Z, or +Y±Z), and from a universally quantified formula with a conjunctive predicate-term deduce a new statement just like the original but without one of the conjuncts of the predicate-term (i.e., from -X±⟨+Y+Z⟩ deduce either -X±Y or -X±Z).

Addition (Add): Any two previous formulae in a sequence can be conjoined to yield a new formula, and from any pair of previous formulae that are both universal affirmations and share a common subject-term a new formula can be derived that is a universal affirmation, has the subject-term of the previous formulae, and has the conjunction of the predicate-terms of the previous formulae as its predicate-term (i.e., from -X+Y and -X+Z deduce -X+⟨+Y+Z⟩).

Note that Add incorporates, in part, It. For example, from -X+Y and -X+Z one can deduce -X+⟨+Y+Z⟩, which, by It, equals -⟨+X+X⟩+⟨+Y+Z⟩.

It should be noted that DON accounts for all valid first-figure syllogisms (and much more besides). As I said earlier, DON amounts to a rule of substitution (the predicate-term of a universal statement can be substituted for the subject-term of that statement in any other statement in which that subject-term occurs positively). The subject-term just mentioned is the middle term of traditional syllogistic. In effect, DON permits the cancellation of middle terms. The same rule accounts not only for syllogistic inference, but for such an inference as *modus ponens* and for instances of Leibniz's Law as well. That modern logic must make use of three different rules for these three kinds of inferences while this logic needs only one is due to the fact that I, unlike most modern logicians, take categoricals (including relationals and singulars), compound statements, and "identity statements" as all sharing a common logical syntax—each is viewed as a pair of (possibly complex) terms connected by a binary connective/formative (split or unsplit).

I will now show how DON operates in these three kinds of cases and then give some sample proofs to illustrate all of the rules spelled out above. First, consider Cesare. Its premises are symbolized as -M-P, -S+M. The conclusion, -S-P, follows directly by DON since -P replaces M in the minor premise (a statement in which M occurs positively, not universally quantified).

Consider next a slightly more complex inference. 'Every animal runs from a bear. All bears are carnivores. Hence, every animal runs from some carnivore.' The premises are formulated as $-A_1+(+R_{12}+B_2)$, -B+C. The middle term here is B, which occurs universally quantified in the second premise and positively in the first. Thus, by DON, C can replace B in the first premise to yield the conclusion $-A_1+(+R_{12}+C_2)$. Before going on, it should be noted that the usual proof for this simple inference using the standard predicate calculus requires twelve steps beyond the premises, makes use of such rules as conditional proof, addition, simplification, *modus*

ponens, universal instantiation, existential instantiation, universal generalization, and existential generalization. Now, the original argument is extremely simple, and virtually any rational person can draw the appropriate conclusion from the given premises quickly and with very little effort. A system of logic making use of a rule like DON can at least claim some degree of psychological reality. The standard system now in place cannot, does not, and would not.

A general *modus ponens* inference can be formulated as -p+q, +p, therefore +q. Again, the conclusion can be seen to follow directly by DON. Here, the middle term is p, which occurs quantified universally in the first premise (recall that we have agreed to call the first part of any split binary connective a quantifier) and positively in the second premise. DON simply allows us in such cases to substitute the predicate of the first premise for p in the second premise. An alternative account of *modus ponens* (following Sommers's method of incorporating statement logic into term logic) would treat the second premise and conclusion as having as their subjects the singular term 'the (actual) world', *W. Thus: -p+q, *W+p, therefore *W+q, a Barbara or Darii syllogism. Recall that a formula like *W+p can be read as 'the (actual) world is a p-world'. Notice that not only *modus ponens* but rules of the sentential calculus such as *modus tollens* and chain argument are also merely instances of DON.

Finally, consider Leibniz's Law. This rule is explicitly a rule of substitution. It says that from premises of the general form 'a is identical to b' and 'b is so-and-so' one can derive the conclusion 'a is so-and-so'. The rule embodies the notion that when two terms stand in an identity relation one can be substituted for the other in any (nonintensional) statement in which the other is used. In other words, one can derive 'Pa' from 'a=b' and 'Pb'. Nonetheless, this rule is merely another instance of DON. We formulate the premises as *a+b, *b+P. The middle term here is b, which (given that it is singular) is indifferent with respect to quantity in the second premise, and, so, allows us to regard it as universally quantified. It occurs positively in the first premise. So, by DON, the conclusion, *a+P, follows directly, since we can simply substitute the term predicated of universal b for b in the other premise.

I offer now a few sample proofs using the system outlined above.

From 'No P is M' and 'Every S is M' derive 'No S is P':

1.	-P-M	P
2.	-S+M	P
3.	-[+P+M]	1, EN
4.	-[+M+P]	3, Com
5.	-M-P	4, EN
6.	-S-P	2, 5, DON

From 'Every circle is a figure' derive 'Every drawer of a circle is a drawer of a figure':

1.	$-C+F$	P
2.	$-(D+C)+(D+C)$	P (tautologous assumption)
3.	$-(D+C)+(D+F)$	1, 2, DON

From 'Every boy loves some girl', 'Every girl adores some cat', 'All cats are mangy', and 'Whatever adores something mangy is a fool' derive 'Every boy loves a fool':

1.	$-B_1+(+L_{12}+G_2)$	P
2.	$-G_2+(+A_{23}+C_3)$	P
3.	$-C+M$	P
4.	$-(+A_{23}+M_3)+F$	P
5.	$-G_2+(+A_{23}+M_3)$	2, 3, DON
6.	$G+F$	4, 5, DON
7.	$-B_1+(+L_{12}+F_2)$	1, 6, DON

The traditional term logic failed to hold the field against the new logic introduced by Frege. For the most part, this was due to its inability to offer adequate analyses for three kinds of inferences—those involving singulars, relationals, and compound sentences. We have seen that this disadvantage in inference power was not inherent in term logic. Sommers, following suggestive hints from his pre-Fregean predecessors, has built a new version of the old logic of terms. As we have also seen, it enjoys the same advantages of expressive and inference power as does the Fregean logic. Indeed, its powers here actually exceed those of the Fregean system, for just as the old logic was faced with three kinds of inferences beyond its capacity, the new Fregean logic is faced with three kinds of inferences beyond its scope.

Consider the simple inference 'Plato taught Aristotle. So Aristotle was taught by Plato'. The standard system formulates this inference as

$$Tpa \ / \ Tpa$$

Unschooled intuition, as well as grammar, sees the conclusion as semantically equivalent but syntactically distinct from the premise. We naturally take a sentence in the active voice and its "passive transform" to be different sentences even though they share the same truth conditions. Taking logical form as nothing but the revelation of truth conditions, Quine has said, "The grammar that we logicians are tendentiously calling standard is a grammar designed with no other thought than to facilitate the tracing of truth conditions. And a very good thought this is" (Quine, 1970: 35-36). Frege had held the same view, and on the basis of it he dismissed the

active/passive distinction. His claim was that since they "express the same thought" the grammatical difference between them "is of no concern to logic" (Frege, 1979: 141). The present system formulates the arguments as

$$*P_1+(+T_{12}*A_2) \ / \ *A_2+(+T_{12}*P_1)$$

It turns out that though the premise and the conclusion entail one another, the two are formally distinct. The transformation from active to passive (or vice versa) is accomplished in this case by an application of Com (twice) and Assoc.

A second type of inference beyond the scope of standard predicate calculus is represented by the argument 'Socrates taught a teacher of Aristotle. So one whom Socrates taught taught Aristotle'. The best that mathematical logic can do in terms of formulation is

$$(\exists x)(Tsx \ \& \ Txa) \ / \ (\exists x)(Tsx \ \& \ Txa)$$

Again, the standard system is powerless to exhibit the formal difference between the premise and the conclusion. In the present system, the inference has the form

$$*S_1+(+T_{12}+(+T_{23}*A_3)) \ / \ +(*S_1+T_{12})+(+T_{23}*A_3)$$

Modern grammarians call this a case of "associative shift." While modern logicians see it as a trivial reiteration, this logic recognizes the formal distinction between premise and conclusion. The conclusion is derived by an application of Assoc.

The third kind of inference that challenges the Fregean logician is represented by the following example: 'Plato taught Aristotle with a dialogue. So Plato taught Aristotle.' The standard formulation is

$$(\exists x)(Dx \ \& \ Tpax) \ / \ Tpa$$

The two relational predicates are distinct; one is a three-place function ('. . . taught . . . with . . .'); the other is a two-place function ('. . . taught . . .'). For the inference to be valid there must be a hidden assumption of an analytic, semantic tie between these two predicates (like the one between, say, 'bachelor' and 'unmarried'). The formalization below retains a more natural syntax and preserves the common-sense view that 'taught' is univocal throughout its two uses here.

$$*P_1+((+T_{123}*A_2)+D_3) \ / \ *P_1+(+T_{123}*A_2)$$

(The subscribed numerals here are not to be confused with bound variables of the predicate calculus. The latter simultaneously keep track of reference and the order of subjects and objects with respect to interpretations of relational predicates. The subscribed numerals perform only the second of these tasks.) The inference proceeds by the application of Assoc and Simp.

The standard logic is essentially mute in the face of inferences involving passive transformations, associative shifts, or simplifications with polyadic predicates (relationals). I conclude this section with an example of a simple inference that is beyond the scope of the standard system in all three ways: 'A man loves a woman. So some lover is a man.' My proof is

1.	$+M_1+(+L_{12}+W_2)$	P
2.	$+(+M_1+L_{12})+W_2$	1, Assoc
3.	$+W_2+(+M_1+L_{12})$	2, Com
4.	$+W_2+(+L_{12}+M_1)$	3, Com
5.	$+L_{12}+M_1$	4, Simp

Notice that each of the intermediate lines in the proof "makes sense" in natural language. Thus:

2. What some man loves is a woman.
3. A woman is what some man loves.
4. A woman is loved by some man.

Names and Other Pronouns

> *First, it is of fundamental importance to grasp that the properness of proper names is a feature—in Saussurean terms—of 'parole', not of 'langue'.*
>
> L.J. Cohen

> *Augustus, meeting an ass with a lucky name, foretold himself good fortune; I meet many asses but none of them have lucky names.*
>
> Swift

Singular terms, especially names and personal pronouns, are a prominent feature of natural-language discourse. Pronouns, in the guise of individual variables, play an important role in the formal language of the predicate calculus. Even Quine's Predicate Functor Algebra, which eliminates such variables, is meant to reveal just what their roles are in the calculus. I have followed Leibniz and Sommers in giving them wild quantity when used as subject-terms. Whatever grounds exist for distinguishing names and pronouns from general terms are semantic ones. Although my path has been, in contrast to the Fregeans, to degrade this distinction in the building of a formal language, the semantics of such terms demands our special attention.

There is both a classical and a modern semantic theory for the standard predicate logic. The former holds that all singular expressions

refer, and that reference is determined by the sense of those expressions. Since general terms are terms with sense, pronouns and names are replaceable by appropriate definite descriptions, whose senses are determined by the senses of the general terms that occur in them. The classical semantics is most closely associated with Frege and Russell. The more modern theory, associated particularly with Kripke and Putnam, holds that a distinction must be made between singular expressions that are "rigid" and those that are not. Pronouns and names are rigid; definite descriptions are flaccid. The former, unlike the latter, do not have their references determined by their senses. Rigid designators refer directly to their referents without detour through senses or meanings. Thus a rigid designator refers to or designates the same object in all of its referential uses (cf. Salmon, 1986). Since the new theory is usually part of a possible-worlds semantics, it is said that a rigid designator refers to the same object in every possible world in which that object exists. A logic of terms requires something like the rigid/nonrigid distinction. But it need not accept the entire modern theory of semantics. It especially eschews possible worlds.

My semantic theory recognizes at least two levels of reference for terms: *denotation* and *reference* (proper). In the normal, nonvacuous case, every term used in a statement, whether the term is charged positively or negatively, has a denotation determined by its *signification* and the *domain* (of discourse) relative to which the statement is made. What a term signifies is a *property*. For example, 'red' signifies the property redness, 'wise' signifies the property of wisdom, 'pious' signifies piety, 'nonsquare' signifies nonsquareness. Whatever is denoted by a term has the property signified by that term. The converse need not hold. Every used term is used in a used (statement-making) sentence, or statement. Every such sentence is used relative to some specifiable domain of discourse. A domain is a nonempty totality of compossible objects. Ordinarily our domain of discourse is simply the actual world. But any world, any part of the actual world, or any set of objects could serve as a domain. When we say 'Some man held a horse on his shoulders' our domain is, ordinarily, the actual world. When we say 'Some man held the Earth on his shoulders' our domain is, presumably, the world of Greek mythology. A term used in a sentence denotes the objects that have the property signified by that term and that are in the domain relative to which the sentence is used. We can think of denotation, then, as the intersection of a domain and the set of objects having a given property. We have seen that normally each term used in a sentence has a denotation. Now, each term used in a sentence is either quantified (i.e., it is a logical subject) or qualified (a logical predicate). Subjects have a mode of "reference," however, not shared by predicates. Every term used in a sentence, whether a subject-term or a predicate-term, has a denotation, but only a subject (quantified term) refers (in the proper sense). The referent of a subject is determined by its quantity and the denotation of its term. A universally quantified subject refers to the entire denotation of its subject-term; a particularly quantified subject refers to some (possibly specifiable) part (perhaps the whole) of the denotation of its subject-term. So, while predicate-terms may denote, they do not refer.

Thus far, no distinction has been made between singular and general terms. In the standard predicate calculus the distinction is all-important. A general term used in a statement makes reference (denotatively) to one or more individuals. A term whose denotation is either unique or not unique to individuals is not a general term. A term whose denotation is not individual is a mass term (e.g., 'wood', 'water', 'wool', 'wine'). Mass terms, those with nonindividual denotation, and singular terms, those with unique individual denotation, contrast with general terms, those with nonunique individual denotation, in having tacit quantity when used as subjects. When a mass term is explicitly quantified it is because the term is being used with an understood, thus implicit, phrase such as 'sample(s) of', 'piece(s) of', 'drop(s) of', 'chunk(s) of', and so on. For example, 'Water is dripping in the sink' is understood as 'Some drops of water are dripping in the sink', and 'Wood is combustible' is understood as 'All samples of wood are combustible'. Terms like 'drops of water' and 'samples of wood' are not mass but general. Singular terms denote uniquely; they denote just one individual object. A singular subject (a logical subject whose term is singular) refers to the object denoted by its term. The implicit logical quantity of a singular subject is always understood to be particular. The canonical form of 'Socrates is wise' is '(Some) Socrates is wise'. Particularly quantified subjects make undistributed reference to the denotations of their terms. Universally quantified subjects refer distributively; they refer to the entire denotations of their terms. Since we know the subject of a given sentence to be singular (with tacit particular quantity), we can infer a corresponding universal sentence from it. This, again, is the wild quantity thesis. Such inferences are informal, depending as they do on our extra-logical knowledge of the denotations of the subject-terms.

We saw above that the denotation of a term is in part determined by its signification. What a term signifies is a property. The term 'red' signifies the property of redness and 'wise' signifies wisdom. Let [T] be the property signified by the term 'T'. Thus, 'red' signifies [red] (= redness) and 'wise' signifies [wise] (= wisdom). A singular term such as 'Socrates' signifies a property as well. The term 'Socrates' signifies [Socrates] (= the property of being Socrates, Quine's *Socratizer*). Indeed, [Socrates], [wise], [Greek], [teacher of Plato], and [philosopher] are some of the properties that Socrates has. Many things have the property [Greek]; not so many things have the property [wise]; a small number of things have the property [teacher of Plato]; just one thing has the property [Socrates]. As long as Socrates belongs to our domain of discourse we will denote Socrates by 'Socrates', since he is what has the property [Socrates]. Indeed, in any domain in which Socrates is located, the use of 'Socrates' will denote (and, when in subject position, refer to) Socrates, for the signification of

'Socrates' is invariable from one domain to another. Names are rigid. As subject expressions, their references are immutable.

Definite descriptions have the form 'the x', where 'x' is a (usually complex) general term. While 'x', as used in a sentence relative to a specifiable domain, denotes all the objects in the domain that have [x], if there is but one such object the term 'the x', similarly used, will denote it. As subject expressions, definite descriptions are not rigid in reference because their constitutive general terms have denotations that may vary from one domain to another. Thus, for example, 'man who fought a duel', when used relative to the domain of *Hamlet*, denotes, among other things, Hamlet but not Hamilton. The same term, used relative to the actual world, denotes, among other things, Hamilton but not Hamlet. Likewise, the use of 'the man who fought a duel' in subject position may refer on one occasion to Hamlet and on another to Hamilton. In summary, since the denotation of a used term is determined in part by its signification (which is invariable) and in part by the domain relative to which the sentence in which it is being used is used (which is variable), its denotation is variable from domain to domain. Definite descriptions are variable in denotation, as the denotations of their constituent general terms are variable. Names are invariable (i.e., rigid) from domain to domain only because the properties that they signify cannot (as those signified by general terms can) be possessed by different objects in different domains. Consider again the term 'man who fought a duel'. Here, it is possible to have two different domains relative to which the denotation of the term is nonempty, yet which are such that the two sets of objects constituting the denotation are disjoint. In contrast, consider 'London'. Here, given any two domains, if the denotation of 'London' relative to each is nonempty it can only be because London is in both (compare the actual world and the world of Sherlock Holmes). So names, but not definite descriptions, are rigid. What of pronouns?

Here, roughly, is one version (Kripke, 1972/80) of how names are introduced into discourse (the so-called causal theory). Names are introduced in an initial "baptismal ceremony" ('I name this child "Socrates",' 'I dub thee "Sir Lancelot",' 'Let's call this place "London",' 'Henceforth this bridge will be called "Pons Asinorum" '). A subsequent user of the name, in the normal course of events, will use that name to designate (denote) the same object only if such use can, in theory, be traced back through intermediary uses, ultimately ending in the initial baptism. As long as the intention of each user was to denote just the object that the preceding user intended to denote, the denotation from the baptism to the present use is constant. Constancy is preserved by a uniformity of shared intentions (to denote). Uses of a name to refer to the same object on different occasions are links in a chain of intentions.

Our theory, derived from Sommers, holds that nominal subjects are simply links in anaphoric chains. Indeed, names, so used, are just special duty pronouns, "pro-pronouns." And generally, most links in an anaphoric chain are pronominal. But how do chains get started? And how are their links bound together? On this theory, initial reference to an object must always be by the (sometimes implicit) use of an indefinite description. We say (or assume) such statements as 'A child is born', 'Here is a newly discovered mountain', 'There is a star'. These sentences have the overall form 'Some X is Y', where 'some X' is indefinite but makes specific reference. (Compare: 'Some man is at the door'—specific—and 'Some man will be the first to walk on Pluto'—nonspecific.) Subsequent reference to the object is pronominal: 'A child is born. She is beautiful.' According to Sommers's theory, pronominal subjects are, like any subject, logically analysable as quantified terms. As singulars they enjoy wild quantity. The denotation of the term of a pronominal subject, the "pro-term," is determined in part by the reference of its antecedent. In our example the antecedent of 'she' is 'a child'; the denotation of the pro-term is also partly determined by the ascription of its antecedent sentence. The antecedent sentence ascribes the property of being born to the referent of its subject. The pro-term of the subsequent anaphoric pronoun, then, denotes a child who is born. The reference of the pronoun is determined by its implicit quantity and the denotation of its pro-term. Since it denotes *all* of what the antecedent refers to, it has universal quantity, but it also inherits the quantity of its antecedent. So, in effect, a pronoun whose antecedent is particular (the normal case) has wild quantity. As Quine has said, "[Pronouns] may have indefinite singular terms as antecedents but they can be supplanted only by definite singular terms" (1976: 46). In our example, 'she' refers to the child who is born. In fact, we could make this explicit by using the definite description 'the child who is born', where the definite description is anaphoric. Thus: 'A child is born. She (or: the child who is born) is beautiful.' And we could go on: 'She has red hair.' In this last statement, 'she' refers to the child who is born and is beautiful, as ascriptions are accumulated from link to link in an anaphoric chain.

We have yet to introduce names into these chains. Thus far, the initial link in a chain is an (often implicit) indefinite description having the logical form 'some X'. Subsequent links are anaphoric, picking up the referents and ascriptions of their antecedents. These links are pronominal or, when the ascriptions are explicit, definite descriptions. Sometimes, though far from always, we make such frequent reference to an object or have such interest in it that, by fiat or custom, we create an expression whose job is to refer specifically to just one object. These expressions are names. We name an object in what Kripke calls a baptismal ceremony. But our introduction of the name there is not the initial link in an anaphoric chain containing uses of that name. Names are (nonvariable) pronouns. Nominal reference is a special kind of pronominal reference.

Names are usually introduced after (or perhaps at the same time as) pronominal reference has been made to the object named. Thus: 'A child is born. She is beautiful. She has red hair. Let's call her Lucy.' Even

where no intervening pronominal links occur between the initial link and the nominal one, we cannot say that the naming of an object is the initial link in an anaphoric chain. Suppose upon the birth of a child I say, 'Let's call her (this, that, it) Lucy'. On such an occasion, the object being named has been picked out by 'her'; and this pronominal reference must be accompanied by an indefinite reference, such as 'a child', in an implicit statement, such as 'A child is born', or 'Here is a child'. Names, then, are like any other pronouns in that when used as subjects their denotations are determined by the referents of their antecedents and the accumulated ascriptions, and, as singular subjects, they have wild quantity. Names differ from other pronouns in that they are introduced only on special occasions to make anaphoric reference to just one specified object. We could do without names (as we usually do in the case of cows and crows). Names are special-duty pronouns—they are pro-pronouns. Names, then, are rigid because they are referentially pronominal; and pronouns are always rigid, in the sense that they are always used to refer to the same object on each occasion of their use in a given anaphoric chain. This is guaranteed by the fact that each pronominal pro-term in a chain is determined by its antecedent's reference and the accumulated ascriptions. These are what bind pairs of links in order to form a cohesive chain. This theory of rigidity contrasts favourably with the causal theory, which binds links in a referential chain by means of common intentions on the part of users of those links.

We have seen that for the Fregean the difference between singular terms and general terms is that the former can never be used as functions (or predicates, as they are still called). According to the Asymmetry Thesis, this difference is of the greatest logical import, so let us briefly review this thesis. According to defenders of Asymmetry: (i) a sentence can be negated by negating its predicate but not by negating any of its arguments (singular terms), (ii) a pair of sentences can be conjoined or disjoined by conjoining or disjoining their predicates but not by conjoining or disjoining their arguments. Thus, according to the thesis, 'Socrates drinks' is negated by 'Socrates is not drinking' (or 'Socrates does not drink'), but not by 'NonSocrates drinks'; 'Kant is serious' is negated by 'Kant is nonserious' (or 'Kant is not serious'), but not by 'NonKant is serious'; 'Socrates eats' and 'Socrates drinks' can be conjoined as 'Socrates eats and drinks', but 'Plato and Aristotle carried the piano' is not a way of conjoining 'Plato carried the piano' and 'Aristotle carried the piano'. A corollary of (i) is that singular terms do not have negations. A corollary of (ii) is that there are no logically compound singular terms. Finally, there is the basic, underlying claim of the Asymmetry Thesis: (iii) singular terms are never predicated.

Many twentieth-century analysts have offered a variety of arguments intended to support the various tenets of the Asymmetry Thesis (we saw in chapter two how Geach and Strawson were prominent among these philosophers). Here are the outlines of some of the more important or common arguments.

(1) Names cannot be negated, because if they could be there would have to be negative objects that negative names enote. But there cannot be such objects.

(2) Names cannot be compounded, because if they could be there would have to be compound objects that such names denote. But there cannot be such objects.

(3) All negation is logically sentential. Any colloquial form of negation that cannot be construed as sentential is not genuine (logical) negation. Name negation cannot be construed as sentential, it is not genuine.

(4) All compounding is logically sentential. Any colloquial form of compounding that cannot be so construed is not genuine. Name compounding is not sentential, so it is not genuine.

(5) All logical predicates are general terms (adjectives, verbs, common nouns, etc.). Names are not general terms, so they are never logical predicates.

Each of these arguments rests on the conclusion of an unstated argument:

(6) All logical arguments (subjects) are singular. Negated and compound names are either singular or general. If they are singular, then they denote impossible objects. If they are general, they are not singular. Therefore, such terms cannot be logical arguments. No logical subjects are negated or compound.

The first premise of (6) is the Fregean Dogma. It is essential to the Asymmetry Thesis.

Some of the claims made in the above arguments cry out for examination. Defenders of the Asymmetry Thesis hold that a name like 'Socrates' cannot be negated because the result, 'nonSocrates', would have to name an impossible object. The reasoning here is that any object denoted by 'nonSocrates' would have to lack all of the properties that Socrates has and have all of the properties that Socrates lacks. But, since the properties that Socrates lacks are not compossible, it is impossible for an object to have all such properties. Thus, such an object is impossible. Suppose Socrates is white, Greek, and male. By the above reasoning, nonSocrates must be nonwhite, nonGreek, and female. So far so good. However, the argument further requires that nonSocrates have all the properties that Socrates lacks. Well, Socrates lacks the properties of being green, red, black, Roman,

French, Japanese, and female (and many other properties besides). NonSocrates may be a black Japanese female, or a yellow French female, or a green Roman female, and so on, but it/she cannot possibly be all of these. NonSocrates, so the argument goes, is impossible—so 'nonSocrates' is not a genuine name.

The Asymmetrist's argument rests on a simple logical fallacy involving De Morgan's Laws. The negation of a conjunction is a disjunction of negations, not, as per the argument above, a conjunction of negations. Consider: Socrates is white. Thus he lacks the properties of being green, red, and black, and so on. In other words, Socrates is nongreen, nonred, and nonblack. What follows with regard to nonSocrates, then, is not that it/she is (per impossible) green, red, and black, but rather that nonSocrates is not nongreen, nonred, and nonblack—that is, non-Socrates is *either* green *or* red *or* black, etc.

There are at least two sources of the confusion over negative names. One is lack of clarity concerning the distinction between properties that an object lacks and properties that do not apply to that object. The number 2 is even and lacks the property of being odd, but it neither has nor lacks the property of being green. The term 'green' simply does not apply to (span, in Sommers's terminology) the number 2. Likewise, Socrates may be white and nonRoman, but he neither has nor lacks the property of being Marxist. Consequently, whatever properties an object named by 'N' has, nonN can neither have nor lack any properties that N neither has nor lacks. 'Marxist' does not apply to Socrates—so it does not apply to nonSocrates either. Whatever nonSocrates might be, it/she is the same sort of thing (i.e., has exactly the same terms applicable to it, is spanned by all the same terms, belongs to the same categories) as Socrates is.

The second, and perhaps more fundamental, source of confusion is the refusal of standard mathematical logicians to recognize the distinction between term negation and sentential negation. We have seen that the logical contrary of a term is semantically equivalent to the disjunction of all terms incompatible with that term. For example, the *non*logical contraries of 'white' are 'red', 'green', 'yellow', 'pink', 'black', and so on. 'Male' has 'female' (normally). 'Greek' has 'Roman', 'French', 'Japanese', and so on. 'Six feet tall' would have infinitely many nonlogical contraries (e.g., 'four feet tall', 'six feet one inch tall', 'six feet two inches tall'). Sentence pairs that differ only in that their predicate-terms are nonlogical contraries are themselves nonlogical contraries. Sentence pairs that differ only in that their predicate-terms are logical contraries are themselves logical contraries. The nonlogical contrary of a given sentence entails the logical contrary of that sentence. The converse need not hold. The logical contrary of a given sentence entails the contradictory of that sentence. The converse need not hold. Fregean logicians today do not recognize the notion of logical term negation. As a result, this way of drawing the contrary/contradictory distinction is not to be found in standard mathematical logic. Now, if terms cannot be negated, then clearly names cannot be negated.

Nonetheless, we are free to abjure the Fregean Dogma and the Fregean refusal to recognize term negation. Ordinary language practice (not to mention the counsel of many contemporary linguists and all traditional logicians) suggests such a departure from contemporary logical cant. There are negative terms, and among them are negated names. We have already seen that the argument that negative names must name impossible objects is logically (thus fatally) flawed. But there is something else plaguing such an argument. The simple fact is that *the negation of a singular term, such as a name, is not itself singular—the negation of a singular term is a general term.*

Consider 'Socrates is not Roman'. As the logical contrary of 'Socrates is Roman', we read this as 'Socrates is nonRoman', which is semantically equivalent to 'Socrates is Greek or French or Japanese or . . .'. What, now, of 'Socrates taught someone other than Plato'? What could 'other than Plato' (logically, 'nonPlato') refer to? Suppose I tell you that someone other than me taught my logic course last year. Logically, I may be construed as saying 'Someone who is not me (other than me, nonme) taught my logic course last year'. To whom am I referring? Well, at least not to myself. In fact, I refer to some member of the set of professors excluding me. But in all probability only one of these other professors taught the logic course. So how could the term 'nonme' be general? One more example: 'Olivier read every Shakespeare play but *Hamlet*' (logically: 'Olivier read every non*Hamlet* Shakespeare play'). My contention is that the negation of a singular term (name, pronoun, etc.) is a general term. Thus, 'nonPlato', 'nonme', 'non*Hamlet* Shakespeare play' are general terms. They are not used to make reference to any particular (possible or impossible) objects. To see how this is so, we need to recall key elements of my semantic theory.

My view is that every used term, in the normal case, denotes objects, and the denotation is the intersection of the domain relative to which the term is used and the extension of the term. When quantified, such terms refer. The reference of a quantified term (logical subject) is determined in part by its denotation and in part by its quantity. Universal subjects make distributed reference to their entire denotations; particular subjects make undistributed reference to a part of their denotations. When I state that Olivier read every non*Hamlet* Shakespeare play, the term 'non*Hamlet* Shakespeare play' denotes all the plays in the Shakespeare corpus—minus *Hamlet,* and the quantifier 'every' shows that my reference is to all of these. The term is *prima facie* general, denoting not a mysterious, purported play by the Bard called "NonHamlet," but the entire body of plays, excluding just one. Now, recall the example 'Some nonme taught my logic course last year'. The term 'nonme' denotes all the people in the domain (say, the faculty) who are not me. But I do not refer to all of them. My use of a particular quantifier shows that reference here is made to one

or more of the faculty members, but not necessarily all. Again, 'nonme' is not a singular term. Finally, consider once more 'Socrates taught some nonPlato'. To what (or whom) does 'nonPlato' refer? Suppose our statement is made relative to the domain of Athenian philosophy students in the fifth century B.C. The expression 'some nonPlato' must refer undistributively to those—minus Plato. Again, 'nonPlato' is clearly not a singular term naming a strange object, but a general term denoting a large number of people (in this case, the fifth-century-B.C. Athenian philosophy students, minus Plato). 'NonPlato' is not an impossible or even strange name—it is no name at all.

Again, the fact is that the negation of a singular term is not singular. Recognition of this point would diminish much of the enthusiasm for asymmetry. Recognition of another, but closely related, point would also contribute to this general loss of enthusiasm: *the conjunction or disjunction of singular terms is not singular.* We saw in chapter two that Strawson has argued that if, say, 'Tom and William' did make a reference, it would have to be to an individual (again, the Fregean Dogma) that possessed all and only those properties that Tom and William both have. But if Tom were short and William tall, then this third individual would be neither short nor tall. If Tom were two-legged and William one-legged, then this third individual would be neither. Such an individual is impossible. So a conjunction of names cannot refer—thus cannot be a name. Similarly, a disjunction of names cannot name. 'Tom or William' would have to name an individual that had all and only those properties that either Tom or William possess. But this would require, for example, that such an individual be both short and tall. In order to preserve the Fregean Dogma, a statement like 'Tom and William played squash (together)' cannot be construed by the asymmetrist as having 'Tom and William' as its subject. Terms are either singular or general. If this term is general, it cannot be the subject; if it is singular, it names an impossible object. Consequently, the statement must be paraphrased so that the 'and' is seen as a sentential rather than a term connective. The statement is logically paraphrased as 'Tom played squash and William played squash'.

Now, the Fregean Dogma is just that—a dogma, and so are such well-entrenched beliefs as the one that demands that all connectives (negation, conjunction, etc.) are logically sentential. Like Euclid's parallel postulate, they can be rejected with impunity. We have already seen that a logic that permits negative names is natural, and so is one that countenances compound names. The key is, again, the realization that the compound of a pair of singular terms is not itself singular. 'Tom and William', for example, is not the name of anyone or anything—even an impossible thing. From *my* logical point of view, 'Tom and William' and 'Tom or William' are quantified terms—logical subjects. They refer distributively and undistributively, respectively, to the denotations of their terms.

Let us say that 'a and b are C' has as its subject 'a and b', and that the subject-term is '{a,b}'. This latter term we will call a term of *explicit denotation.* All used terms have a denotation—but it is almost always implicit, unstated. Consider the statement 'Every logician is wise' (made relative to the actual world). The term 'logician' here denotes Aristotle, Chrysippus, Abelard, and so on. The number of denoted items is quite large in this case. In 'Every Canadian province has a park', the term 'Canadian province' denotes British Columbia, Alberta, Ontario, Quebec, and so on. Here the number of objects denoted is only ten. Notice that we *could* use terms of explicit denotation in place of 'logician' or 'Canadian province'. We could say 'Aristotle, Chrysippus, Abelard, Ockham, Leibniz, Frege, Russell . . . and Quine are wise', for example. But clearly there is much practical advantage in using terms of implicit denotation—and there is no alternative in cases where the denotation is infinite (e.g., 'Every prime greater than 2 is odd'). The advantage virtually disappears, however, when the number of denoted objects is very small. 'Every author of *Principia Mathematica* was British' has little to recommend it over 'Russell and Whitehead were British'.

Now, the denotation of 'author of *Principia Mathematica*' is Russell and Whitehead and no one else. Let '{Russell, Whitehead}' be a term denotatively equivalent to 'author of *Principia Mathematica*'. The first is denotatively explicit; the second is denotatively implicit. A logical subject is a quantified term. The reference of a subject is determined by the denotation of the term and the quantifier. Given that they are made relative to the same domain, we know that 'Every author of *Principia Mathematica* was British' and 'Russell and Whitehead were British' are equivalent. They share a common predicate and their subject-terms are denotatively equivalent. The quantifier in the first case is explicit ('every'), indicating that the subject refers to the entire denotation of the subject-term. In the second sentence there is apparently no quantifier. But, since the two sentences are logically equivalent, the logical quantity of the second must, like the first, be universal. The universal quantity here is indicated by 'and'. The two sentences have the general forms

(1) Every author of *Principia Mathematica* was British.
(2) (Every) {Russell, Whitehead} was British.

But in (1) the term 'author of *Principia Mathematica*' is simply a convenient shorthand for '{Russell, Whitehead}', just as 'logician' is a (very) convenient shorthand for '{Aristotle, Chrysippus, Abelard, . . . Quine}'. (2) is the logical version of the colloquial 'Russell and Whitehead were British'.

If 'and', as in the example above, indicates universal quantity, then it is reasonable to expect its dual, 'or', to indicate particular quantity. Consider 'Some (one of the) author(s) of *Principia Mathematica* was an earl'. Replacing the term of implicit denotation with a denotatively equivalent one of explicit denotation yields 'Some {Russell, Whitehead}

was an earl'. This is rendered more naturally as 'Russell or Whitehead was an earl'. Terms like 'Tom and William', 'Russell or Whitehead', and so on, are not singular terms. They are general terms—indeed, quantified terms (subjects).

Of course, it is possible to use a term of explicit denotation that happens to denote just one object. Consider 'Socrates is wise'. Let us say that '{Socrates}' is a term whose denotation is Socrates. It is a term of explicit denotation. A subject of the form 'every {Socrates}' refers to the entire denotation of '{Socrates}', and a subject of the form 'some {Socrates}' refers to a part of the denotation of '{Socrates}'. Now, the entire denotation of '{Socrates}' is Socrates, and the only part of the denotation of '{Socrates}' is Socrates. So, whether universally or particularly quantified, the reference in each case is the same object, Socrates. This is a further explanation of why there is no need for singular subjects to be explicitly quantified and no logical need to replace a singular term with its denotatively explicit equivalent. Singular subjects have wild quantity.

every {Socrates} = some {Socrates} = {Socrates} = Socrates

Truth and What 'There' Is

If what is is what is said, then the more we talk, the more being there is.
Umberto Eco

"There is something better than logic." "Indeed? What is it?" "Fact."
Mark Twain

As scarce as truth is, the supply has always been in excess of the demand.
Josh Billings

Every sentence is a (complex) term. In the normal case, every used term signifies a property and denotes those objects (if any) that both have that property and are also in the domain of discourse relative to which the term is used. Thus, every sentence (used to make a true statement) both signifies and denotes. What a true statement (i.e., sentence used to make a statement) signifies is a constitutive characteristic of the domain relative to which it is used. If the sentence is (used to make) a statement, then it is used with the implicit accompanying (truth) claim that the domain is so characterized. A true statement denotes its domain. A false statement denotes nothing; nor does it signify anything; it is doubly vacuous. Compare the term 'red' and the term 'present king of France'. The first signifies the property of being red and denotes whatever has that property in the domain at hand (say, the actual world). The second term expresses the concept of being a present king of France. But since nothing corresponds to that concept in the actual world, it is vacuous; it denotes and signifies nothing. A true statement denotes whatever has the property it signifies. So if the domain at hand has that property (that constitutive characteristic), then the statement denotes the domain. Otherwise it denotes nothing. "It is just true statements that have a corresponding entity" (Davidson, 1969: 74).

The actual world is Mars-ish and red-ish (in this case, and many others, we actually have the word 'reddish') and logician-ish and shy-ish because it contains such things as Mars, firetrucks, Quine, and me as constituents. It is un-ghost-ish and un-unicorn-ish because it does not contain such things as ghosts and unicorns. The actual world is red-ish and nonred-ish and un-ghost-ish because it contains firetrucks and lemons but no ghosts. In general, to say that a domain, D, is P-ish is to say that some (at least one) P-thing belongs to, constitutes (in part at least), D. To say that D is un-P-ish is to say that no P-thing is a constituent of D. Note that for any D and any P, D is P-ish or D is un-P-ish, but it need not be the case that D is either P-ish or nonP-ish. The domain of natural numbers is either red-ish or un-red-ish because either it has a red constituent or it has no red constituent; but it is neither red-ish nor nonred-ish, since it has no red constituent and also no nonred (blue or green or pink or . . .) constituent.

To make a truth claim is implicitly to characterize the relevant domain constitutively. When I state, relative to the actual world, 'Mars is red', I implicitly characterize the actual world as being in part constituted by red Mars, as being red-Mars-ish, as being [red Mars]. When I state 'Some logician is shy' I characterize the domain, normally the actual world, as shy-logician-ish [shy logician]. Notice that for 'Mars is red' to be true, all that is required is that something that is both Mars and red be in the world; there is no need for states or facts also to be in the world. Suppose I state 'Mars is male'. Whether my implicit claim here is true or false will depend on the domain with respect to which I make my statement. If that domain is the actual world, then the claim is not true. For the actual world has no constituent that is both Mars and male. But if the domain is the world of Greek mythology, my claim is true, since that domain is constituted by, among other things, a male Mars.

To say of an object that it exists is, in effect, to say that it is a constituent of the domain at hand. Existence, as Kant and so many others have argued, is not a (real) property. What existence is is a constitutive property—not of objects, not of concepts (as Frege thought), but of domains, totalities of objects. Recognition of this leads to an interesting thesis concerning the term 'there'. Indeed, we can now say what 'there' is. Our thesis can be stated in simple terms: the English word 'there', as used in such statements as 'There is an X', 'There are X's', 'There is no X', 'There are no X's', is nothing more than the simple locative adverb, equivalent to 'in/at that place'. The received view among logicians and philosophers has generally been that in addition to the locative use of 'there' (as in 'There is the book I was looking for', 'The lighthouse is there, on the

other side of the cape', 'My car is not there, it's been stolen') there is an *existential* use of 'there' . To use 'there' in this second way (e.g., 'There is a Kantian joke', 'There is no god', 'There are no unicorns', 'There are honest politicians') is to affirm or deny existence of some thing or things. Thus, to say 'There is a Kantian joke' is just to say 'A Kantian joke exists', the phrase 'there is' being reduced to 'exists'. In like manner, one can generate 'No god exists', 'No unicorn exists', and 'Honest politicians exist'. There is no denying that this way with 'there' leads to the growth of Plato's Beard, taking, as it does, the use of 'exists' in these paraphrases as the predication of a property (existence/nonexistence). But there have been good arguments advanced against treating 'exist' as a predicate. Hume gave one in the *Treatise* (I.ii.6 and I.iii.7), and Kant gave an even more famous version in the "Transcendental Dialectic" of the *Critique* (II.iii.5). Others have argued this since.

Modern logicians are hardly prepared to deny the Hume-Kant position. Existence is not a property that any thing has or lacks. Still, there is an existential use of 'there'. Their way with 'there' is to treat it as a "higher" function—namely, a quantifier. To say 'There is an X' is to say 'There is at least one thing such that it is X'. The phrase 'there is at least one thing such that' is standardly treated as a Fregean "second-level function". To say 'There is an X' is just to say that something falls under the concept referred to by 'X'. Accordingly, existence may not be a property of objects but it is surely a property of concepts.

My claim is that there is no existential use of 'there'—all (normal) uses are locative. Admittedly, such a thesis cannot stand alone. If 'there' is always locative in statements such as 'There is/are an/no X(s)', then the question immediately and naturally arises: Where? Using 'there' locatively, 'There is an X' is convertible into 'An X is there'. Thus: 'A Kantian joke is there', 'A god is not there', 'Unicorns are not there', 'Honest politicians are there'. So, where is the Kantian joke, the honest politician; from where is god missing; from where are unicorns missing? Here I need my second thesis: In statements of the form 'There is an X', 'There' is always used to locate objects in the same place—the world. The locative sense of 'there' can always be expressed by the wordier 'in/at that place', where the demonstrative 'that' is interpreted differently according to the context of its use. The same paraphrase applies to 'there' in 'There is an X', and so on. Thus: 'In that place is an X'. But in such cases 'that place' is always used to designate the same locale, the world. Thus: 'There is no god' ('No god is in the world', 'No god is a constituent of the world', 'The actual world is un-god-ish'); 'There are honest politicians' ('Some honest politicians are in the world', 'Some honest politicians are constituents of the world', 'The world is honest-politician-ish'). The use of 'there' is always locative, and the "existential" use of 'there' locates objects in the world.

A New System of Diagrams[1]

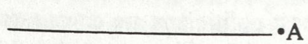
> *To find a lucid geometric representation for your . . . problem could be an important step toward the solution.*
>
> G. Polya

> *Be he a Triangle, Square, Pentagon, Hexagon, Circle, what you will—a straight Line he looks and nothing else.*
>
> E.A. Abbot

> *Novelty, by itself, is no drawback to a scheme; in some cases (as with milk, eggs, and jokes) it is a positive advantage.*
>
> Lewis Carroll

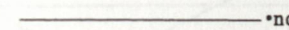
Euler and Venn diagrams are simple and effective devices for illustrating syllogistic validity. Their potential is limited, however, since they cannot apply to arguments with more than four terms. Attempts at extending the scope of plane figure diagrams (e.g., by Carroll, 1958, 1977) have been only marginally successful.[2] Aristotle probably used some sort of diagram method in teaching the syllogistic, and many ancient commentaries made use of linear diagrams, though our understanding of just how they worked is sketchy.[3] If the ancient syllogists used linear diagrams rather than planar ones, and if they diagrammed not only simple syllogisms but sorites, polysyllogisms, and compound syllogisms, then it is likely that there is a satisfactory linear method of logical diagrams that can readily go beyond the virtual four-term limit on plane diagrams. Leibniz tried to use line diagrams in analysing syllogisms,[4] and a century later Lambert attempted linear diagrams for syllogistic.[5] In what follows I will describe such a linear diagram method, illustrate some of its uses, and extend the method to relationals and compounds statements.[6]

Rather than follow the nineteenth-century practice of representing each term of an inference as a set of points constituting a closed plane figure, let us follow the ancient suggestion of representing such terms as points of a straight line segment. (Topologically, we might think of the line as a covering space on a Venn circle.) We can think of the place in which a given diagram lies as constituting the relevant domain of discourse. A term such as 'animal' (symbolized by 'A') will be represented as a straight line segment, the extent of which is undetermined. More precisely, the line represents the denotation of the term. Each such line segment will be labelled at its right terminus.

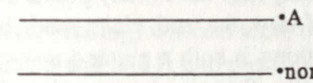


Terms may be negated or unnegated (i.e., implicitly positive). In either case, their diagrammatic representation is a straight line (segment). Thus 'nonA'

will be diagrammed as



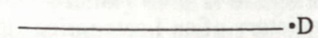
Since nothing could ever satisfy both a term and its negation, their linear representations can have no point in common. In other words, the two lines representing such terms must be parallel.



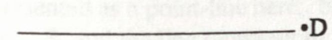
This diagrams the logical truth that no A is nonA.

A limiting case of a line segment is a single point. Singular terms will be represented quite naturally by such lines (point-lines). For example, a term such as 'Fido' will be diagrammed as a single point.

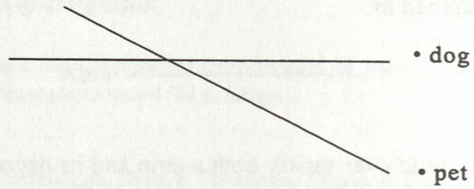
If Fido is a dog, then we want the point-line representing Fido to be one of the points constituting the line representing dogs. If



represents the term 'dog', we will place the point representing 'Fido' at the left terminus of this line (since we have agreed to label each line at its right terminus and a point-line has no other point to its left).

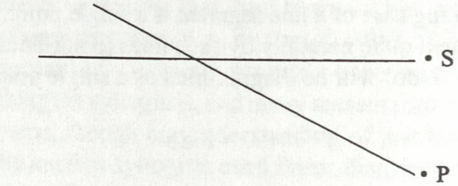


I will now show how categorical sentences in general are represented by linear diagrams. But first a preliminary condition: a line consisting of no points is no line, so no terms are empty. Every term is represented as a line of one or more points. We have seen how to diagram a sentence such as 'Fido is a dog'. Suppose, however, that we want to diagram 'Some pet is a dog'. Here, what needs to be illustrated is the claim that there is at least one thing common to both pets and dogs. The lines for 'pet' and 'dog' must have at least one common point—they must intersect.



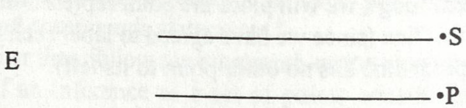
Notice that we represent 'pet' and 'dog' as having a single common point. Yet, for all we know, they may have many points in common. Nonetheless, from a logical point of view, the truth claim made by 'Some pet is a dog' is just that at least one thing is both a pet and a dog. This is what we have diagrammed. Generally, then, an I categorical ('Some S is P') will be diagrammed as

I



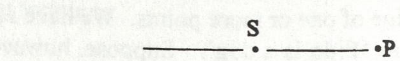
If two lines do not intersect, then they must have no common point; they must be parallel. The contradictory of an I categorical, therefore, must be represented by parallel lines. An E categorical ('No S is P') will be diagrammed as

E



Universal affirmations claim that whatever is denoted by the subject-term is denoted by the predicate-term. So the subject-term line must be represented as a (possibly proper) part of the predicate-term line. An A categorical ('Every S is P') will be represented, then, as

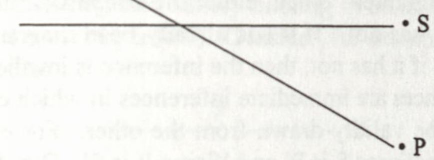
A



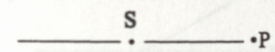
Notice that if every S is P *and* every P is S, then the number of points between the right terminus of S and the right terminus of P will be zero.

To be very clear, then, our diagrams for universal and particular

affirmatives are stipulated to be understood in such a way that



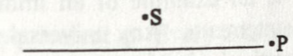
permits interpretation (or reading) wherein more than one S is P (more than one point shared by line S and line P, and possibly all points on S are on P, and even *vice versa*, and



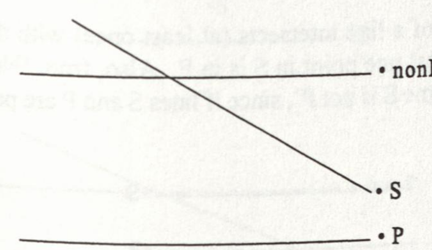
permits interpretation wherein all the points on P are points on S as well. So, one line crossing another at a single point is to be interpreted to mean that *at least one* point is shared by the two lines. (This parallels exactly a single 'x' on a Venn diagram interpreted as 'at least one'.) And one line, say S, partially coinciding with another, P, is to be interpreted to mean that all points on S are points on P and possibly no points on P are left over.

The contradictory of an A categorical claims that at least one thing satisfies the subject-term but not the predicate-term. So, an O categorical ('Some S is not P') must be diagrammed as an S-point outside the P-line.

O



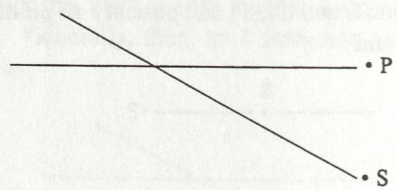
Note that 'S' is represented as a point-line here. But, for all we know, there may be more than one S, and the line representing them may or may not be parallel to the P-line. Indeed, to say that some S is not P is to say that some S is nonP, which can be diagrammed as



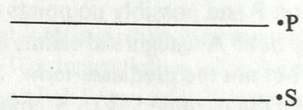
The simpler diagram for O consists just of the P-line and the point of intersection of the S- and nonP-lines.

Line diagrams represent inferences in the usual way. First, the premises are diagrammed. Then, either the conclusion has already been diagrammed or it has not. If it has already been diagrammed, then the inference is valid; if it has not, then the inference is invalid.

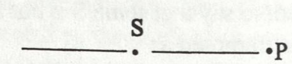
Equivalences are immediate inferences in which each of a pair of propositions can be validly drawn from the other. For example, simple conversion equates 'Some S is P' and 'Some P is S'. Our diagram method illustrates this by representing both sentences by a pair of intersecting S- and P-lines.



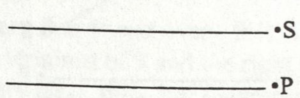
Universal negatives are likewise simply convertible. Both 'No S is P' and 'No P is S' are diagrammed by parallel S- and P-lines.



Subalternation is an example of an immediate inference between two nonequivalent statements. Any universal statement will validly entail its corresponding particular just because no term is empty. Diagrammatically, whenever there is a line there must be at least one point in that line. For example, we can derive 'Some S is P', from 'Every S is P', where the premise is diagrammed as

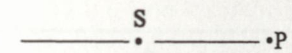


Since every part of a line intersects (at least once) with that same line, it follows that at least one point in S is in P. Also, from 'No S is P' we can validly derive 'Some S is not P', since if lines S and P are parallel (given the premise)

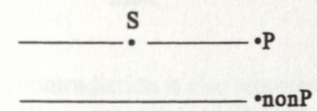


and since every term is nonempty (every line consists of at least one point), there must be at least one point in line S outside of line P.

Obversion is an example of immediate inference relying on the fact that a term and its negation satisfy nothing in common (so that their line representations must be parallel). Consider, for example, 'Every S is P'. It is diagrammed as

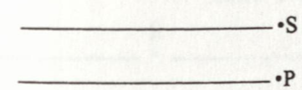


But we also know that 'nonP' is logically contrary to 'P', so that a nonP-line is parallel to the P-line. By adding this we have

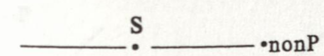


Since every point on P is outside of nonP, and since every point on S is on P, it follows that every point on S is outside of nonP. In other words, lines S and nonP are parallel (i.e., 'No S is nonP', the obverse of our premise, 'Every S is P').

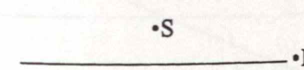
However, the true importance of obversion is seen when applied to E and O forms. 'No S is P' has been diagrammed thus far as



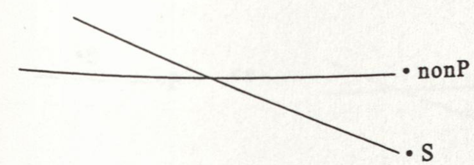
By obversion, 'No S is P' is equivalent to 'Every S is nonP', thus:



Likewise, 'Some S is not P' is equivalent to 'Some S is nonP'. Thus, both



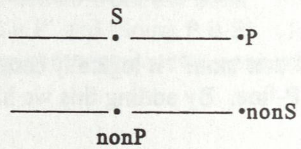
and



can be used to diagram an O categorical.

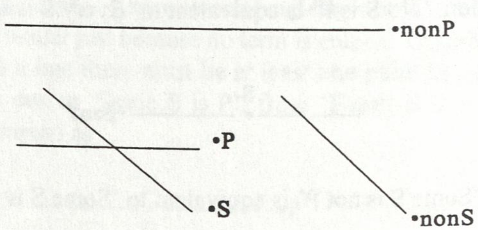
Now, an obverted A statement can be converted. The resulting statement can then be obverted to yield the contrapositive of the original. The contrapositive of 'Every S is P' is 'Every nonP is nonS'. Diagrammatically, then, the *full* representation of 'Every S is P' must be

A (full)



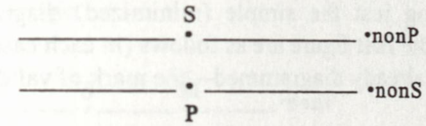
This represents such equivalent statements as 'Every S is P', 'Every nonP is nonS', 'No S is nonP', 'No nonP is S', 'No nonS is P', 'No P is nonS', 'No S is nonS', 'No nonS is S', 'No P is nonP', and 'No nonP is P'. These last four are tautological and are instances of the law of noncontradiction. A *full* representation of any statement will necessarily represent the law of noncontradiction as well. Consider the I statement 'Some S is P'. Conversion and obversion on I demand that a *full* representation must exhibit such equivalent statements as 'Some P is S', 'Some S is not nonP', and 'Some P is not nonS'. Thus:

I (full)

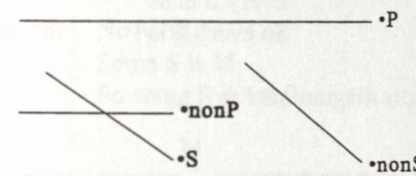


Universal and particular negations can also be given *full* representations in order to exhibit logical equivalences.

E (full)



O (full)

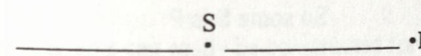


Note that the law of noncontradiction is also represented (twice) by each *full* diagram.

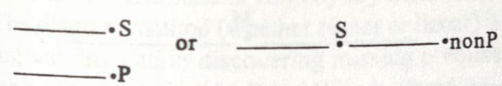
The *full* representation of a categorical will always be a diagram consisting of two pairs of parallel lines. However, for most purposes of logical reckoning, the simple A, E, I, and O diagrams are sufficient. These are the results of "minimizing" (Gardner, 1982: 72) the *full* diagrams, which will usually represent far more information than we need.

Here are our minimized, simple diagrams for the four categoricals.

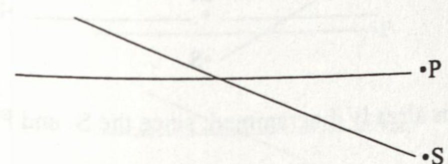
A



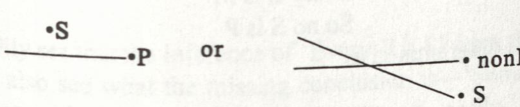
E



I



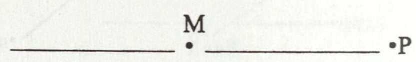
O



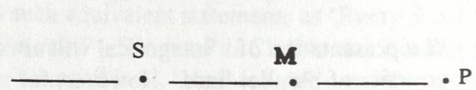
The classical simple syllogisms are easily diagrammed by my linear method—using just the simple (minimized) diagrams. The premise diagrams for the first figure are as follows (in each case the conclusion can be seen to be already diagrammed—the mark of validity):

Barbara: Every M is P
Every S is M
So every S is P

Here, the major is diagrammed first as



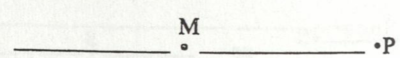
The minor is then added to get



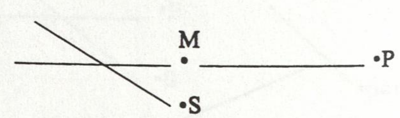
From this the conclusion can be read directly.

Darii: Every M is P
Some S is M
So some S is P

We diagram the major as



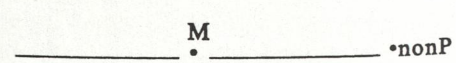
The minor is then added:



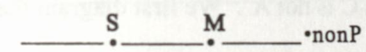
The conclusion is already diagrammed, since the S- and P-lines intersect.

Celarent: No M is P
Every S is M
So no S is P

The major is diagrammed as

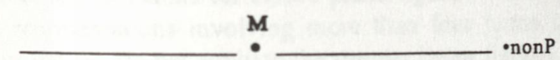


Adding the minor, we get

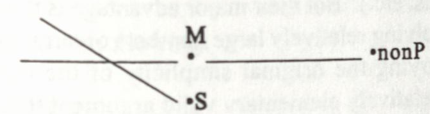


Ferio: No M is P
Some S is M
So some S is not P

The major is

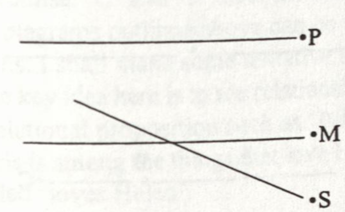


Adding the minor:

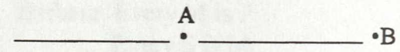


Similar diagrams can easily be constructed for all 24 valid classical syllogisms from AAA-1 to EIO-4. The method is complete. It is sound as well. An exhaustive (and exhausting) check of each of the 232 classical invalid forms shows that none is valid by my method.

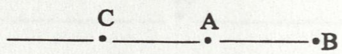
The diagram method (whether planar or linear) is most effective in determining validity and in discovering missing premises or conclusions. Consider the premise pair 'No P is M' and 'Some M is S'. These are diagrammed linearly simply as



We can readily see that the inference of 'Every P is S' from this is invalid. But we can also see what the missing conclusion is—'Some S is nonP'. Enthymemes with missing premises are most easily resolved by diagramming the explicit premise along with the contradictory of the conclusion. What follows, then, will be the contradictory of the missing premise. For example, let the explicit premise be 'Every A is B' and the conclusion be 'Some C is not A'. We first diagram the premise as



Next, we add the contradictory of the conclusion (viz., 'Every C is A').

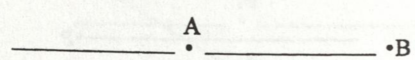


From this we conclude 'Every C is B', which is the contradictory of the tacit premise 'Some C is not B'.

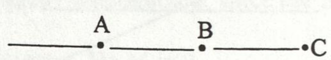
Thus far, we have seen that linear diagrams can do virtually what planar diagrams can do. One minor advantage they may enjoy is that they are faster to construct (since lines and points are easier to draw than circles, squares, ellipses, etc.). But their major advantage is their ability to represent inferences involving relatively large numbers of terms (viz., more than four) without destroying the original simplicity of the diagrams. Here is an example of a relatively elementary valid argument that Venn diagrams are powerless to represent in a simple, perspicuous manner.

Every A is B
Every B is C
No C is D
Some D is E
So some E is not A

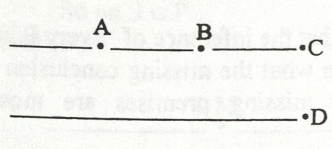
Diagramming the first premise gives us



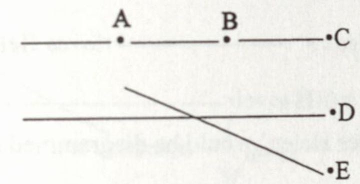
Adding the second:



Adding the third:



Finally, we add the fourth:



That's all—three lines, five labels. The conclusion is already diagrammed. Sorites of any number of terms can be diagrammed using the linear method. The geometric restrictions on closed plane figures, which prevent perspicuous representations involving more than four terms using simple continuous figures, do not apply to the simpler linear figures.

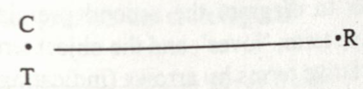
Identity statements are easily diagrammed by my method. A statement of the form 'A is (identical to) B' claims that the A-point is on the B-line. Since the B-line is a point-line, this means that 'A' and 'B' label the same point:

A • B

An argument such as the following:

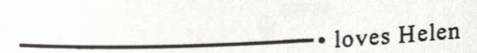
Tully is Cicero
Cicero is Roman
So Tully is Roman

would be diagrammed as

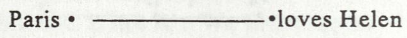


where, by the first premise, 'C' and 'T' label the same point-line.

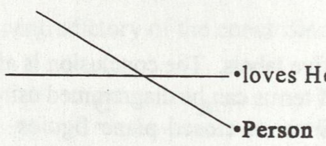
The simple diagrams outlined above can be extended to represent relational propositions. I shall make some tentative suggestions as to how this can be done. The key idea here is to see relational expressions as terms. Consider a simple relational proposition such as 'Paris loves Helen'. The claim here is that Paris is among the things that love Helen. Let us diagram these by a line labelled 'loves Helen'.



Paris is one of them. So:

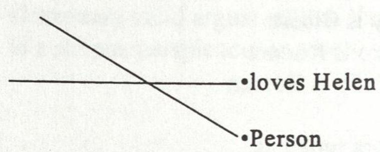


'Some person loves Helen' would be diagrammed as an I categorical.

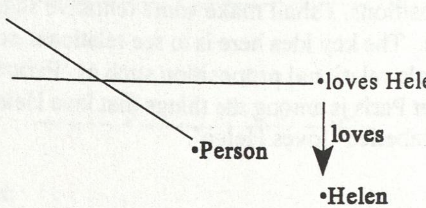


(Notice that the converse here is 'Something that loves Helen is a person'.)

So far, 'loves Helen' is treated just like any other term. It is represented by a single line segment representing things that love Helen. Often in inferences, however, it is necessary to analyse such relational terms, abstracting from them one or more of their constituents for independent treatment. Consider the argument 'Someone loves Helen, but Helen is a vamp. So someone loves a vamp'. The first premise can be diagrammed as

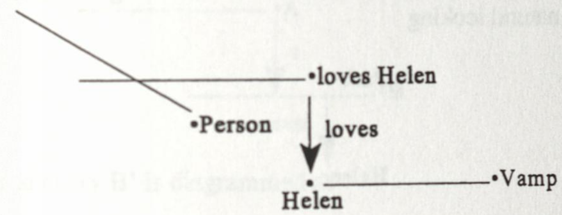


But now, in order to diagram the second premise, we need a way to represent the relative term, 'loves', and the object term, 'Helen', separately. Let us represent relative terms by arrows (indicating the "direction" of the relation) connecting their relata. Then 'loves', in this case, would be represented by an arrow from lovers of Helen to Helen.



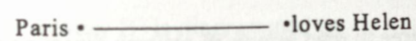
'Helen' is now extracted from the relational 'loves Helen' and we are free

to diagram our second premise, 'Helen is a vamp'.



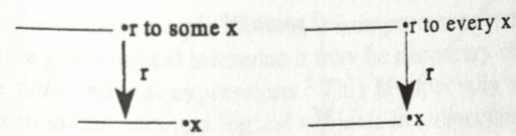
Given that from this we can see that every lover of Helen loves Helen, the conclusion, 'Someone loves a vamp', can be read directly.

Look once more at our diagram for 'Paris loves Helen'.

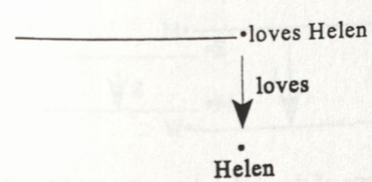


We know that whatever loves Helen loves Helen. In fact, we can say, generally:

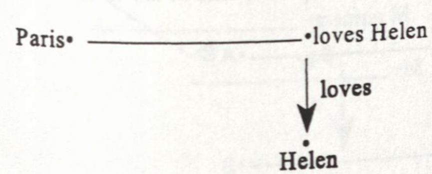
Whatever is r to some/every X is r to some/every X, i.e.:



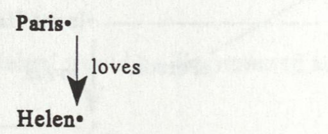
It is this tautology that permits the tautological



so that 'Paris loves Helen' can be rendered as

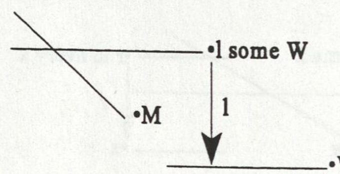


This *full* representation, however, diagrams more information than we often need in computing logical inferences. But it can be simplified (by suppressing tautological information, as with categoricals in general) to the more natural looking

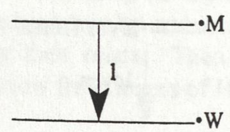


or simply

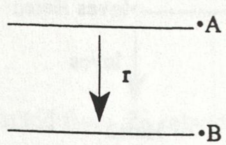
If we agree to read arrows in reverse direction as representing the converses of the relations they represent when read in their indicated direction, we can take the preceding diagram to represent both 'Paris loves Helen' and 'Helen is loved by Paris'. This same process can be used to simplify diagrams for statements such as 'Some man loves some woman'. Its *full* representation is



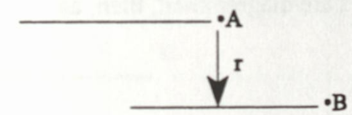
Simplifying, by suppressing tautological information, we get



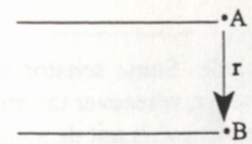
Notice that the locations of the end points of a relational arrow indicate the quantities of the relata. The quantity is universal when the relational arrow meets the term-line at the line's right terminus; otherwise the term is particular in quantity. For example, 'Some A is r to some B' is diagrammed as



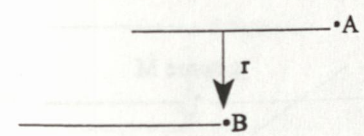
'Every A is r to some B' is diagrammed as



'Every A is r to every B' is diagrammed as

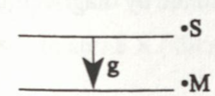


And 'Some A is r to every B' is diagrammed as:

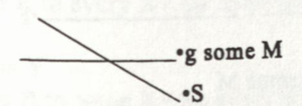


In spite of these simplifications, it is important to remember that in the context of a given logical inference it may be necessary to restore some or all of the *full* relational expressions. This is especially so when those relational expressions occur as logical subjects in subsequent premises or conclusions. For example, a proposition such as 'Some senator gives away some money' could be diagrammed in one of three ways:

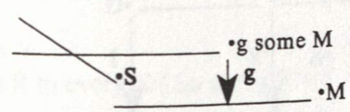
(i)



(ii)

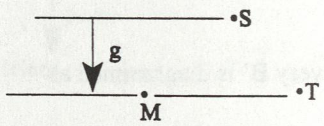


(iii)



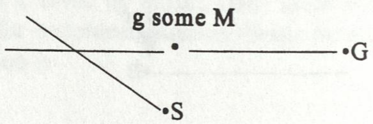
We can use (i) if the relative term 'gives away' occurs elsewhere without the logical object 'money'. Thus, suppose the second premise is 'All money is tainted'. Our premises are diagrammed, then, as

(i.1)



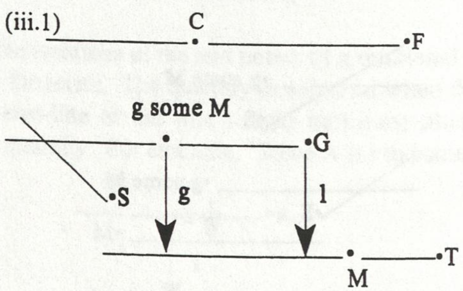
from which we might conclude 'Some senator gives away something tainted'. We can use (ii), however, whenever the analysis of the relational expression 'gives away some money' is not demanded by any subsequent premise or conclusion. Suppose our second premise is 'Whoever gives away some money is generous'. Then we can diagram our premises as

(ii.1)

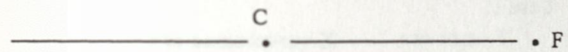


And from this we could read the conclusion 'Some senator is generous'. Finally, we are in need of the *full* representation, (iii), when the relational expression occurs subsequently both analysed and unanalysed. For example, suppose our second premise is 'All money is tainted', our third premise is 'Whoever gives away some money is generous', and our fourth premise is 'Whoever is generous loses some money'. The conclusion, 'Some senator loses something tainted', is diagrammed by diagramming the premises thus:

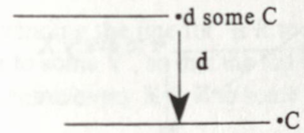
(iii.1)



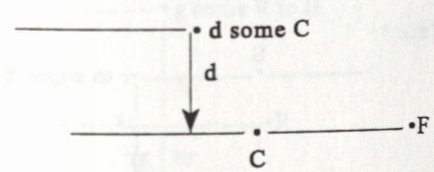
Consider next the famous inference 'Every circle is a figure. So whoever draws a circle draws a figure'. The premise is easy enough:



We also know that whoever draws a circle draws a circle. So:



Together, these give us



And here we can see 'Whoever draws a circle draws a figure'.

We can develop a useful general rule out of the preceding example, call it Rule R, based on four kinds of cases. These cases are argument forms that each have the premise 'Every X is Y'. Then, two kinds of conclusions occur with four variations in the remaining premise, as follows:

Case 1: Every X is Y
Some S is R to some X / So some S is R to some Y

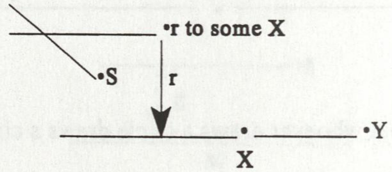
Case 2: Every X is Y
Some S is R to every X / So some S is R to some Y

Case 3: Every X is Y
Every S is R to some X / So every S is R to some Y

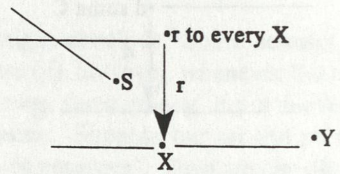
Case 4: Every X is Y
Every S is R to every X / So every S is R to some Y

So, in general, the conclusion has one and the same grammatical predicate, '. . . is R to some Y', even when the same predicate in the second premise has an 'every' in it. All of these cases seem valid intuitively. Each is confirmed to be valid when placed on linear diagrams:

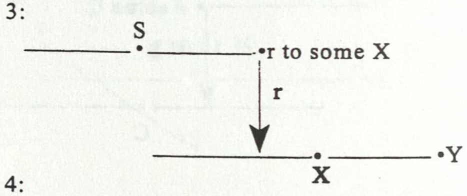
Case 1:



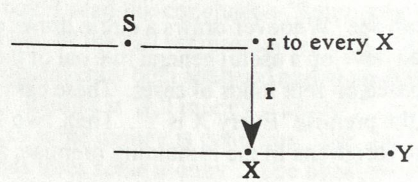
Case 2:



Case 3:



Case 4:

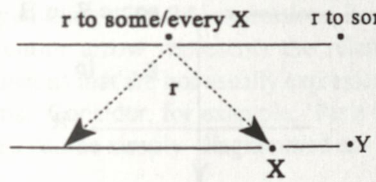


It is important to note that as obviously valid as cases 2 and 4 appear in our diagrams, they are not valid in the predicate calculus. To prove them valid in the predicate calculus would require adding a premise tantamount to the existential import embedded in our diagram method (viz., every line has at least one point). The premises required for 2 and 4 are 'There are X's' or 'There are Y's'. Now, Rule R is simply the generalization from these cases—to wit:

Rule R: If every X is Y, then whatever is R to some/every X is R to some Y.

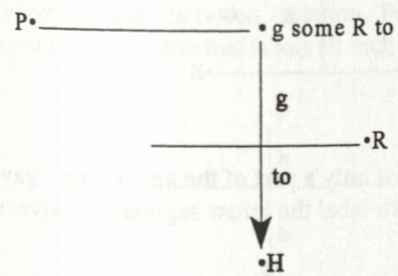
Diagrammatically, we state it as follows:

Rule R:

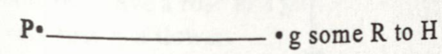


Thus, Rule R permits extending the line for 'is R to some/every X' so that it is a sub-portion of 'is r to some Y', so that the top line above can be read 'Everything that is R to some/every X is R to some Y'.

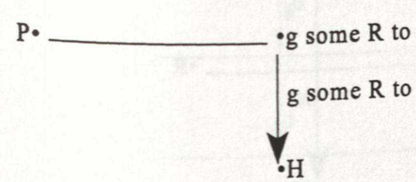
Now let us diagram the proposition 'Paris gave a rose to Helen' as



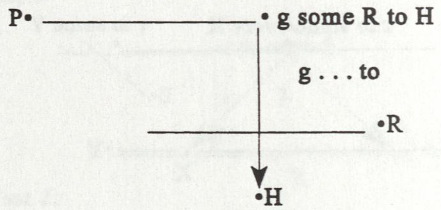
I have arrived at this representation in the following way. First, diagram the proposition simply as



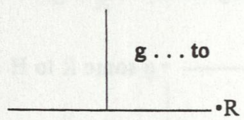
Next, analyse the relational expression 'gave a rose to Helen' as a relative term, 'gave a rose to', and its logical object, 'Helen':



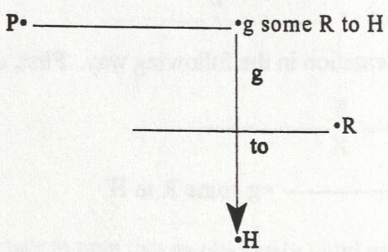
Now, the relative term 'gave a rose to' is itself a relational expression. We analyse it as a relative term, 'gave . . . to', and its logical object, 'a rose'. Thus:



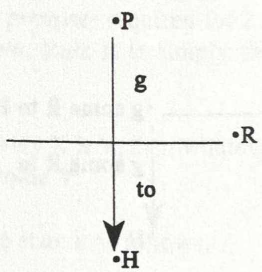
Here, the arrow labelled 'gave a rose to' has been replaced by



Since the vertical line is now only a part of the arrow from 'gave a rose to Helen' to 'Helen', we can re-label the arrow segments to give us:

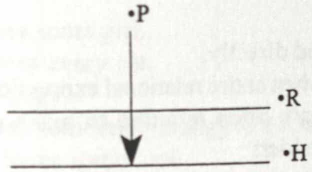


As before, we can often, unless the context demands otherwise, simplify this as:

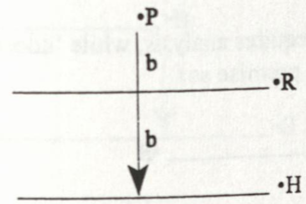


And this diagrams as well all the equivalent converse relationals, passive transforms, such as 'Helen was given a rose by Paris', 'A rose was given to Helen by Paris', 'Helen was given by Paris a rose', and 'A rose was given by Paris to Helen'.

In diagramming relational expressions, it must be kept in mind that ultimately the entire arrow represents the relation. This is especially important for relations that are not usually expressed in natural language by multi-word terms. Consider, for example, 'Paris is between a rock and a hard place'. This can be simply diagrammed as



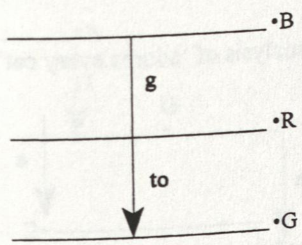
But there is no way to label independently the arrow segments. The entire arrow represents the 'between' relation. Perhaps the most perspicuous diagram would thus be one that labels all such arrow segments by 'between':



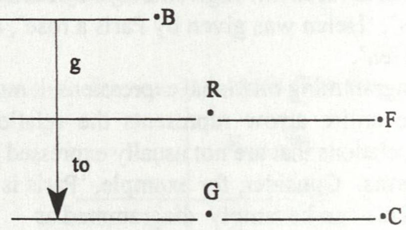
Our analysis of relationals with more than one object is useful for diagramming such inferences as

Some boy gave a rose to a girl.
Every rose is a flower.
Every girl is a child.
So some boy gave a child a flower.

The first premise is diagrammed (simply) as



Adding the next two premises gives us

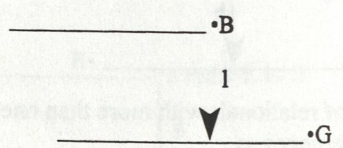


The conclusion is read directly.

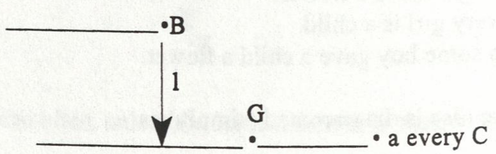
Of course, when entire relational expressions occur more than once in an argument, we are often required to make use of their unanalysed representations. Consider:

Every boy loves some girl.
Every girl adores every cat.
Whoever adores every cat is a fool.
So every boy loves a fool.

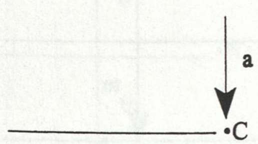
Here 'loves some girl' requires analysis, while 'adores every cat' does not. So we diagram the first premise as



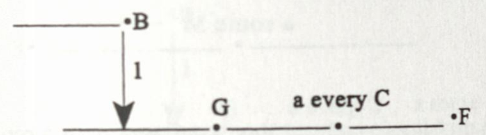
Adding the second premise yields



(We could add the analysis of 'adores every cat', viz.:



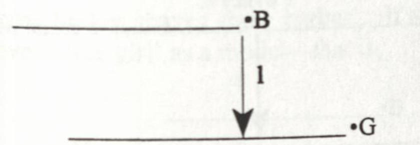
but it is unnecessary in this context.) Finally, we add the last premise:



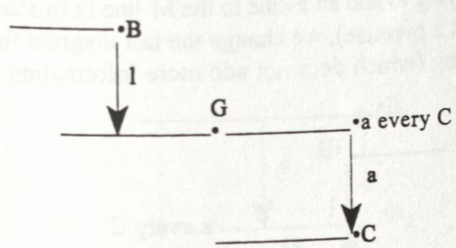
Let us consider one last example:

Every boy loves some girl.
Every girl adores every cat.
Every cat is mangy.
Whoever adores something mangy is a fool.
So every boy loves some fool.

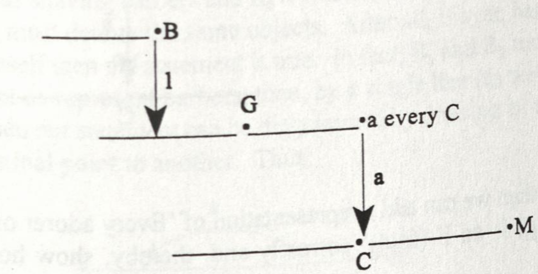
In this case, unlike the preceding one, both relational expressions must be analysed. But, as we will see, 'adores every cat' must have an unanalysed representation as well. The first premise is diagrammed as



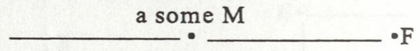
Adding the second premise, we have



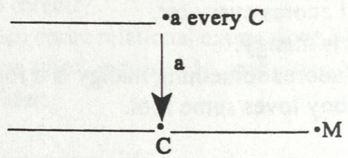
The third premise gives us



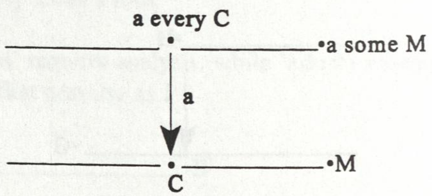
Now, the fourth premise is diagrammed as



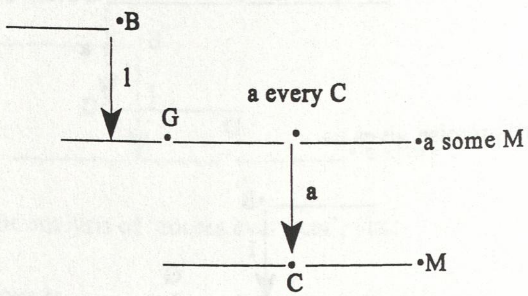
Clearly what is missing to connect the two diagrams is a representation of 'Whatever adores a cat adores something mangy'. And, in fact, this does hold, given the third premise and our Rule R, for Rule R explicitly sanctions changing a diagram like



into

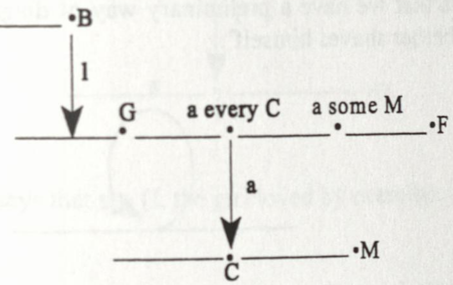


So, instead of trying to add an F-line to the M-line (a mistake, since 'Every M is an F' is not a premise), we change the last diagram for this argument into the following (which does not add more information and is merely a rearrangement):



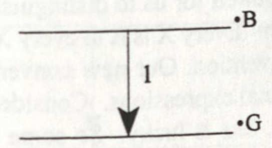
For then we can add a representation of 'Every adorer of some M (mangything) is an F (fool)' correctly and, thereby, show how the conclusion,

'Every B (boy) loves some F (fool)' is already represented:

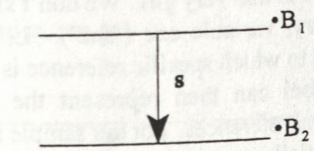


And notice that this final diagram also contains a very clear representation of the intermediate conclusion that many might think is the most important one for the argument—namely, 'Every girl adores something mangy'.

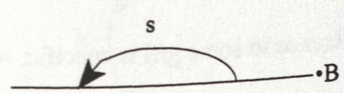
We have adopted the convention that any arrow that touches the right terminus of a line touches every point on that line; an arrow that touches a line only at a point left of the right terminus touches that line at no other point. Yet the presence of repeated references raises the possibility of counter-examples to the second part of our convention. Consider the statement 'Some barber shaves some barber'. If we diagram this using 'Some boy loves some girl' as a model—that is,



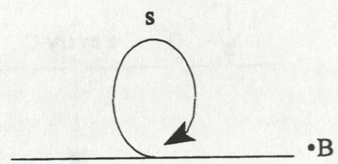
we get this:



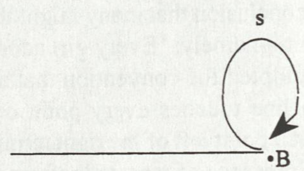
where B_1 represents shaving barbers and B_2 represents shaved barbers. But clearly, B_1 and B_2 must denote the same objects. After all, if even just one barber shaves himself then the statement is true. In fact, B_1 and B_2 must be identical lines. Let us represent barbers, then, by a single line (as we have always done). Then our statement can be diagrammed by drawing an arrow from one nonterminal point to another. Thus:



Here the distance between the base and the head of the arrow may be equal to or greater than zero; equal to zero if some barber shaves himself. This last suggests that we have a preliminary way of diagramming the statement 'Some barber shaves himself':

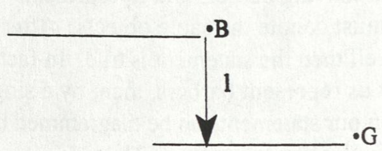


and the statement 'Every barber shaves himself':

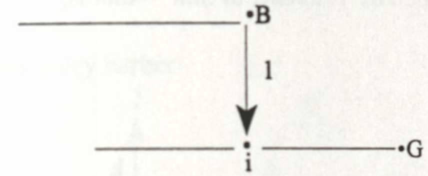


But how, given our right terminus convention, do we diagram 'Every barber shaves every barber'?

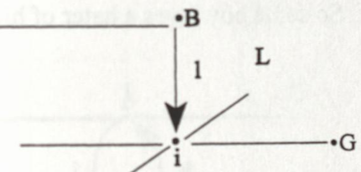
What is required for us to distinguish diagrammatically between statements of the form 'Every X is R to every X' and 'Every X is R to itself' is an additional convention. Our new convention will involve the representation of pronominal expressions. Consider the inference 'Some girl is loved by every boy. She is lucky. So some boy loves something that is lucky'. Here, specific reference has been made to some girl (i.e., to a certain girl rather than to some girl or other). Then a pronoun is used to make subsequent reference to that very girl. We don't know her name, but we can give her an arbitrary, variable one ('she'). Let us agree to label every unnamed individual to which specific reference is made with a small roman numeral. That label can then represent the pronoun in subsequent, anaphoric pronominal references. For our sample inference, we can diagram the first premise initially as



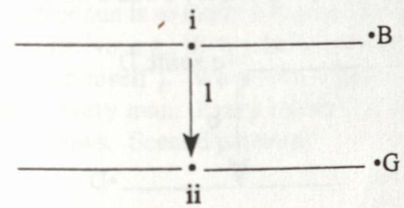
But, since the reference to some girl is specific, we will label it:



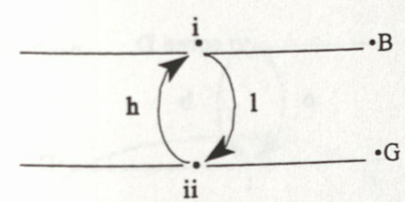
The second premise says that she (I, the girl loved by every boy) is lucky. Adding this yields



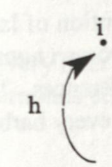
Consider next the argument 'Some boy loves a girl. She hates him. So he loves a hater of him'. We diagram the first premise, adding the pronoun labels for subsequent use:



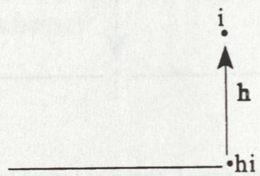
Adding the second premise, we get



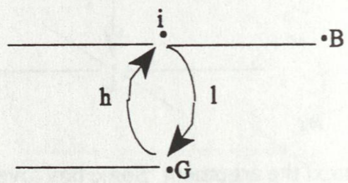
The conclusion is diagrammed here once we recognize that 'hates him'—that is,



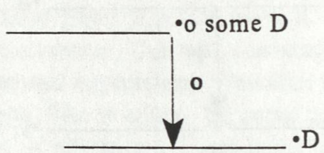
is a simplification of 'is a hater of him'—that is,



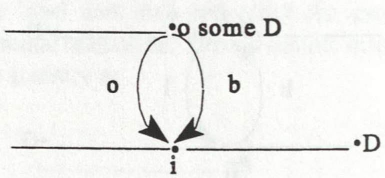
This same simplification allows us to diagram 'Some boy loves every girl. They hate him. So some boy loves a hater of him':



Next, consider the statement 'Every owner of a donkey beats it'.[7] Analytically, every owner of a donkey owns a donkey:



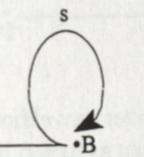
It, the donkey so owned, is beaten by its owner. So



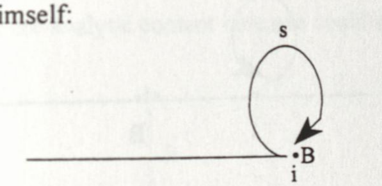
We are adopting the convention of labelling individuals to which specific reference is made by small roman numerals, which are then used to represent subsequent pronominal references. This convention permits us to diagram, now, 'Every barber shaves every barber' and 'Every barber shaves

himself' as follows:

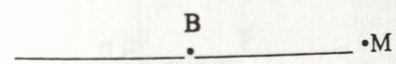
Every barber shaves every barber:



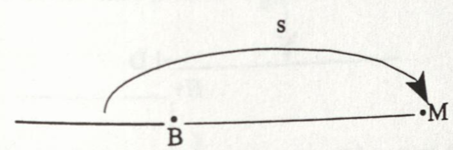
Every barber shaves himself:



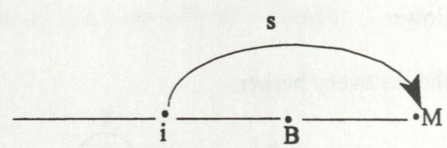
Now, since 'Every barber shaves every barber' entails 'Every barber shaves himself', we might justify this by a rule (called "i-insertion" by an anonymous reader) analogous to existential instantiation in the standard predicate calculus. Such a rule allows us to pronominalize at will by marking any point on a given diagram line with a roman numeral. It must be noted that once a pronoun is so marked it cannot subsequently be ignored (otherwise one might derive, e.g., 'Every barber shaves every barber' from 'Every barber shaves himself'). We would diagram the valid argument 'Some barber shaves every man. Every barber is a man. So some barber shaves himself' as follows. Second premise:



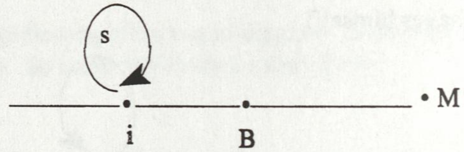
Adding the first premise:



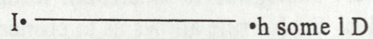
(i.e., 'Some barber shaves every man'). Given our pronominal convention (i-insertion) and our right terminus convention, we can read the conclusion directly from



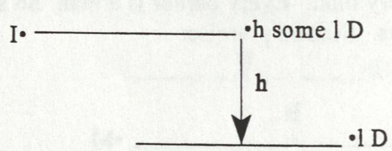
In particular, the latter convention permits movement at the arrowhead to the left (with the i-point as its left limit). In other words,



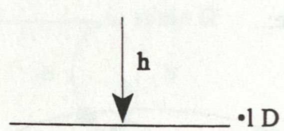
Next, consider the statement ‘Iago is a hater of a lover of Desdemona’. Leaving ‘hater of a lover of Desdemona’ unanalysed, we have



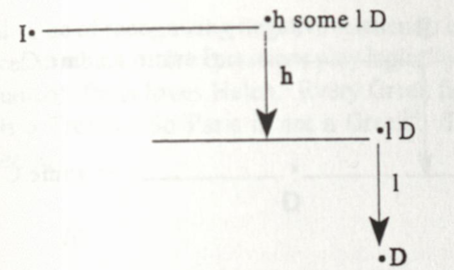
Here Iago (I) is an individual member of the set of things that hate (h) some lover of (l) Desdemona (D). We could analyse the relation here first as



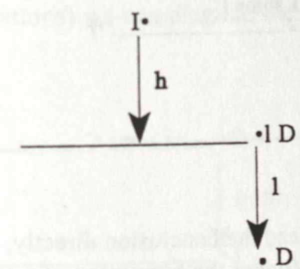
where



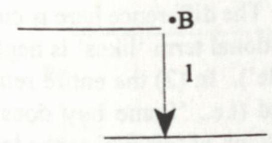
represents ‘hates a lover of Desdemona’. Further analysis yields



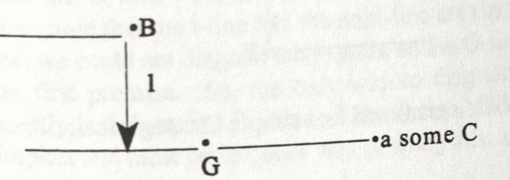
By suppressing some of the analytic content here, we could simplify this as



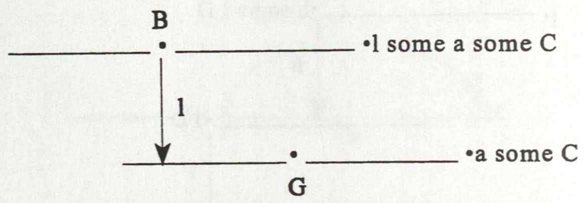
Finally, consider the argument ‘Every lover of an adorer of a cat is a fool. Every boy loves some girl. Every girl adores some cat. So every boy is a fool’. We diagram the second premise simply as



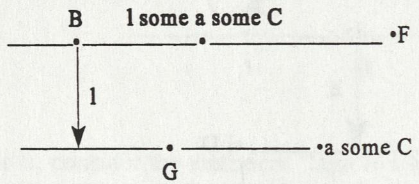
There is no need to analyse ‘adores some cat’, so we can add the representation of the third premise to give us



Now, by Rule R, we can add

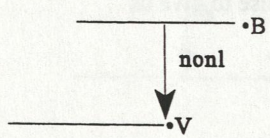


Finally, we add the first premise to get

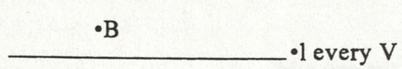


from which we can read the conclusion directly.

Let us note (under the prompting of our anonymous but friendly reader) that negative relationals can be diagrammed by our method. First, we must recognize that, as with nonrelational terms, relational terms have corresponding negatives. Consider 'Some boy does not like every vegetable'. Given no contextual clues, the sentence is ambiguous (in at least two ways), between (1) 'Some boy dislikes every vegetable' and (2) 'Some boy fails to like every vegetable'. The difference here is due to the scope of the negation. In (1) only the relational term 'likes' is negated (i.e., 'Some boy does not-like every vegetable'). In (2) the entire relational predicate 'like every vegetable' is negated (i.e., 'Some boy does not-(like every vegetable)'), an O form. We can think of 'dislike' as the logical contrary of 'like'. Such relationals are diagrammed just as nonnegative relationals are diagrammed. Thus, (1) can be diagrammed as

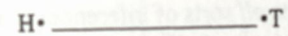


(2) can be diagrammed as a simple O categorical:



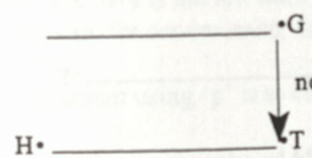
The real value of recognizing negative relationals is seen when we approach inferences in which such expressions play logically effective roles. Consider the argument 'Paris loves Helen. Every Greek fails to love every Trojan. Helen is a Trojan. So Paris is not a Greek'. The third premise is diagrammed as

(i)



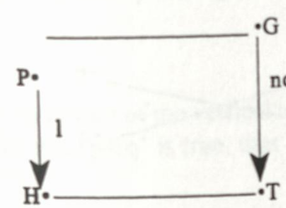
Now (without accounting for the relations between contrary and contradictory negative relations) we can diagram the second premise (along with the third) as

(ii)



Adding the first premise, we have

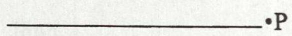
(iii)



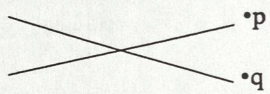
At this point, the question naturally arises concerning how we know that the P-point is *not* on the G-line. For if it is not, then our conclusion is diagrammed. First, note that the l-line and the nonl-line are (indeed, must be) parallel. Now, we could not diagram the P-point on the G-line without contradicting the first premise. So, the only way to diagram all three premises consistently is to keep the P-point off the G-line. Diagram (iii) represents the simplest and most perspicuous way of doing this, and from it the conclusion is easily read.

Let us turn now to the diagramming of compound statements. Compound statements of forms ‘p and q’, ‘p and not q’, ‘if p then q’ and ‘neither p nor q’ could be exhibited using Venn-type diagrams, where the circles are labelled ‘p’ and ‘q’ and each is taken to represent all states of the world for which the labelling statement is true. Truth tables and truth trees have generally supplanted the use of diagrams for analysing inferences involving compound statements. Nonetheless, my linear diagram system can extend to compounds, permitting an easy geometric representation for inferences involving such statements. Besides, the initial idea that there ought to be a single diagram system for *all* sorts of inferences is a sound one and mirrors my Sommersian claim that a single logic of terms suffices for both analysed and unanalysed statements.

We begin with some preliminaries. Noncompound statements (symbolized by appropriate lower-case letters) will be diagrammed by line segments labelled at their right termini by statement symbols. Thus, for example, a statement symbolized by ‘p’ will be diagrammed as

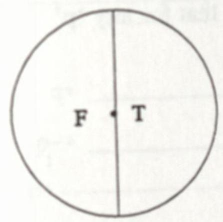


Each point on the line could be taken as a p-state. Again, to assert or state ‘p’ is to claim that the state(s) that make it true (the p-states) are among the states that obtain—that is, characterize the domain of discourse—the world. Thus, to assert both ‘p’ and ‘q’—that is, to assert ‘p&q’—is to assert that at least one of the states that obtains is both a p-state and a q-state. In other words, ‘p&q’ is true whenever the p- and q-lines intersect:



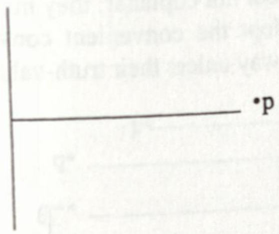
What is wanted, of course, is a way to represent states that obtain. Let us imagine that all line segments are in a single plane. For practical purposes, we can assume that the plane is finite in area so that it has a central point (it is easiest to think of the plane as having a circular perimeter). Assume as well that the plane is bisected along its vertical axis, resulting in a positive (right) sub-plane and a negative (left) sub-plane. Let us label the central point ‘T’, and think of lines extending from T to the right at any angle except ninety degrees vertically. We will call the right sub-plane of our plane the ‘T-field’. Let ‘F’ also label the central point. Lines from F extend to the left in just the way that lines extend to the right from T. We will call the left sub-plane the ‘F-field’. T will represent those states that obtain. To make a truth claim by the use of a statement is to claim that the

state of affairs expressed (signified) by that statement obtains (i.e., that the statement is true, that the line representing it lies in the T-field). T is truth; F, naturally, is falsehood. In the left, or F-field, lines will be labelled at their left rather than right terminus. A very rough picture of our plane would look like this:

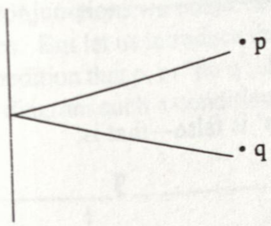


The important vertical line here is the left limit of the T-field and the right limit of the F-field. We can, for convenience, think of it as a single line that is the T-line or the F-line.

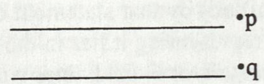
To make a statement using ‘p’ is to claim that ‘p’ is true. Thus:



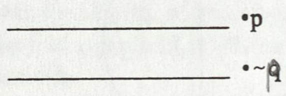
(Here we take the mid-point of the vertical to be the T-point.) To assert that p and q is to claim that ‘p&q’ is true, that T is on each line, and that both lines intersect at T:



To deny a conjunction (e.g., ~(p&q)) is to claim that no p-state is a q-state, that the p-line and q-line do not intersect, that they are parallel:

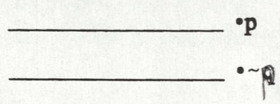


Since no statement and its negation are ever both true together, the law of noncontradiction guarantees that for any 'p'

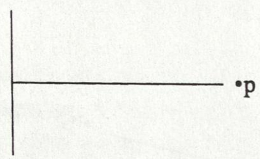


Excluded Middle is guaranteed by the bifurcation of our plane, so that any line segment must lie in either the T-field or the F-field, either to the left or to the right of the vertical.

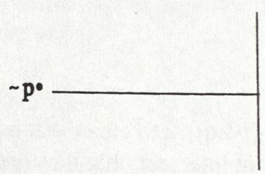
To say that two line segments are parallel is not to suggest that they lie in the same field. Since every line segment in the T-field must have T as its left terminus (and F as its right terminus if in the left field), no two line segments in a given field are ever parallel. Any two parallel line segments must, in fact, be colinear but not coplanar; they must lie in opposite fields. Nonetheless, we will adopt the convenient convention of representing parallel lines in the usual way unless their truth-values are known. Thus, in general we know that



But, knowing that 'p' is true—that is,

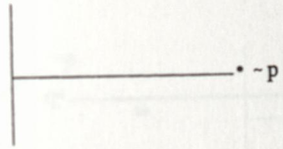


we likewise know that '~p' is false—that is,

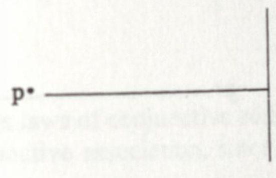


This suggests that we can represent the negation of a statement, say 'p', in two equivalent ways:

~p:

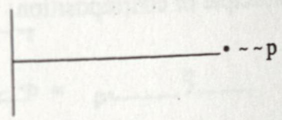


~p:

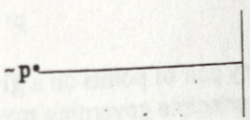


The law of double negation can be diagrammed in three steps:

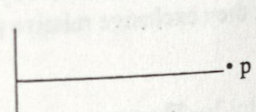
(1)



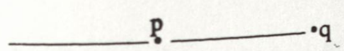
(2)



(3)

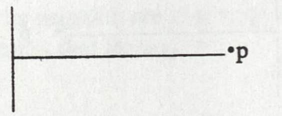


With negation and conjunction, we could define all other possible truth-functional connectives. But let us introduce the conditional on its own. To claim that q on the condition that p, or 'p⊃ q', is to claim that all p-states are q-states. So we can diagram such a conditional as

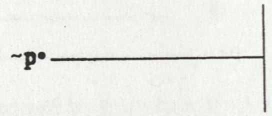


(Notice how the common diagrams for I categoricals and conjunctive statements, and for A categoricals and conditional statements, reinforce Sommers's claim that the logic of compound statements is part of a logic of terms.)

I now introduce some other important principles of equivalence, which will apply to our diagrammatic analysis of inferences to come. We saw above that



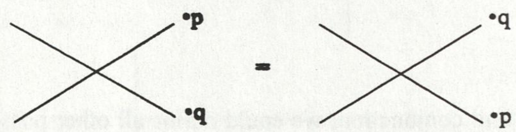
and



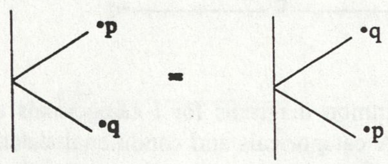
are equivalent. If we take T and F to be contradictions of one another, then we have, in effect, allowed a pair of points on a line segment to exchange places while reversing their signs of polarity (negated/unnegated). Indeed, this is just the principle of contraposition:

which holds for any pair of points on a given line segment.

Another principle governing my diagrams holds that any pair of intersecting line segments can each rotate by any degree around the point of intersection so that they exchange relative positions. So, in general,

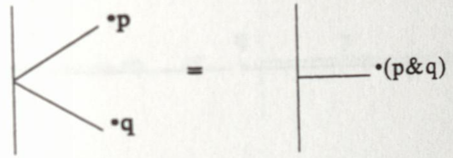


and specifically,

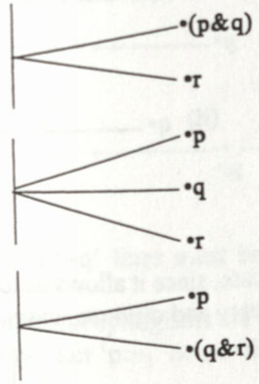


This "principle of rotation" guarantees commutability for conjunction.

Finally, a third "principle of composition/decomposition" holds that

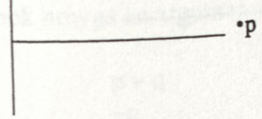


This principle guarantees laws of conjunctive addition and simplification. It also guarantees conjunctive association, since the following three are equivalent:

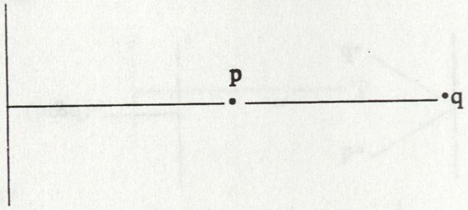


We will now examine some examples of inferences in order to illustrate how they are analysed by the use of linear diagrams. Again, the diagramming of all premises is followed by an inspection of the resulting diagram to see whether or not the conclusion has been diagrammed already. If so, the inference is valid; otherwise, it is invalid.

A simple *modus ponens* argument form is easily diagrammed to show its validity. First, we diagram the unconditional premise, 'p', as

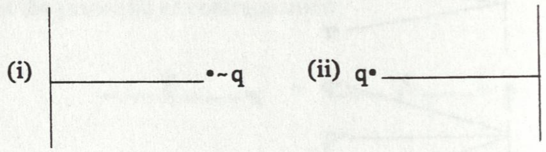


The conditional premise demands that all p-states be q-states, so that the p-line must be a (possibly proper) part of the q-line. Adding this gives us

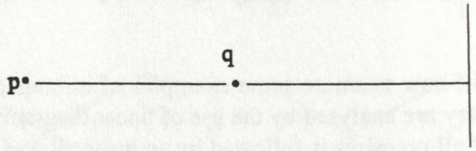


Since the T-point extends at virtually any angle to the right, 'q' is on a line intersecting T. So the conclusion, 'q', has already been diagrammed.

While the diagram for a *modus ponens* argument form falls in the T-field, one for a *modus tollens* argument form will fall in the F-field. The premise '~q' can be diagrammed in two equivalent ways:

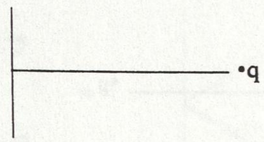


We choose the second way in this case, since it allows us to add the diagram for the conditional premise in an easy and obvious way:

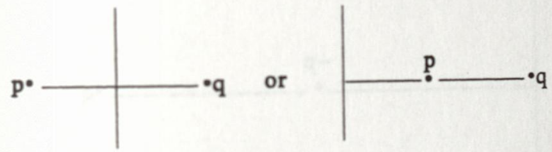


(Using (i) and contraposition would not have meant much more difficulty.)

Consider next an example of affirming the consequent. Diagramming our unconditional, 'q', we have

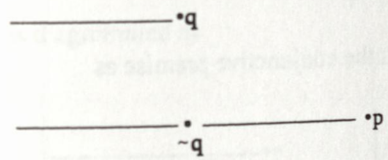


According to the conditional premise every p-state is a q-state, so that the p-line must be a part of the q-line. So, either



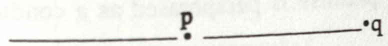
But, as we cannot know which, we cannot have already diagrammed the conclusion.

A second way to illustrate the invalidity of affirming the consequent would be to diagram the unconditional and then the contrapositive of the conditional—that is,

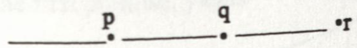


Since the 'q' and '~q' lines must be parallel, the conclusion cannot be diagrammed.

Hypothetical syllogisms are diagrammed in an especially simple and obvious way. Let 'p⊃q' be our first premise.



Adding our second conditional premise, 'q⊃ r', gives us

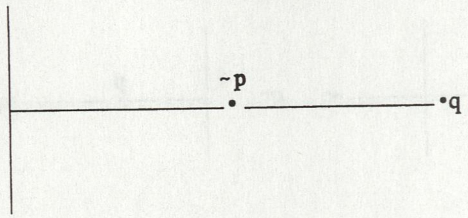


And we see at once that our conclusion, 'p⊃r', is already diagrammed.

Let us look now at an argument of the form

p v q
~p
q

First, we paraphrase the disjunction as a conditional ('~p⊃ q'). Our diagram, then, is a familiar one:

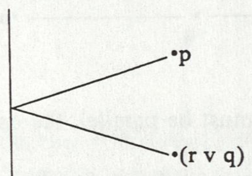


illustrating validity.

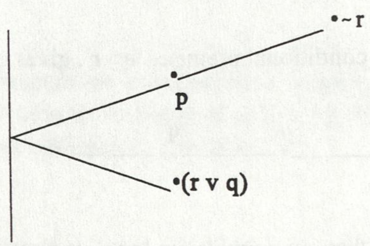
Let us consider next a (slightly) more complex example:

p & (r v q)
~p v ~r
~q

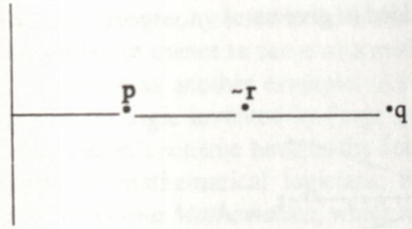
First, we diagram the conjunctive premise as



Next, the second premise is paraphrased as a conditional, 'p⊃ ~r', and diagrammed:



Paraphrasing 'r v q' as '~r⊃q' now requires

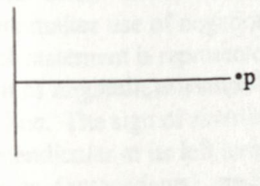


which shows that the argument is invalid, since the conclusion is not —indeed, cannot be—diagrammed.

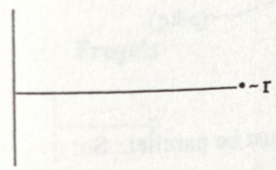
Finally, one last example:

(p & q) ⊃ r
~r
p
~q

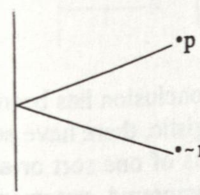
The third premise is diagrammed as



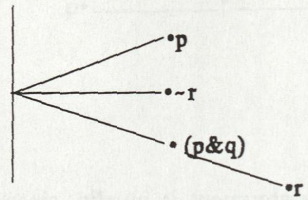
The second premise is diagrammed as



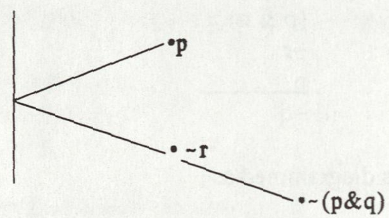
which, added to the first premise, yields



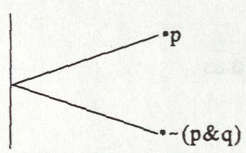
The first premise can be added to give us



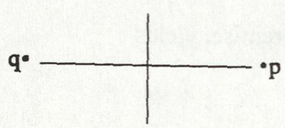
which we know is equivalent by contraposition to



From this we know, by *modus ponens*, that



Thus the p- and q-lines must be parallel. So:

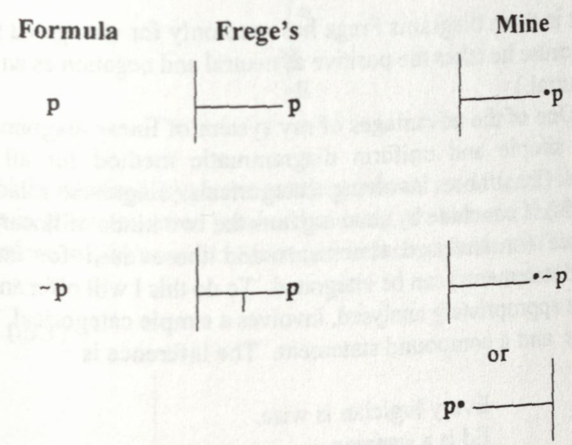


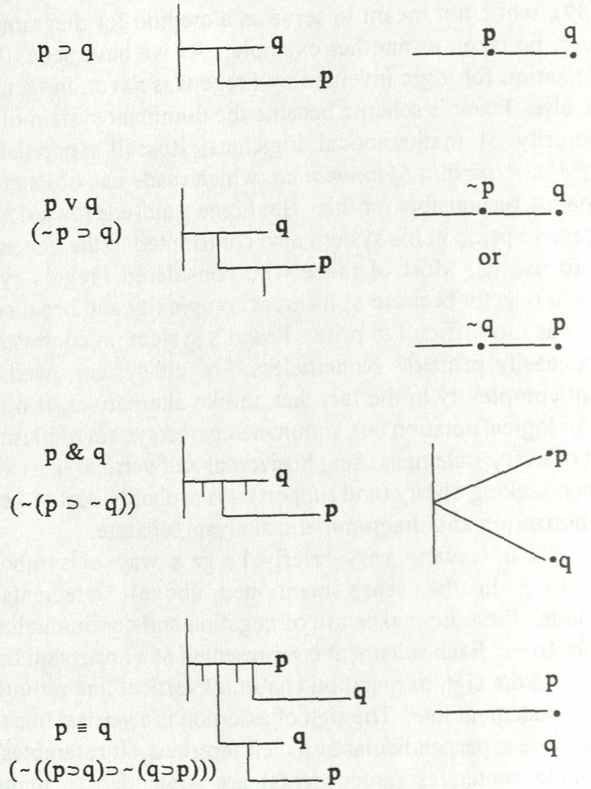
The argument is valid since the conclusion has been diagrammed.

Unlike the case for syllogistic, there have actually been a number of attempts to use linear diagrams of one sort or another to display and analyse inferences involving compound statements. A truth tree, for example, could be viewed as a kind of linear diagram. Frege's well-known

but rarely used ideography introduced in the *Begriffschrift* (cf. Murawski, 1988-89), while not meant to serve as a method for diagramming, could, however, be taken as another example. As we have seen, the system of symbolization for logic invented by Frege was never, in fact, accepted by anyone else. Peano's scheme became the dominant system of notation for the majority of mathematical logicians; Russell's popularity and the influence of *Principia Mathematica*, which made use of Peano's notation, were mainly responsible for this. But Frege's attitude toward his critics and his excessive pride in his system also contributed to the general refusal by others to use it. Most of those who considered Frege's symbolization claimed to reject it because of its great complexity and because, they often said, it was too difficult to print. Peano's system, in contrast, was simple and was easily printed. Nonetheless, Frege's system owed much of its apparent complexity to the fact that, unlike alternatives, it was not only a system of logical notation but, simultaneously, a system of illustration of the content of every statement using horizontal and vertical lines. Suffice it to say, when seeking clarity (and support) it is probably best to keep the tasks of symbolization and diagrammatic analysis separate.

Let us review very briefly Frege's way of symbolizing (i.e., diagramming in the sense mentioned above) statements and their compounds. First, he makes use of negation and conditionalization as his only primitives. Each statement is represented as a horizontal line (as in my system), and the sign of negation is a small vertical line orthogonal to and below the statement line. The sign of assertion is a vertical line to which the statement line is perpendicular at its left terminus. It resembles my T-line. Subordinate sentences (antecedents) are subtended to main sentences (consequents). Here is a comparison of some of Frege's diagrams with my own.





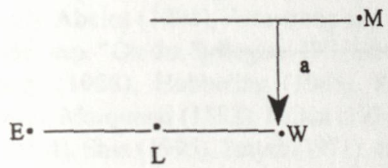
(Note that in these diagrams Frege has need only for unnegated sentences. This is because he takes the positive as neutral and negation as an operation on the neutral.)

One of the advantages of my system of linear diagrams is that it allows a simple and uniform diagrammatic method for all kinds of inferences (i.e., those involving categoricals, singulars, relationals, or compounds). I conclude by showing how the two kinds of linear diagrams (those used for analysed statements and those used for unanalysed, compound statements) can be integrated. To do this I will offer an inference that, when appropriately analysed, involves a simple categorical, singulars, relationals, and a compound statement. The inference is

Every logician is wise.
Ed is a logician.
Some mathematician admires whatever is wise.
If Ed is admired by any mathematician, then some mathematician is a fool.
Therefore, some mathematician is a fool.

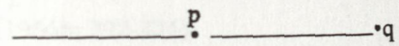
The first three premises are diagrammed together as

(i)

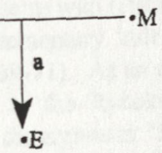


For now, let us diagram the last premise, the conditional, as

(ii)

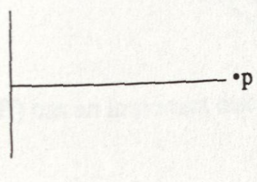


Using the rules and conventions established for diagramming analysed statements (especially: if an arrow terminus touches a line it touches every point on that line to the left, and a converse of a relational results from reading its terms in an order other than tail-to-head), we can conclude from (i):



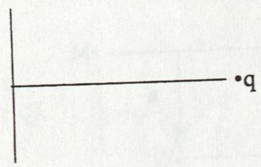
(i.e., 'Ed is admired by some mathematician'). Indeed, since all the premises in (i) are asserted (claimed as true), we can add the T-line to (i), and therefore to (iii) as well. Since (iii) is the antecedent of (ii)—that is,

(iii.1)



by *modus ponens* from (iii.1) and (ii) we get

(iv)



our conclusion.

Notes for Chapter 4

[1] A substantial portion of this section comprises Englebretsen (1992). It appears here by permission of the editors of the *Notre Dame Journal of Formal Logic* and the University of Notre Dame.

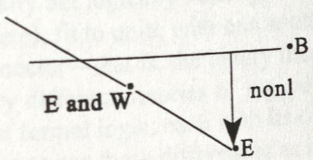
[2] See especially Abeles (1991), Armstrong and Howe (1990), De Morgan (1966, esp. "On the Syllogism II"), Edwards (1989), Gardner and Harary (1988), Hubbeling (1965), Karnaugh (1953), Keynes (1906), Marquand (1883), Peirce (1931-35, "Existential Graphs" in vol. 4), Shin (1995), Smyth (1971), and, of course, Venn (1971). A thorough survey of diagram systems for logic is found in Gardner (1982). A system of diagrams motivated by purely ontological rather than logical considerations is found in Smith (1992).

[3] Discussions of possible Aristotelian diagrams are found in Ross (1965: 301-2) and in Kneale and Kneale (1962: 71-72). See also Flannery (1987).

[4] See Leibniz (1966b: 292-324).

[5] Lambert (1965: 111-50). Critiques of Lambert's system are found in Venn (1971: 504-27), Peirce ("Existential Graphs" in 1931-35, and Keynes (1906: 243-47).

[6] Rybak and Rybak (1976, 1984a, 1984b) use a version of Karnaugh Maps to analyse inferences involving more than four terms, singular terms, compounds, and relationals. However, their system is far from simple and perspicuous, requiring arguments with many terms to be "split" (1976: 473), large maps to contain discontinuous cells (1976: 472), and arguments with relationals to require, in addition to a series of supplementary rules, a complex "streaming procedure" (1984b: 269-71). As an example of how my system compares favourably to the Rybaks', consider their diagram (1984b: 272, fig.1) for the argument: 'Some botanists are eccentric women. Some botanists do not like any eccentric person. Therefore, some botanists are not liked by all botanists.' The linear diagram is simpler, faster to construct, and far easier to read:



[7] Geach (1962: 116ff) has an important discussion of such sentences.