3 Lines of Reason

3.1 New Lines (Smyth and Pagnan)

> [T]he syllogism admits of a graphical representation which is as suggestive as a diagram of geometry.
>
> De Morgan

We saw above that, as Wesoły's research suggests, Aristotle might well have used some sort of diagrams using labelled line segments to illustrate the forms of perfect syllogisms. What is certain is that by the 17th century logicians had begun projects designed to use diagrams not only to reveal the forms of syllogisms but to provide illustrations of syllogistic reductions. It appears that Leibniz (followed then by Lambert) was the first logician to attempt such a system of diagrams for syllogisms based on line segments rather than closed plane figures such as circles. Yet, in the long run, systems of logical diagrams based on the use of closed figures (such as those of Euler, Venn, and Peirce) won out. As we have seen, there was a significant decline in interest in logic diagrams during the first several decades of the 20th century. Nonetheless, we have also seen that there has been something of a "renaissance of diagrammatology" of late. Not surprisingly, attempts to develop systems of logic diagrams based on the use of line segments have been part of that renaissance. One might consider M. B. Smyth's "A Diagrammatic Treatment of Syllogistic" (Smyth 1971) as an early setting of the stage for this part of the renaissance.

Smyth proposed a system of "directed graphs" that could effectively be used to represent standard categorical propositions, with the restriction that they contain no complex, negative, or empty terms (Smyth 1971, 483). The use of such a system allows one to "read off" from a graph of an arbitrary finite set of such propositions all logical consequences of that set. Soundness and completeness proofs for the system are briefly sketched (Smyth 1971, 485). Smyth also explores what he terms "the general structure of valid syllogisms" (Smyth 1971, 485–488). So, what is a directed graph?

A directed graph is a diagram in which each premise of an argument is represented. Each term of a proposition is represented by a point ("vertex") at one end of a line segment that may or may not contain a directional arrow, an interruption, or another term. This is best understood by looking at how the standard categoricals are represented.

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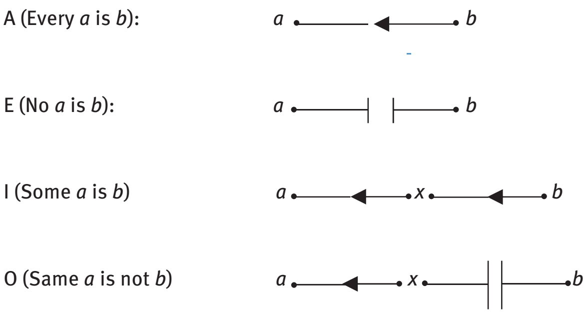


Figure 3.1: Directed Graphs of A, E, I, and O

It's obvious that Smyth's notational and graphic system is hardly simple. And there is more. In diagramming an argument, Smyth was intent on guaranteeing that the directed graph generally conforms to a shape that has a number of distinct "branches." This is achieved by representing terms that are positively connected to one term (i. e. by line segment containing a directional arrow) but is negatively connected to another term (i. e. by an interrupted line segment) as a line segment that is *bent* from the horizontal at its midpoint. Such terms are labelled on a graph at their midpoints. An example of a directed graph from Smyth (Smyth 1971, 484) illustrates this.

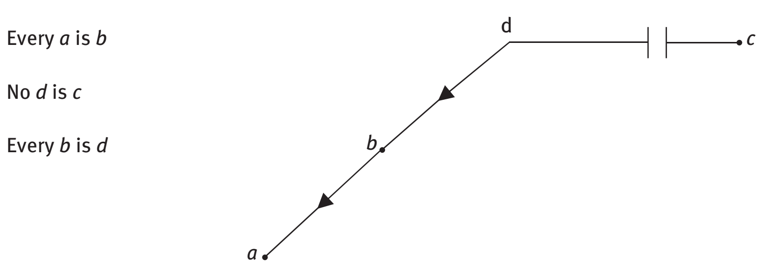


Figure 3.2: Directed Graph of an Argument

The conclusions that can logically be drawn here are those propositions that can be read off. How? A vertex is a *descendant* of another vertex if there is a progression of directed line segments leading from the latter to the former. Every vertex is a descendant of itself. Two terms that are on interrupted branches of a graph are mutually excluded.

(i) If *a* is a descendant of *b* in a graph, then A*ab* can be inferred.
(ii) If *a* and *b* are mutually excluded in a graph, then E*ab* can be inferred.
(iii) If *a* and *b* have a common descendant, then I*ab* can be inferred.

(iv) If *a* has a descendant that is excluded from *b* in a graph, then O*ab* can be inferred.

So, the conclusions that can readily be seen to have already been diagramed (thus read off) are those that just happen to be determined by the above rules. In the case of the three premises diagrammed in Figure 3.2, one can initially draw the following conclusions:

A*ad*, by (i)
E*ac*, by (ii)
E*bd*, by (ii)
and, trivially: A*aa*, A*bb*, A*cc*, A*dd*, E*ca*, E*db*.

It will be useful later on in this chapter to make comparisons between Smyth's directed graphs and other similar linear diagram systems. To that end, consider for now directed graphs for the four Aristotelian perfect syllogisms.

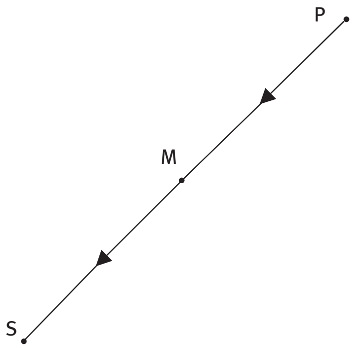


Figure 3.3: Directed Graph of Barbara

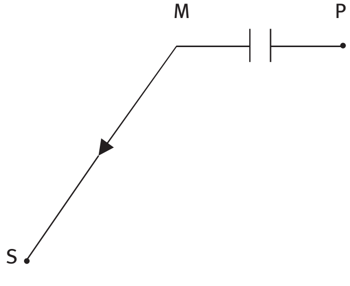


Figure 3.4: Directed Graph of Celarent

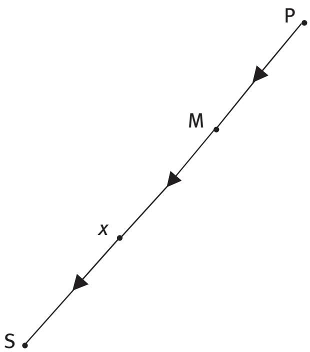


Figure 3.5: Directed Graph of Darii

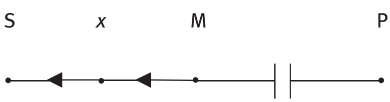


Figure 3.6: Directed Graph of Ferio

The following should be noted. The use of what Smyth calls "arcs" (line segments containing directional arrows) is never clear. Thus, the graphic representation of 'Some *a* is *b*' looks as if it could be read as 'Every *a* is *x* and every *x* is *b*' unless the role of the unknown singular term, *x*, is more fully specified. Also, in considering directed graphs, the limits imposed on them should be kept in mind: no negative terms, no empty terms, and no complex (thus no relational) terms.

In the period following Leibniz and Lambert, it would seem that Smyth was a pioneer in the area of linear logical diagrams. His work appeared only in a brief essay in 1971. It would be two more decades before another attempt at a system of diagrams for logic based on the use of points and line segments (i.e. linear diagrams). That system was first presented in 1992, and, like Smyth's essay, this one also appeared in the *Notre Dame Journal of Formal Logic*. The next several sections of this chapter will focus on that system. But, first, for its value in its own right and its value for comparisons, we will jump ahead to the 21st century and look at the system of linear diagrams developed by Ruggero Pagnan.

In recent years, Pagnan has been engaged in a program that is rich in promise (Pagnan 2010, 2012, 2013a, 2013b). The heart of this program is a system of linear logical diagrams that is meant to be at once algebraic and graphic. The result is a heterogeneous formal system of logic. We offer below a brief summary of Pagnan's system, highlighting some of its main elements, and provide example diagrams to illustrate it. There is of course more to his program than this summary of his system suggests. For example, he has shown (Pagnan 2013b) how his account of syllogistic can be seen as a part of an intuitionistic version of the sequent calculus of linear logic, and he extends the account, given in earlier papers, of De Morgan's syllogistic with "complemented" (negated) terms. In each of his papers (especially Pagnan 2010), with due credit to Smyth, who had done the same for his own system, Pagnan illustrates how his system can extend to n-term syllogisms. However, it is his core system for diagramming syllogisms (admitting negated terms) that requires our attention now.

Pagnan calls his system "SYLL" a system that is heterogeneous in that it makes use of both graphic and linguistic syntactical objects. Terms are represented by letter labels (called "term-variables") and their relations in a given statement by a series of directed arrows (→,←) and "bullets" (dots: ●). Its diagrammatic representations are linear, making no use of any closed curves. In effect, a well-formed SYLL diagram of a categorical statement is a pair of term letters separated by a finite list of arrows, themselves separated by a bullet or a term letter. The four standard categoricals are represented as follows:

A: S → P,
E: S → ● ← P,
I: S ← ● → P.
O: S ← ● → ● P.

According to De Morgan, "In the form of the proposition, the copula is made as abstract as the terms: or is considered as obeying only those conditions which are necessary to inference" (De Morgan 1966, ix). As Pagnan notes, the series of arrows and bullets between such pairs of terms amounts to an *abstract logical copula*, (Pagnan 2012, 35). One could read the categorical diagrams as, correspondingly, 'P belongs to every S', 'not-P belongs to every S', 'P belongs to some thing that S belongs to', and 'not-P belongs to some thing that S belongs to'. Pagnan also permits statements to be equivalently represented by the "reversals" (mirror images) of their usual diagrams, making use of either kind of representation throughout Pagnan 2012 and 2013b. Thus an A categorical could be represented as: P ← S, so that the arrow here could be read as 'belongs to every'. Note that, reversed or not, the bullets – and their positions relative to the directions of the adjacent arrows – carry substantial logical weight. For example, → ● X (or its reversal) can be taken (though Pagan doesn't say so) as diagramming 'not-X'; a bullet between the tails of two arrows could be taken as the representation of a particular (existential) quantifier; and X → … (or its reversal) can be taken as diagramming '… belongs to every'.

In SYLL, syllogistic inference amounts to concatenation and reduction. “Two or more syllogistic diagrams can be concatenated and reduced, if possible, by formally composing two or more consecutive and accordingly oriented arrow symbols separated by a single term-variable, thus deleting it” (Pagnan 2013a, 36). If such a concatenation and reduction is possible, then the syllogism is valid; otherwise it is invalid. *Concatenation* and *reduction* works, essentially as follows. The two premises are diagrammed and placed adjacent to each other in such a way that the term common to each (viz., the middle term) is the right-most term of the premise on the left and the left-most term of the premise on the right. Then the two tokens of the middle term are amalgamated into a single token, resulting in a new, single diagram. Finally, the middle term is deleted representing the conclusion and pairs of commonly directed arrows are amalgamated. Often, in practice, the first step is avoided by simply diagramming the premises together while amalgamating the two middle term tokens. For illustration, here are the syllogistic diagrams of the four perfect syllogisms.

Barbara	Celarent	Darii	Ferio
S → M → P	S → M → ● ← P	S← ● → M → P	S ← ● → M → ● ← P
___________	______________	_____________	__________________
S → P	S → ● ← P	S ← ● → P	S ← ● → ● P

Figure 3.7: SYLL Diagrams of the Four Perfect Syllogisms

Imperfect syllogisms can be given similar graphic treatment, reducing each of them to a perfect syllogism by applying rules of conversion, premise re-ordering, and term substitution. In each case, a necessary (but not sufficient) condition for the validity of any diagrammed syllogism is that the diagram of the conclusion contains as many bullets as the conclusion (Pagnan 2013a, 36).

So-called strengthened (sometimes weakened) syllogisms can be treated in SYLL as well. According to Pagnan, existential import is not implicit in any categorical other than one of the form ‘Some X is X’ (‘Something is X’, ‘There is an X’, ‘Some thing is X’, ‘There exists an X’, etc.). This is always the form of a contingent statement, and the diagram for which is X ← ● → X. Its contradictory, ‘No X is X must be contingent as well. Moreover, since ‘Some X is not X’ is a contradiction, ‘Every X is X’ must be tautologous. Pagnan calls X→ X and X ← ● → X the “laws of identity” (Pagnan 2013a, 40). Strengthened syllogisms require an implicit premise of this form. Consider the fourth figure syllogism Baramtip.

The two universal premises must be supplemented by an implicit premise of the form ‘Some S is S’. Diagrammed in SYLL, this yields:

S ← ● → S → M → P

____

S ← ● → P

Figure 3.8: SYLL Diagram of Baramtip

Note the new rule applied here. Two tokens of a given term letter flanking ← ● → can be amalgamated. Thus … X ← ● → X.. simply reduces to … X … ..

Indirect reductions, as usual, assume the contradictory of the conclusion and then derive an explicit contradiction. Such a contradiction will have the form X ← ● → ● ← X. For example, here is a diagrammatic proof that an E statement and the corresponding I statement are contradictory by deriving a contradiction from the pair of them:

S ← ● → P   P → ● ← S

____

S ← ● → ● ← S

Figure 3.9: SYLL Proof that E and I are Contradictory

Note that in this derivation that 1) the E premise is used in its equivalent revered form and that 2) the conclusion has the form of a contradiction, viz. X ← ● → ● ← X.

One of the ways Pagnan wants to extend traditional syllogistic is by incorporating “complemented” (negated) terms (Pagnan 2013a, 49), which he introduces in the context of his partial adoption of De Morgan’s *spicular notation.* De Morgan used the lowercase form to represent a term’s negation (thus ‘a’ is the negation of ‘A’). This would allow a slight simplification in Pagnan’s formalism. An E categorical could now be represented as S → p and an O categorical could be represented as S ← ● → p. De Morgan’s spicular notation uses left and right parentheses to symbolize quantity and dots to indicate affirmation or denial. In the spicular system of logical notation, X) and (X are meant to formalize ‘every X’ while X(and)X formalize 'some X'; an even (or no) string of dots between parentheses (whatever the orientation of the parentheses) indicates affirmation and an odd string of dots indicates denial. For example, an E categorical would be symbolized as S)●(P and an O categorical would become S(●(P. Using De Morgan's notation for term negation, the four standard categoricals can be more simply represented as follows;

A: S))P
E: S))p
I: S( )P
O: S( )p

Pagnan claims that his diagrammatic system supports De Morgan's formal system:

> The possibility of making a distinction between a term being universally or particularly quantified, as well as between affirmative and negative modes of predication is supported by the diagrammatic formalism [SYLL] … together with the possibility of handling complements of terms. Indeed, we can look at the symbols ●, → and ← as to fundamental ones. In a diagram built as a combination of such fundamental symbols, a term-variable X is universally quantified if it enters in it as X → or ← X, whereas it is particularly quantified if it enters in it as X ← or → X. The complement of X is represented as X → ● or ● ← X, both of which may be abbreviated as x. (Pagnan 2013a, 50)

For the most part, Pagnan is right here. We have already noted how the right combination of arrows and bullets can represent term negation. But Pagnan (and sometimes De Morgan) is wrong in thinking that a combination of an arrow adjacent to a term always indicates quantity, for this would commit him to the quantification of predicate-terms. For example, the universal affirmation, X → Y, would have to be interpreted as 'Every X is some Y'. What Pagnan's arrows (and De Morgan's parentheses) actually indicate is the *distribution value* (distributed or undistributed) of the term to which they are attached. Put simply, terms at the tails of arrows are distributed; terms at the heads of arrows are undistributed. In order to see this more fully one must get clear about both distribution and the notion of a logical copula in a logic of terms (such as syllogistic). These things will come under much closer scrutiny in subsequent sections of this chapter. But, before going on to those sections, a few general comments and observations about SYLL are in order now.

The SYLL system guarantees that all diagrams are unambiguous. It can accommodate n-term syllogisms. It represents the symmetry of E and I form statements in an explicit and obvious way. SYLL accurately takes a strengthened syllogism to be an enthymeme whose tacit premise has the form 'Some X is X', 'There is an X', 'An X exists', etc. (X ← ● → X). SYLL dispenses with the traditional subject/predicate distinction. As well, SYLL is able to represent term complementation (term negation). Finally, SYLL implicitly (but unadmitted by Pagnan) is able to represent the distinction between distributed and undistributed terms. These are all important characteristics that are wanted in any viable formal account of syllogistic logic. Nonetheless, SYLL does have some unwelcome characteristics. SYLL provides no means of symbolically or graphically representing the contradictoriness of a pair of contradictory statements – no way to represent sentential negation. It provides no way to represent relational terms. Furthermore, it makes no provisions for the representation of singular terms (e.g., proper names, definite descriptions, anaphoric pronouns, etc.). Finally, there is a feature that is most disturbing about SYLL. As we have seen, the difference between linguistic expression and graphic representation is a matter of degree rather than kind. Pagnan's diagrams seem to fall on the scale much closer to the linguistic end than to the graphic end. They look more like logical formulae than logical diagrams. S ← ● → P looks closer to ∃x(Sx & Px) than to Euler's two overlapping circles. Indeed, the string of symbols ← ● → looks to be simply a symbolic rendering of Aristotle's 'belongs to some'. Pagnan's diagrams *are linear*. But are they *diagrams?* SYLL "diagrams" certainly don't appear to be graphic in any significant sense. SYLL proofs don't seem to rely on visual inference. SYLL might easily be construed as nothing more than an alternative symbolic system, on a par with the standard symbolic system of first-order monadic predicate logic.

So what *is* wanted in a linear (and two-dimensional) diagrammatic system for logic? In the remainder of this chapter we will present such a system of logical diagrams, one that will go well beyond previous systems in its graphic capacities. It is the system referred to earlier, the one first presented in 1992a. It is the system called *Englebretsen Diagrams* (Rauf 1996, 397–408).

3.2 In Logical Terms: Term Functor Logic

> All our logics are now but a shadow of what I should wish and what I see from afar.
> Leibniz

When Venn developed his system of logical diagrams, he did so with the intention of providing a graphic analogue to Boole's algebra of logic. The system of Englebretsen Diagrams (from now on (ED) (Englebretsen 1992a, 1996) was likewise meant to provide a graphic analogue to Fred Sommers' term logic (Sommers 1967, 1969. 1970, 1973, 1975. 1976a, 1976b, 1976c, 1976d, 1981, 1982, 1983a, 1990, 1993, 2000, 2005a, 2005b, and Sommers and Englebretsen 2000). Sommers' version of term logic, which he came to refer to as Term Functor Logic (TFL), was explored, defended, emended, and exploited in many places by Englebretsen (for example, Englebretsen 1981, 1996.2002, 2005, 2015, and especially 2016b; also Englebretsen and Sayward 2011). The key to understanding any term logic such as Aristotle's syllogistic logic or Sommers' TFL is its account of logical syntax.

The grammatical form of any natural language statement is determined by the grammatical conventions of the pertinent linguistic community. Such conventions (grammar rules) exhibit a variety of features that are meant to help encode information. Different languages make do with featuring different kinds of information. Thus some languages exhibit a variety of tenses, others systematically encode gender, and so forth. As well, natural languages often exhibit redundancy of information (for example, English marks number (singular/plural) on both subject expressions and main verbs). Much of this sort of information is immaterial to the interests of logicians. The logician seeks those features of a statement that are involved in determining the statement's role in logical inference, etc. In short, the logician looks beneath the "surface" grammar of natural language statements in hopes of finding their *logical grammar*, their logical forms. When Aristotle sought logical form, he initially followed his teacher Plato by taking his clues about what the logical form of a statement might be from select features of his native Greek. Plato and the early Aristotle took the basic logical form of any statement to be a noun connected to a verb (e.g., 'Theatetus walks'). A logically formed statement cannot be constructed from a pair of nouns, nor can it be formed from a pair of verbs. One noun and one verb are required. The logical form is the grammatical form minus any accidental features that are of no logical concern. How does a noun connect with a verb to form a statement rather than just a pair of words? How is unity achieved? According to Plato, the noun and the verb "mix" in the sense that they are simply *fit* for one another. Think of the verb as a board with a round hole and the noun as a round peg fitting that hole. As it happens, two-and-a-half millennia later, that turned out to be just the idea Frege had when he came to the problem of determining logical form and the so-called "problem of propositional unity." Of course Frege took his clues about logical form not from Greek grammar (or even the grammar of his German). His clues came from the mathematical notion of functions. A function applies to one or more "arguments" to yield a new expression that has a value. For example, the square root function applied to the argument 4, yielding an expression ($\sqrt{4}$), has the value ± 2. For Frege, the logical form of a simple statement was a function (essentially a predicate), which is "unsaturated" or incomplete (having holes) along with an appropriate number of arguments, “names” (essentially singular denoting expressions) that are “saturated” or complete and fitting into those holes. The result is a statement whose logical unity is guaranteed by the pegs filling the holes so that the statement is complete. Thus a statement is, in turn, itself a peg, an expression fit for filling the holes in “higher” predicates (e..g., sentential “truth functions” such as ‘if…then’ and ‘or’). As well, a statement, being the result of a function applied to an appropriate number of arguments, has a value. For Frege the value of a statement is its truth-value (what Frege called “the True” and “the False”). Theories of logical syntax such as these require that statements be construed as unified strings of terms … but there must be two fundamentally distinct kinds of terms: nouns and verbs (or names and predicates).The logician was thereby committed to a heterogeneous logical vocabulary, lexicon. But Aristotle eventually abandoned any such a theory.

By the time he composed *Prior Analytics*, Aristotle realized that mediate inference must involve at least two premises that share a term in common. Consequently, at least three terms are involved in such inferences. Moreover, and most importantly, Aristotle saw that at least one of those terms had to play a different logical role in at least two different statements. Since a noun can never play the role of a verb, nor can a verb ever play the role of a noun (from a logical point of view, they are fundamentally distinct kinds of terms), logical form cannot rest on such a distinction (see Englebretsen 1982b, 1986b). Aristotle’s solution was to adopt a logical lexicon that is homogenous; it simply consists of *terms*. Those terms certainly have grammatical features and grammatical roles to play, but these are beside the point of logic. Any term can play any logical role that any other term can play. So, how is a statement more than just a sting of terms? How is it a unit – logically? Here is an example of Aristotle’s genius. Pairs of terms are bound together to form a statement, a unit fit for the role of premise or conclusion, by a *logical copula*. A logical copula binds pairs of terms together. It literally facilitates the copulation of pairs of terms. A logical copula is a special-duty expression (a *formative*, a *syncategorematic* expression, a *logical constant*) whose job in a statement is manifold: it unites the terms, it determines the *quality* of the statement, and it determines the *quantity* of the statement. And it does this all at once. This is Aristotle’s theory of logical syntax. It is what makes syllogistic logic possible. On this theory of logical syntax, a statement is not the result of nouns/names fitting verbs/predicates; it’s a product of pairs of expressions (terms) being bound together: not pegs fitting holes but blocks glued together.

For Aristotle there were four kinds of glue, English versions of which are: ‘belongs to some’, ‘belongs to every’, ‘belongs to no’, and ‘does not belong to some’. Statements made by flanking any of these with a pair of terms are *cate-*

gorical. Thus, for example: 'Humour belongs to some speakers', 'Reason belongs to every man', 'Reason belongs to no fish', 'Humour does not belong to some speakers'. Syllogistic logic is the logic of categoricals. Notice that these English paraphrases are understandable but awkward (that was also true for Aristotle's Greek paraphrases). Aristotle could have eased this awkwardness by not merely paraphrasing but by fully *symbolizing*, rendering categoricals in an artificial language meant to display the logical form free of any natural language expressions. Aristotle did not go that far. However, he did take an important step in that direction. He allowed letters to stand in the place of natural language terms. In other words, he introduced symbolic *variables*, such as A, B, Γ, etc. to stand for different terms in different logical contexts. To this degree, Aristotle's syllogistic logic was a *symbolic logic*. Term variables certainly relieve some of the awkwardness of the categorical paraphrases ('H belongs to some S' is marginally better than 'Humour belongs to some speakers'), but not much. Late medieval logicians helped here. They *split* the copula: one fragment went with one term and another fragment went with the second term; Next, they rearranged these terms and fragments so that they look more like natural language statements (a kind of logicized Latin (for a brilliant account of such a language, which he calls "Linguish," see Parsons 2014). Thus, for example, 'H belongs to some S' became 'Some S is H'. This splitting of the copula reveals two of its functions in the statement by assigning each to a different fragment. Here, 'some' indicates quantity and 'is' indicates quality (but see Englebretsen 1990b and 1997). However, it is important to remember that quantity and quality do not characterize the terms to which the *quantifier* and *qualifier* are assigned. Quantity and quality are features of the categorical statement as a whole. The copula is split into two fragments – but they are just that, fragments of a single quantifier. Traditional logicians countenanced both split and unsplit versions of the copulae. Consequently, categoricals with their copulae unsplit could be rendered fully symbolic by allowing the copula to be symbolized using lowercase letters corresponding to the A, E, I, O that labeled categorical forms in general. Thus: SaP, SeP, SiP, SoP.

Term Functor Logic, TFL, rests on a theory of logical syntax that amounts to the traditional theory … up to a point. Traditional syllogistic logic could not easily accommodate three kinds of expressions: singular terms, complex (including relational) terms, and compound (truth functional) statements. Aristotle was well aware of all of this. He may have had reasons to think that singular statements are rarely encountered in science, but in both *Analytics* one can find many examples of them (*Prior Analytics* 43a34–35, 47b24–25, 47b32–33, 67a33–37, 68b41 ff, 69a2, 69b12–15, and *Posterior Analytics* 78b4–10, 90a5–25, 93a30–33, 93a36–67) (see Englebretsen 1980b). Moreover, there is good reason to believe that Aristotle understood that just as singular terms can be predicated in natural language, the same holds for the logical forms of such sentences (see *Topics* 15234 ff). One consequence of this is that syllogistic requires no special account of "identity statements" (Sommers 1967, 1969, 1976c, 1976d, 1982, 1990, 2000b, Englebetsen 1982a, 1985 f, 1996, 2015). Post-Fregean logic takes great pride in the notion that it can easily account for the logic of statements involving relational terms, marking its primary advantage over traditional logic. Yet Aristotle, though unable to fully incorporate the logic of relationals into his syllogistic, was at least aware that a way to do so was required. He made a start by providing examples of inferences involving relationals and attempted to formulate rules governing them (*Topics* 114a18 – 19, 114b40 – 115a1 – 2, 119b3 – 4, and *Prior Analytics* 48b11 – 24; see also Bocheński 1968, 68 – 69). Yet, he was never able to see how relational statements contain more than one referential term (viz. subject- and object-terms) (see Thom 1977 and Englebretsen 1982d). Finally, one should not be surprised to learn that Aristotle was confident that he could account for the logic of so-called truth-functional statements. He promised to carry out the task but apparently never did. Aristotle gave examples of inferences involving such unanalyzed (into terms) statements (*Prior Analytics* 53b12 – 24, *Posterior Analytics* 57a36 – 37, 75a2 – 4; see also Bocheński 1968, 70 – 71). What was missing was the idea that entire sentences can be treated as (complex) terms, which allows the logic of statements to be treated as merely a special branch of a logic of terms (see Sommers 1993 and Englebretsen 1980c).

Medieval logicians made some progress with all three of these challenges (Parsons 2015). But it wasn't until Leibniz's efforts along these lines that real progress was made. Medieval logicians tended to construe singular statements as implicitly universal in quantity, thus fit for roles as premises or conclusions of syllogisms. However, Leibniz realized that the quantity of such statements could be either universal or particular, depending on the logical environment in which they were being used (Leibniz 1966, 115). As we will see, this was far from a minor adjustment that strengthened syllogistic logic. Leibniz also saw that a relational expression, a relational term along with its object term (e. g., 'loves a philosopher') can be taken logically as a single complex term. This allowed him to analyze inferences involving such relationals as straightforward syllogisms. He actually provided a proof of the inference 'Painting is an art; therefore, he who learns painting learns an art' (Leibniz 1966, 88 – 89). Three centuries later, De Morgan dealt with a now more famous version of an inference of the same logical form. Finally, and most importantly, Leibniz offered valuable insights into how the logic of compound statements (e. g., conjunctions, disjunctions, conditionals), the so-called "hypothetical" statements, could be incorporated into the logic of terms (see especially Castañeda 1976). He saw that entire *statements* can be construed as *terms*. "[T]he categorical proposition is the basis of the rest, and modal, hypothetical, disjunctive and all other propositions presuppose it" (Leibniz 1966, 16). "[A]bsolute and hypothetical truths have one and the same laws and are contained in the same general theorems, so all syllogisms become categorical" (Leibniz 1966, 78).

> If, as I hope, I can conceive all propositions as terms, and hypotheticals as categorical, and if I can treat all propositions universally, this promises a wonderful ease in my symbolism and analysis of concepts, and will be a discovery of the greatest importance. (Leibniz 1966, 66)

Unfortunately (as with his systems of logical diagrams), all these Leibnizian insights were generally unknown for a very long time. Some of them were independently acquired by others in the 19th and 20th centuries (e.g., the treatment of relationals by De Morgan and Peirce). Eventually all of them were more clearly articulated and strengthened in Sommers' development of TFL (Sommers 1976a, 1976b, 1982, 1993, 2000, Englebretsen 1981, 1982c, 1985a, 1987a, 1988, 1996, 2015). Before continuing, it should be noted that Sommers' TFL was not the only system of term logic developed in the 20th century. For it turns out, perhaps ironically for some, that one of the most prominent champions of MPL, Quine, also formulated a version of term logic, Predicate Functor Algebra (PFA) (see Quine 1936a, 1936b, 1937,1959, 1960a, 1971, 1976a, 1976b, 1981a, 1981b and Noah 1980, 1982, 1987, 1993, 2005; see also Bacon 1985). Of course, he did so only to highlight the crucial role of individual variables and the quantifiers that bind them in MPL by eliminating them. Once eliminated, their roles had to be assigned to new formal elements, "predicate functors." which recursively apply to predicates to form new predicates. The result was a logical syntax admitting only predicates (which, since they are now the only kind of non-formative expression, might just as well be called *terms*) and functions on them. Once rules for the logical manipulation of formulas in the new language are provided, the result is PFA.

Quine built his version of term logic in order to highlight certain fundamental features of standard predicate logic. Sommers, by contrast, built *his* version of term logic (TFL) in order to reveal the logic of natural language. Consequently, TFL's system of symbolization is simple, natural, and perspicuous. It preserves a number of old (but now often forgotten) logical insights. The system of symbolization for TFL rests on Aristotle's notion of logical syntax in terms of copulated term pairs; it admits the medieval practice of splitting copulae; it borrows and expands on Leibniz's insight that singular terms, when in subject or object roles, have arbitrarily universal or particular quantity ("wild" quantity in Sommers 1969 and Englebretsen 1986a, 1988); it refines De Morgan's idea that the logical copula should be as abstract as possible (De Morgan 1850 and 1966, ix, 51).

TFL also incorporates the Leibnizian view, noted above, that the logic of compound statements (truth-functional logic) can be construed as a part of the logic of terms (Leibniz 1966, 16, 66, 78). Finally, and crucially, TFL entrenches the twin theses that terms, logically, come in oppositely charged pairs and that logical formatives are signs of opposition. Thus:

> To reason therefore is the same as *to add* or *to subtract*, … Therefore, all reasoning reduces to these two questions of the mind, *addition* and *subtraction*.(Hobbes 1981, 177)

> So just as there are two primary signs of algebra and analytics, + and −, in the same way there are as it were two copula, 'is' and 'is not'… (Leibniz 1966, 3)

> I think it reasonably probable that the advance of symbolic logic will lead to a calculus of opposite relations, for mere inference, as general as that of + and − in algebra. (De Morgan 1966, 26)

> We shall take this suggestion of Leibniz quite seriously, and see where it leads. (Sommers 1976a, 20)

> Both terms and propositions come in opposed pairs. Opposed terms are called logical contraries. … Opposed propositions are called contradictories. (Sommers 1982, 169)

> It is a surprising but little known fact that familiar logical words we use in quotidian deductive reasoning behave in a natural language like English in just the way that '+' and '−' signs behave in algebra and arithmetic. (Sommers 2005a, 59)

The formal symbolic language of TFL consists of countably many uppercase letters that are variables for natural language terms, the two functors (syncategoremata) + and −, and parentheses pairs as needed for grouping. Terms always come in oppositional pairs (e. g., 'happy'/'unhappy', 'massive'/'massless', 'colored'/'colorless', 'in the car'/'not in the car'). Term letters, then, are always logically charged positively or negatively (e. g., +H/−H, +M/−M). In such cases, the signs of term charge are unary, applying to one term. But the same plus and minus signs can be binary (applying to a pair of terms) as well. Such binary signs are either unsplit logical copulae or the fragments generated by a split copula. For example, Aristotle's standard I and O categoricals could be paraphrased as 'P belongs to some S', and 'nonP belongs to some S', and symbolized as 'P+S' and '−P+S'. Note that in this symbolization for O the '−' is unary and the '+' in each is binary (an unsplit copula). Since A and E are the contradictories of O and I, these latter can be negated to yield '−(−P+S)' and '−(P+S)'. Note that the result of copulating a pair of terms is a new (more complex) term, a term which itself can be negated (as when the particular forms are negated to yield the new universal forms. Also, notice that, just as in algebra and arithmetic (not to mention natural language) unary pluses are normally left tacit unless explicit use is needed. Moreover, again as in algebra and arithmetic, the plus and minus signs are systematically ambiguous between their unary and binary uses (compare positive and negative numerical expressions, on the one hand, with addition and subtraction, on the other).

Our versions of the A and E categorical forms look unfamiliar. We could rectify that by algebraically distributing the outside minuses into the complex expression inside the parentheses. The results would be: A: 'P−S' and E: '−P −S'. In these formulae, the first minus in E is unary and the other minuses are binary; they could be read as 'belongs to every'. We could make the universal forms even more natural looking by splitting those negative copula, in effect formulating 'belongs to every' as 'every … is …'. Here the 'every' is a (universal) quantifier and the 'is' is a positive qualifier. So, now, we can apply the splitting procedure to A above to yield '−S+P' and to E above to yield '−S+ −P' ('Every S is nonP'), which, in turn can be taken as '−S−P' ('No S is P'). In statements with split copulae, the *subject* consists of a quantifier and a term (the "subject term") and the *predicate* consists of the qualifier and a term (the "predicate term"). No term, by itself is either a subject or a predicate. Singular terms are terms understood as denoting just one individual (e.g., proper names, definite descriptions, anaphoric singular pronouns). When a singular term is a subject term it arbitrarily admits either a universal or a particular quantity. If, in such a case, the appropriate quantity is either undetermined or logically unimportant, the quantity is symbolized by '*'. Thus 'Socrates is wise' would be symbolized as '*S+W'.

Any pair of terms can be conjoined to form a new complex term. Such conjunction is affected by either the split or the unsplit positive copula. For example, 'rich and famous' is symbolized with the unsplit + as 'R+F' and its equivalent 'both rich and famous' is symbolized with the split copula as '+R+F'. Since *any* term can be conjoined with any other term to form a complex term, singular terms and compound terms can be conjoined with any other term to form a complex term. Examples are 'Tom and Jerry', 'rich and famous but unhappy'.

Relational terms are terms; in fact, they are compound terms with unsplit copulae. Consider 'Plato teaches some mathematicians'. It has three terms, but every statement is, from our logical point of view, a pair of copulated terms (i.e., a complex term). Here, 'Plato' is the subject term. So we can make a start at symbolization with '*P teaches some mathematicians'. Now 'teaches some mathematicians' is a relational expression, consisting of a *relative term*, 'teaches' and 'some mathematicians'. The entire relational expression is a complex term consisting of a pair of terms joined by an unsplit copula, which happens to be, in this case, 'some' (as in 'belongs to some'). The full symbolization would be '*P+(T+M)'. When a relational has another relational as one of its terms, as in 'Every candidate made some promises to every voter', it can just as easily be symbolized. Thus, in this case: '−C+((M+P)−V), in which the relational term is itself a relational term.

Entire propositions are complex terms. They are the results of copulating pairs of terms (which themselves might be general terms, singular terms, conjoined terms, relational terms, or even propositions). Consider 'Some philosophers are logicians and every logician is rational'. Its two conjuncts can easily be symbolized as '+P+L' and '−L+R'. Moreover, we know we can take the 'and' here as an unsplit copula to form the conjunction: '(+P+L)+(−L+R)'. Suppose our logical context does not require that propositions be analyzed into their sub-sentential terms. Let us use lowercase letters (e. g., p, q, r, …) as letter variables for them. Thus 'p+q' for the above example. Since 'p' and 'q' stand for unanalyzed propositional terms (compound terms) they have a charge. In this case the charge for each is positive (so tacit). Such terms could have negative charge. Thus '−p' would be the negation (viz., contradictory) of 'p'. Since we can reformulate any complex of propositions (conditionals, disjunctions, etc.) entirely in terms of negation and conjunction, we can symbolize all such truth-functionals using only propositional variables along with appropriate complements of pluses and minuses. Thus, since 'p+ −q' is the contradictory negation of '–(p+ −q)', which is the form of 'Not both p and not-q', which in turn is equivalent to 'If p then q', we might well symbolize the latter as '−(p+ −q)'. Better still would be to drive in the external minus to give us '−p+q', taking this to be the most appropriate symbolization for 'If p then q'. We can deal with disjunctions (e. g., 'p or q') in a similar manner to yield '–(−p + −q)' or '− −p − −q'.

Notice that the symbolization of 'If p then q' (−p+q) shares the same form as the universal affirmation 'Every S is P' (−S+P). Indeed, the same holds for all of the four standard categorical forms. 'Some S is P' (+S+P) and 'Both p and q' (+p +q); 'No S is P' (−S−P) and 'Neither p nor q' (−p−q); 'Some S are not P' (+S−P) and 'p but not q' (+p−q). In fact, as Leibniz, Kant, Boole, Peirce, and others have noticed, the logic of terms and the logic of unanalyzed propositions are either isomorphic, or identical, or the latter is a "special branch" of the former (Sommers 1993). It is only the purely formal features (reflexivity, symmetry, transitivity) of logical copulae (whether applied to pairs of terms or pairs of propositions) that are of logical import. It seems De Morgan saw this when he wrote that the copula should be construed as abstractly as possible.

One might object to the symbolic system of TFL by pointing to the ambiguity of the plus (+) and minus (–) signs. They are allowed to play two different roles in the syntax: they are marks of term charge (e. g., +A/−A) and they are fragments of split copulae (in effect, marks of quantity and of quality). Now, while ambiguity is often an obstacle to clear and efficient communication in natural language, ambiguity can be a source of expressive power. This is clearly seen in the use of formal languages such as in mathematics. Consider Arabic numeration (compared with Roman numeration). The expression '222' makes use of three instances of the numeral '2'. But in each instance the position of the numeral signals that it is to be understood differently from the other two tokens of the numeral. The first is understood as '$2x10^2$, the second as $2x10^1$, and the third as $2x10^0$, i.e., 200 + 20 + 2. And there is more. The simple plus and minus signs are systematically ambiguous. They are allowed to play two different roles: they are marks indicating whether a numerical expression is positive or negative (e.g., +42/−42) and they indicate the operations of addition and subtraction. In other words, these pluses and minuses are sometimes unary operators (applied to single numerical expressions) and sometimes binary operators (applied to pairs of such expressions). This kind of ambiguity is a very good thing in mathematics. It is also a very good thing in the symbolic system of TFL.

Needless to say, it's one thing to build a symbolic logic and quite another to build a diagrammatic logic. A symbolic system adopts a number of conventions for interpreting its expressions. Natural languages are symbolic systems. Artificial systems of fully symbolic formal logic are meant to facilitate the translation (via appropriate intermediate paraphrases when required) of natural language statements into well-formed (grammatically correct) formulas of the system's artificial language. Such symbolization aims to reveal logical form, the "canonical" form according to logicians like Quine. The advantage to be gained by symbolization is the increased transparency of form and ease in the process of carrying out various logical tasks. "If we were to devise a logic of ordinary language for direct use on sentences as they come, we would have to complicate our rules of inference in sundry unilluminating ways" (Quine 1960b, 158). A sufficiently well-constructed *diagrammatic* system (in particular, one that is heterogeneous) enjoys these same advantages. But, it also enjoys the added advantage of graphically *showing* logical forms. Thus, it allows for *visual inference*, the drawing of a conclusion simply by looking at a diagram without further manipulation.

3.3 Term Lines

> Some schemata are visibly verifiable …
>
> Quine

> As a practical method of appraising syllogism, rules are less convenient than the method of diagrams … The diagram test is equally available for many arguments which do not fit any of the arbitrarily delimited set of forms know as syllogisms.
>
> Quine

Since Descartes' development of analytic geometry, mathematicians have learned that the system of real numbers is isomorphic with the geometrical line. In 1995, Hillary Putnam (Putnam 1995) showed how Peirce had eschewed this lesson by holding a conception of *line* (and *point*) that shared much in common with that of ancient geometers as well as Aristotle (cf. Roeper 2006; Shapiro and Hellman 2015; Linnebo, Shapiro and Hellman 2016). Here is how Putnam summarizes Aristotle's conceptions of lines and points:

> [T]he Aristotelian view that points are simply conceptual divisions of the line … the line is an irreducible geometrical object, not a collection of more elementary objects. … For [Aristotle], points do not *belong* to lines, although they *lie* on them; that is, they are divisions of them (and also terminations of them, in the case of line segments and curves with endpoints). (Putnam 1995, 4–5)

He goes on to say that according to the ancient view, say Aristotle's or Euclid's (as opposed to the modern view),

> the endpoints [of a line segment] are to be regarded, not as *members* of the line segment … but as loci distinguished by the fact that an object we have constructed or considered *ends there*. … The endpoints … are abstract properties of the line segment itself. (Putnam 1995, 5)

Accordingly, one can think of geometrical points as demarcated, *delineated*, literally, de-lined, conceptually abstracted, from line segments. To use Putnam's word, they are "distinguished" by us according to what "we have constructed or considered." Such conceptual abstraction simply sets them into relief for our attention. Thus, geometrical objects are not constructed from more elementary geometrical objects (ultimately, points). Rather, such more elementary geometrical objects are delineated from less elementary geometric objects. Such delineation does not create, bring into being, geometrical objects; it is merely the result of our *focus*, our conceptual attention to what was previously undelineated. Any point to the left of the right terminal point of a term line is *delineatable* (though very few are actually delineat*ed*).

While geometrical objects are abstract, they can be represented graphically. Such representations are just that – representations. They are physical, perceptible objects. Nonetheless, as representations, they *show* various properties of abstract geometrical objects as well as certain relations that hold among those geometrical objects. It is this feature of such representations, diagrams, that makes them so instructive and useful in geometrical reasoning. Indeed, the same can be said for diagrams used in logic. Russell wrote that a good notation is like a live teacher. One could add that a good system of diagrams is just as much like a live teacher.

The ED system makes use of the following *graphic* elements*:* straight line segments, points, vectors, a rectangular border. Straight line segments represent individuals denoted by a given term. Uppercase term letters are used as straight line segment labels, the charges of which are indicated by + or −. Positive charge is often suppressed. Points represent specified individuals. Lowercase term letters are used as labels for such points and may be labelled as wild in quantity (*). Vectors (directed arrows) represent relations between or among individuals or sets of individuals; they are labelled just like non-relational terms. Rectangles demarcate pertinent domains of discourse (sometimes "worlds"). The following *conventions* apply: the right-most endpoint of a straight line segment is labelled and that label applies to every point on that line segment to its left; any proper part of a straight line segment is a either a straight line segment or a point; any point on a straight line segment can be indicated to represent a specified individual; vectors are labelled by adjacent term letters; vectors share points with at least two line segments. 'Straight line segment' will be abbreviated as 'term line or 'line'. Vectors need not be straight. Demarcation of domains of discourse by use of rectangles will often be suppressed and left as understood unless context requires otherwise. Nonetheless, it must always be understood that any diagrammatic representation is relative to a specifiable domain. This is because the statements we make are themselves always made relative to some specifiable domain. Normally, such domains of discourse consist of things in the actual world or some salient spatially or temporally understood part of that world. But a domain can be a merely possible world or a fictitious world. This notion of domains will be explored more closely when it comes to diagramming unanalyzed statements (propositional logic). Finally, it should always be remembered that only in the context of a diagram do lines, points, and vectors have meaning (i. e., represent anything).

Systems of logical diagrams exploit the fact that the relations that can hold between pairs of sets (inclusion, intersection (overlap), and exclusion) are easily represented by pairs of geometric plane figures that can themselves be arranged into these same relations. Systems such as those devised by Euler and Venn, for example, use closed figures such as circles, for this purpose. Leibniz and Lambert made a start at using open figures – straight line segments. The ED system exploits the use of line segments as well. Any pair of straight line segments can be arranged into the relations of inclusion, overlap, and exclusion. Moreover, given such an arrangement, the relations between what the lines represent can readily be seen. What is true of any viable system of logical diagrams, whether using closed or open plain figures, is that it rests on the analogy between set relations and geometric relations. Peirce expressed this view in many places (e.g., Peirce 1931–1958, 2.277) and perhaps more clearly than anyone when he wrote:

> [A]ll of deductive reasoning … involves an element of observation; namely, deduction consists in constructing an icon or diagram the relation of whose parts shall present a complete analogy with those parts of the objects of reasoning, of experimenting upon this image in the imagination, and of observing the results so as to discover unnoticed and hidden relations among the parts. (Peirce 1931–1958, 3.363)

Of course, before "observing the results" we need a diagram with "parts" that can stand in relations.

Let T be a term variable. The number of individuals denoted by a given term in any particular context, universe of discourse, is often undetermined. The individuals that constitute the denotation of T are represented by a term line, which, correspondingly, is often undetermined. The term line representing T (viz., T's denotation) is labelled by T placed near its right terminus point:

Figure 3.10: A Term Line

If T is singular, denoting just one individual, it is diagrammed as a single point:

Figure 3.11: A Singular Term Point

A term having no denotation, an empty term, can be diagrammed in two alternative ways: it can be inscribed in the diagram attached to nothing else, or it can simply not appear in the diagram.

Note that the tokens of T above are not accompanied by a sign of positive or negative term charge. But we know that every term does have, from a logical point of view, such a charge. We will assume that any term or term variable not accompanied by such a sign will be understood as implicitly positive (thus T is understood as +T). Needless to say, a negative term, nonT (−T), will be graphically expressed by an appropriately labelled term line.

Figure 3.12: A Negative Term Line

Note as well that a term line represents the entire denotation of the corresponding term. Consequently, one could read a term line such as that in Figure 3.10 as a graphic representation of ‘every T’. Likewise, the term line in Figure 3.12 represents ‘every nonT’ (it does not represent ‘no T’.

As noted above, the relations that hold between the lines and points of a diagram (inclusion, intersection, exclusion) are meant to mimic the relations that hold between objects represented by those lines and points. This “relation-based” approach contrasts with the “region-based” approach taken by closed figure diagram system such as those based on Euler or Venn diagrams (for more on this distinction see, for example, Mineshima, Okada, and Takemura 2009, 2010, 2012, and Sato, Mineshima, Takemura 2011). It’s time now to show how this is done in ED.

The inclusion of (the extension of) one term in another is simply represented by the line representing the first term being made a (proper) part of the line representing the other. Thus a universal affirmation of the form ‘Every S is P’ is represented as the following (keeping in mind that the label on a term line is meant to apply to the entirety of the line to its left):

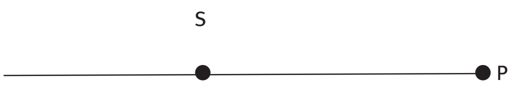


Figure 3.13: Universal Affirmative Line Diagram

Universal affirmations are diagrammed by making one line a proper part of a second. Sometimes, however, two lines will each be (non-proper) parts of one another. This will be so when they are meant to represent a pair of co-extensive terms. For example, to use Quine’s well-known example, every creature with a kidney is a creature with a heart and every creature with a heart is a creature with a kidney. This could be diagrammed by giving both terms variable letters that can both label a common line. Let K and H (obviously) be the two labels:

Figure 3.14: Line Diagram for Two Co-extensive Terms

A universal negative (‘No S is P’) is simply diagrammed with the two labelled lines sharing no point:

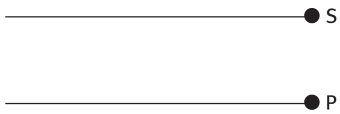


Figure 3.15: Universal Negative Line Diagram

A particular affirmation requires that the two terms of the statement both denote at least one individual in common. The representation of such a statement ('Some S is P') consists of the two lines representing the two terms sharing at least one common point – i. e., intersect:

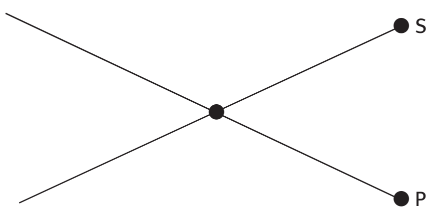


Figure 3.16: Particular Affirmative Line Diagram

As it turns out, there are two alternatives for diagramming particular negative statements ('Some A is not B'). Such a statement may be construed as either (i) claiming that some S is not (= isn't) a P (predicate denial) or (ii) claiming that some S is a nonP (predicate term negation). The first is entailed by the second and is treated so in TFL, (which simply amounts to accepting the traditional rule of *obversion*). The two versions are graphically represented as follows:

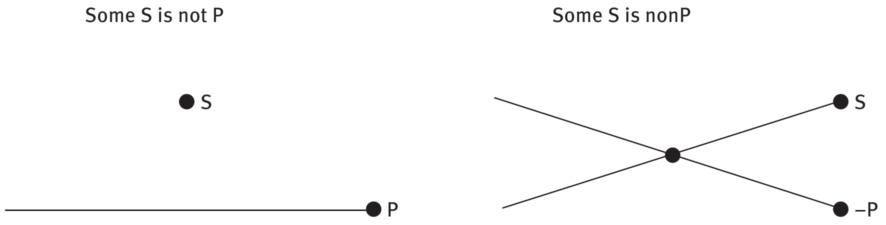


Figure 3.17: Diagrams for Predicate Denial and Term Negation

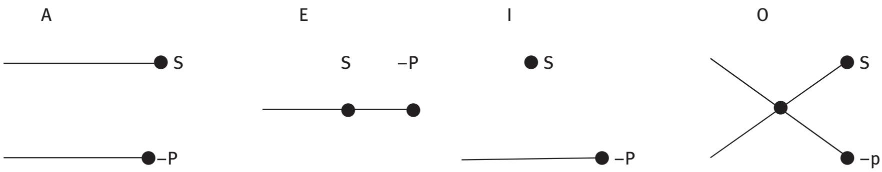


Figure 3.18: Line Diagrams for Obverted Categoricals

Obversion takes term negation seriously. Thus, any categorical statement can be diagrammed by line inclusion or line intersection since any universal negative can be construed as a universal affirmative whose predicate term happens to be negated and any particular negative can be construed as a particular affirmative whose predicate term happens to be negated.

Singular affirmations, such as ‘Socrates is a philosopher’, have the form ‘*s + P’, which can be represented by a line diagram having a delineated point labelled s on a line labelled P. As well, a singular negative (‘Socrates is not a Roman’/ ’Socrates is no Roman’/’Socrates is nonRoman’) would correspondingly be represented with the delineated point representing Socrates on a line labelled – R:

Figure 3.19: Line Diagrams for Affirmative and Negative Singulars

Note that, given the convention that any label on any line applies to the entire line to its left, the delineated point representing the singular cannot be placed anywhere on the line other than the left terminus.

The rule of obversion is important. Yet it depends on an even more important, more fundamental principle – the *principle of noncontradiction.* Consider an individual such as Obama. He is American (and he’s proven it). He is nonCanadian. Since he is nonCanadian, he is not a Canadian. Not everything that is not a Canadian is nonCanadian. The moon is neither Canadian nor nonCanadian; it *is not* a Canadian, however. The number of planets is not green, but it isn’t nongreen either. Whatever is nongreen is either red or blue or white or yellow or …. Colour terms simply do not sensibly apply to numbers. Terms of citizenship do not sensibly apply to astronomical objects. Terms like ‘drinks procrastination’ (as well as its contraries) do not sensibly apply to the quality of quadruplicity. In general, if an individual, x, is nonP, then x is not P. But the converse doesn’t follow. What does follow from all of this is that if a pair of contradictory statements cannot hold at the same time and in the same respect, then the same is true for a pair of contrary statements (statements such that one affirms a property and the other affirms a contrary of that property of the same thing). Here is Aristotle in

Metaphysica 1005b19 – 20: “[T]he same attribute cannot at the same time belong and not belong to the same subject and in the same respect.” Then, at 1011b13 – 22:

> [C]ontradictory statements are not at the same time true … it is impossible that contradictories should at the same time be true of the same thing. For of contraries, one is a privation not less that it is a contrary .. it is also impossible that contraries should belong to a subject at the same time.

All this is as close as Aristotle got to expressing in a clear, unambiguous manner, the principle of noncontradiction. Keep in mind that for Aristotle the contradictory of a statement is not what is today called its negation but rather a corresponding statement having those same terms in the same order but a different quantity and a different quality. Here is what seems clear enough. Contradictories cannot both hold at the same time. Statements of the two forms ‘Every S is P/nonP’ and ‘Some is isn’t P/nonP (where ‘S’ is either singular or general) are contradictories and cannot hold at the same time. But as well, pairs of statements that are of the same logical form but such that the two predicate terms are contraries cannot hold at the same time. For Aristotle, the *privative* of P is nonP. As we have seen, whatever is nonP has some property contrary to P. If nonP is the *logical contrary* of P, then that other property is one of the *nonlogical contraries* of P. Red, blue, etc. are the nonlogical contraries of green; being nongreen amounts to being one of these. So green has many contraries but only one logical contrary. Some properties have just one nonlogical contrary (even/odd); others have an infinite number of nonlogical contraries (1 meter long/1.1 meter long/…). In summary, then, contradictory pairs cannot both hold at the same time and, consequently, pairs that attribute contrary properties of the same thing cannot hold at the same time. For, a statement attributing a nonlogically contrary property entails one that attributes its corresponding logical contrary, which, in turn, entails the contradictory of the statement that attributes the original property. For example, ‘Every X is red’ entails ‘Every X is nongreen’ and ‘Every X is nongreen’ entails ‘Every X is not green’ and ‘Every X is not green’ is the contradictory of ‘Some X is green’.

Linear diagrams adhere to the principle of noncontradiction in the following way. Given a specifiable domain, for any term line, P, there will be a (possibly tacit) term line, nonP, such that the two lines share no point in common. This guarantees that, for example a universal affirmation and its corresponding particular negative are contradictory. In order to represent both statements in the same diagram one would be required to do the impossible: make at least one pair of lines that share no point in common (say, the P and nonP lines) intersect.

According to TFL, a contradiction always has the form of a particular that is algebraically equal to zero: +S−S; a tautology always has the form of a universal that is algebraically equal to zero: −S+S. Consider, for example, an attempt to diagram simultaneously a universal affirmation and its corresponding particular negative:

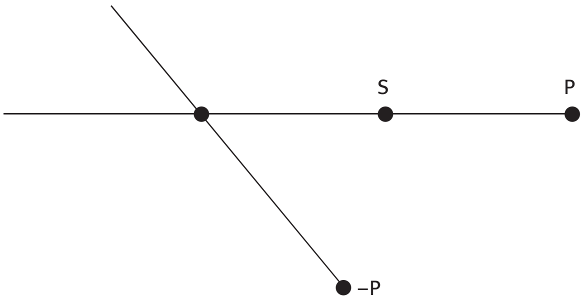


Figure 3.20: An Attempted Diagram for a Contradictory Pair

Note that one of the statements represented by this diagram is that some nonP is P (+(−P)+P), a particular algebraically equal to zero. Moreover, the principle of noncontradiction demands that no two lines representing a term and its negation have any point in common. That means in the diagram above, the term line for –P must be at once both intersecting and not intersecting the P term line. Diagrams like this are similar to Escher drawings; they give only an illusion of what is actually impossible. Such an impossible diagram would be, to use Marc Champagne's delightful neologism, a "contrapiction" (Champagne 2016). Tautological forms are quite another matter.

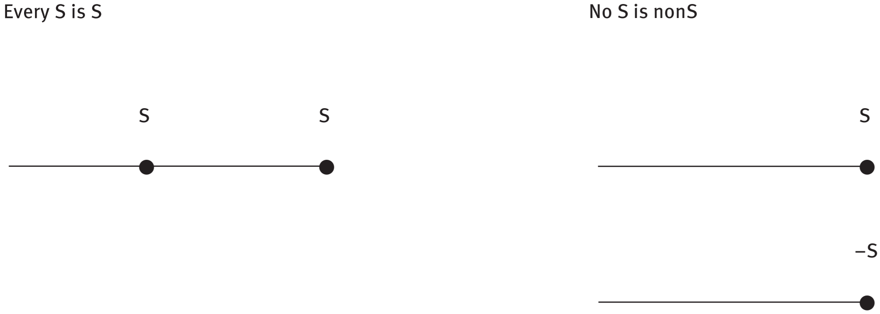


Figure 3.21: Diagrams for Tautologies

Suppose we represent each term line along with its contradictory term line. By virtue of the law of noncontradiction the two lines must not share any point. Following this practice, the *full* diagrammatic representation for an A categorical would be:

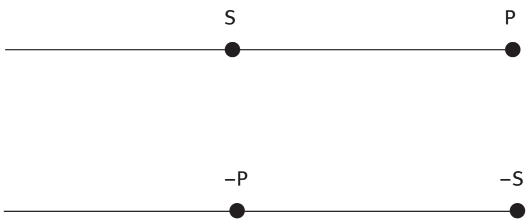


Figure 3.22: Full Diagram of A

Such a diagram represents all of the following: 'Every S is P', 'Every nonP is nonS', 'No S is nonP', 'No nonP is S', 'No nonS is P', 'No P is nonS', 'No S is nonS', 'No nonS is S', 'No P is nonP', 'No nonP is P'.

Any categorical can be given such a full representation.

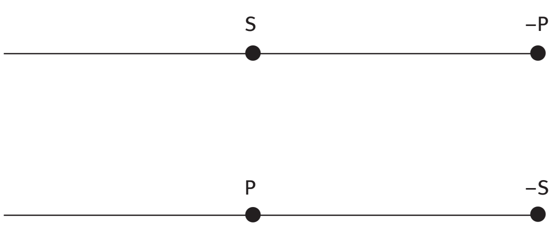


Figure 3.23: Full Diagram of E

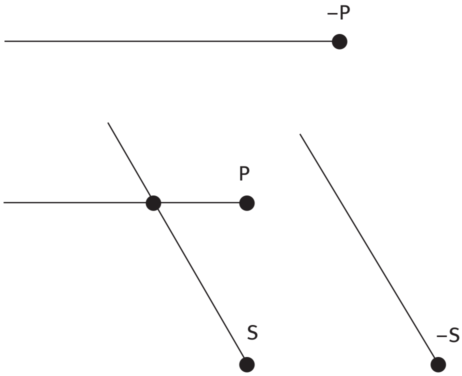


Figure 3.24: Full Diagram of I

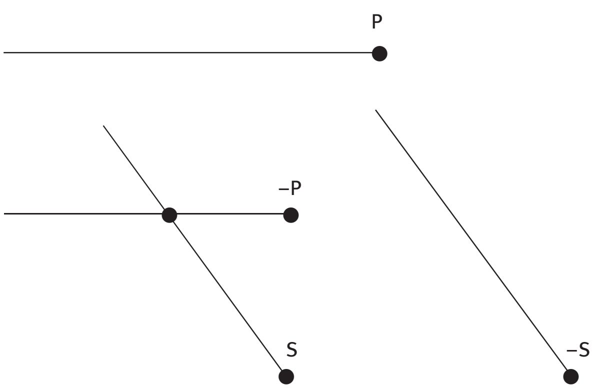


Figure 3.25: Full Diagram of O

Notice that a full diagram always consists of two pairs of non-intersecting lines. As well, just as we saw in the case of a full representation of A, full diagrams in general always represent a number of propositions, many of which are tautological or redundant. In practice, when engaged in logical reckoning diagrammatically, *simple* rather than *full* diagrams are adequate. In effect, a simple diagram for a categorical merely ignores the representations of the law of noncontradiction. Gardner called this simplifying process "minimizing" (Gardner 1982, 72). Examples of simple, minimized diagrams for A, E, I, and O categoricals are seen in Figures 3.12, 3.14, 3.15, and 3.16 above.

The traditional logical relations among categoricals, usually represented on a square of opposition (Englebretsen 2015, ch. 5, and 2016a) are preserved and graphically exhibited by ED. We have seen how contradictories (e. g., A/O and E/I) are treated. The contrariety of A and E is demonstrated by the impossibility of simultaneously diagramming both (a contrapiction).

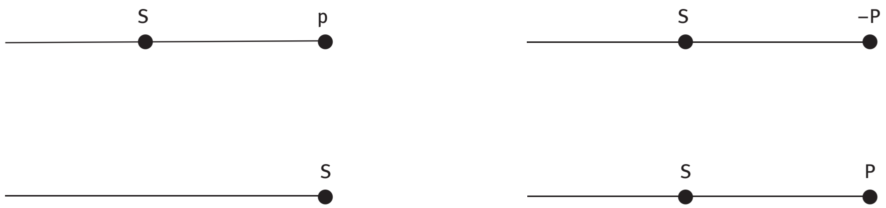


Figure 3.26: Contrapictions of A/E Contrariety

The relation of subcontrariety requires that I and O be logically compatible. This is graphically shown by a diagram expressing both categoricals simultaneously:

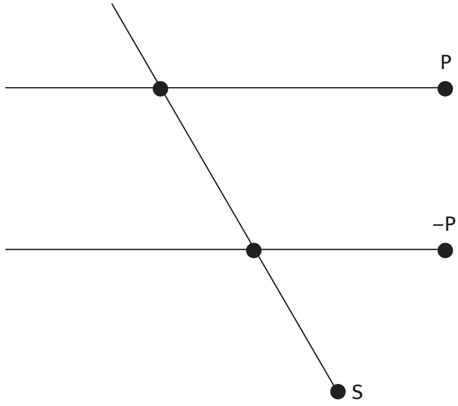


Figure 3.27: Diagram of Subcontrariety

An understanding of the logical relation of subalternation depends on how one is to understand the notion of existential commitment. The controversy stems from the question of whether Aristotle limited the syllogistic to inferences whose statements involved only nonempty terms. If the answer is positive, then all of the traditional relations illustrated on the square of opposition – in particular, subalternation – hold. If the answer is negative, then all bets are off. Modern predicate logic seems to follow the Boolean line that only existentially quantified statements express ontological commitment (in the sense that they are false whenever nothing in the universe of discourse satisfies the functions/predicates applied to the variables so quantified). From this point of view, it follows that under such circumstances of emptiness the corresponding universal statement is true. So, the modern view, generally, is that subalternation fails to hold in some cases (viz, the so-called vacuous cases). However, even if the traditional positive answer is accepted, there are still questions. Most importantly: How is the nonempty character of a term expressed? One answer is that the very use of a term ensures this. Such a view is most often attributed to Aristotle; Aristotle simply *assumed* that terms of a syllogism are never empty. Łukasiewicz even expanded the list of outlawed terms to include not only empty terms but singular and negative terms as well (Łukasiewicz 1957, 72). The idea that Aristotle's syllogistic eschews empty terms is then attributed to his medieval followers (Kneale and Kneale 1962, 59 – 60). Nonetheless, whether Aristotle did in fact hold such a view of empty terms, there is some reason to believe that at least some late-medieval logicians believed that the nonempty character of syllogistic term cannot simply be assumed but must be expressly asserted. Parsons has argued that this is best done by asserting of a term, T, that some thing is T, which he equates with asserting that some T is T (Parsons 2014, 65 – 66). According to this view, while 'Every S is S' may be a tautology, 'Some S is S' (= 'Some thing is S' = 'An S exists' = 'There is at least one S') is contingent. We saw that Pagnan adopted this position for his SYLL and it is a central part of TFL, where it is formulated as +S+S, which is neither universal nor algebraically equal to zero, thus contingent.

It seems that our choice is either to simply assume that no empty terms can be used in our inferences or to require that the nonempty character of a term being used be explicitly expressed. However, a compromise is possible. One can take terms to be nonempty but only explicitly express this when demanded by inferential circumstances. This means that subalternation can be taken to be a matter of immediate inference even though, in some circumstances, it can be taken as mediate, an enthymeme whose missing premise states that its subject term is nonempty. It can be argued that Aristotle too allowed that subalternation can be mediate.

> Now, in order to avoid the existential-import problem, one should assume that the term whose existence is not explicit does exist. Thus, if the problem relates to the term, *x*, one should assume that the premise 'There exists an *x*' is true. This has been called the "Aristotelian proviso" (Alvarez and Correia 2012, 304).

TFL recognizes *Aristotle's Proviso*. Subalternation is generally expressed as 'Every S is P, therefore some S is P' (−S+P ∴ +S+P) and, when appropriate, as

'Every S is P, (some S is S), therefore some S is P (−S+P, +S+S ∴ +S+P). This distinction between *implicit* and *explicit* expression of existential import (thus subalternation) can be readily exhibited in the graphic system of ED.

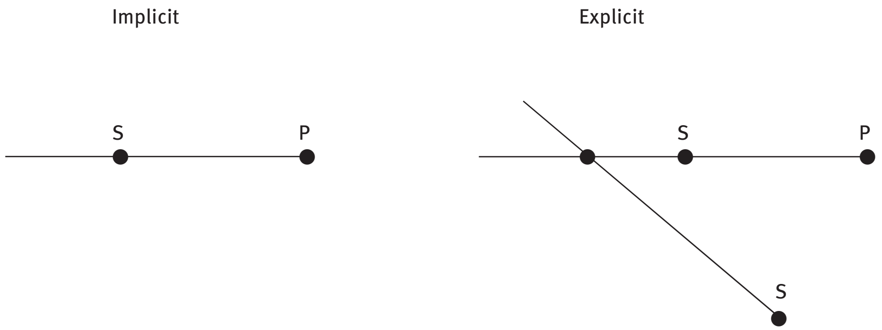


Figure 3.28: Two Versions of Subalternation

One can say that by the lights of the graphic system ED a term is to be taken as nonempty when it is represented as a term point or a term line with a specified, delineated point. *To be is to be delineated.* When existential import is taken implicitly, the very fact that a term is used (and can be diagrammed by a term line or term point) is sufficient. The labelled right terminal point of the S line shows that S and P lines share at least one point. In the case where existence is made explicit for the S term, the intersection of the two S lines explicitly shows a point (other than the right terminal point of S) that is shared by both the S and P lines. Later we will see the usefulness of having a way to make existence diagrammatically explicit.

Thus far we have seen how the graphic system ED can be used to diagram categorical statements (including singulars), exhibit existence (nonempty terms), and display the logical relations illustrated by the square of opposition. The real *raison d'etre* of any system of formal logic, whether diagrammatic or linguistic, is to provide a method to assess arguments for validity or invalidity (a decision procedure) and to provide a way to prove the former (a proof procedure). So, our next step now is to offer a decision procedure for ED. Traditional term logic worked out a fairly simple way to determine the validity/invalidity of any syllogism. The idea was that all, and only, valid syllogisms obey the "rules of syllogism":

Rules of Syllogisms

1. At least one premise must be universal.
2. At least one premise must be affirmative.

3. No term may be distributed in the conclusion if it is not distributed in the premise in which it occurs.
4. The middle term must be distributed at least once.
5. Any term distributed/undistributed in the premises must be distributed/undistributed if it occurs in the conclusion.

Notice how much the notion of term distribution is involved here. Now, as it happens, the theory of term distribution is contentious, though the various versions of the theory seem to have originated by Aristotle's remarks in Chapter 7 of *De Interpretatione*. This is partly because so many different definitions of *distributed* have been offered and partly because the notion of term distribution has so often been grounded in semantic rather than syntactic features determined by the term's role in a given sentence. It was sometimes said that a term used in a given sentence is distributed just in case it makes reference to its entire denotation. At other times it was held that the distribution of a term in a given sentence was a matter of its supposition. At still other times the claim was that a term used in a given statement is distributed in that statement just in case that statement entailed a universal statement in which that term was the subject term. Denotation, reference, supposition, even entailment in this case are semantic notions, matters of sense or meaning or interpretation, and are themselves often unclear and invite controversy (the idea of distribution aside). Still, there is general agreement that for categorical statements subject terms of universals, but not particulars, are distributed and predicate terms of negatives, but not affirmatives, are distributed. Peter Geach was no friend of any notion of term distribution. He famously wrote, "Now we need only look at the doctrine of distribution with a little care to see how incoherent it is" (Geach 1962, 4). Geach has, of course, not gone unchallenged (see especially Parsons 2006). In fact, over the years since 1962, a fairly steady stream of friends of the doctrine of distribution have come to its defense (Makinson 1969, Williamson 1971, Sommers 1971, Katz and Martinich 1976, Friedman 1978 Sommers 1982, Rearden 1984, Englebretsen 1985 g, Wilson 1987, Hodges 1998, Sommers and Englebretsen 2000, Parsons 2006, Hodges 2009, Alvarez and Correia 2012, Martin 2013).

A decision procedure does depend, in part at least, on determining whether a given term is distributed or undistributed (its distribution value) in a given statement (viz., in a premise or conclusion of an argument). As it happens, it is possible to determine the distribution value of a term in a statement without the involvement of any semantic notions. There is a purely syntactic, indeed simply mechanical, method for doing so (for more on the contrast between semantic and syntactic accounts of distribution see Martin 2013, 135–139). Sommers indicated how this mechanical procedure could be done, at least in TFL's algebraic formal language (Sommers 1982, 181):

> [I]n TFL the question whether a given term is distributed or undistributed in a proposition is the question of whether its algebraic value in that proposition is negative or not. ... In determining the distribution value (or valence) of the elements of an expression we simplify its algebraic representation by driving the minus signs in as far as possible. The result will be an algebraic expression in which each element is either negative or positive. Negative elements are distributed, positive elements are undistributed.

Much more recently a similar idea was proposed by Wilfrid Hodges (Hodges 2009, 603), who wrote, "Briefly, a term in a sentence is distributed if it occurs only negatively, and undistributed if it occurs only positively." Keeping in mind that by the lights of TFL universal quantity is formulated as a minus sign, the mechanical determination of a term's distribution value in a statement is easily summarized as follows:

> Distribution Value
> A term is *undistributed* in a statement just in case the total number of universal quantifiers and negations in whose range it occurs is even (including zero), otherwise it is *distributed*.

How are the distribution values of terms represented in the linear diagrams of ED? Such distribution values are readily observable in such diagrams. Quite generally, these values are exhibited by the following diagrammatic features:

> A term, T, is distributed in a diagram if and only if its term line contains no labelled delineated point to the left of its right terminus, otherwise it is undistributed. If a term, nonT (−T), is distributed/undistributed in a diagram, then T is undistributed/distributed in that diagram.

One can verify this for the four standard categorical forms by looking at Figures 3.12, 3.14, 3.15, and 3.16 above. We will eventually see how this is the case for any kind of statement formulated in TFL and diagrammed by ED.

Now, as it happens, the determination of the distribution value of a term is essential when it comes to *deductions* in a term logic, whether Aristotle's syllogistic, traditional syllogistic, or TFL. But the latter can make use of a second mechanical device, other than the one for determining distribution values, in deciding argument validity. While traditional logic made use of the five Rules of Syllogisms, TFL get by with just two:

Rules of Syllogisms (for TFL)

1. The algebraic sum of the premises (including tacit premises) must equal the conclusion.

2. The number of particular conclusions must equal the number of particular premises.

That's it. Deciding validity or invalidity is even easier once arguments are diagramed.

Rule of Syllogisms (for ED)

Diagramming only the premises (including tacit premises) together must exhibit the conclusion.

That means that if either the conclusion has not been revealed simply by diagramming the premises together or more than a single diagram is required for the premises (i.e., they cannot all be diagrammed together), then the argument is invalid.

So, the time has come to turn from argument validity/invalidity decision to (valid) argument deduction, *proof*, the incremental stepwise construction of justified inferences that begins with an argument's premises and culminate with its conclusion. A deductive argument consists of a collection of a finite number of premises and a conclusion. There may be statements that are taken to be among such a collection but which need play no role in the deduction; they are redundant. There may be premises that do play a role in the deduction but are not explicitly stated among the collection of explicit premises; they are tacit (suppressed, understood) premises, premises that are either tautological or else generally accepted on other grounds; these premises are *hidden*. The conclusion is a statement explicitly made or one that is missing and must be found via the process of deduction. A deductive argument is made with the accepted, but usually unexpressed, claim that if all of the premises are accepted as true the conclusion must thereby be accepted as true. The claim is not that the premises *are* all true or that the conclusion *is* true. The process of deduction can be used to establish the understood claim. It can also be used, when necessary, to discover a tacit premise (particularly one that might not be generally accepted), so-called *lost* premises. Once a decision has been made that an argument is valid, one can *prove* that it is valid. This is done in formal, but non-diagrammatic, systems by constructing a finite list of statements (called *lines*). The list consists initially of the premises followed by additional statements, each of which is *justified* by a *deduction rule* applied to one or more of the preceding lines. The final statement of the list is the conclusion, which is also justified. Since every new line in a proof must be justified, having an acceptable set of rules is essential.

When, in his *Prior Analytics*, Aristotle set out to form a system of syllogistic proof, he took the first figure syllogisms to be "perfect" in the sense that they are "complete" and therefore require nothing further to exhibit their validity immediately. In particular, this means that no additional premises are required and that no additional middle terms need to be introduced; they need no proof. He took syllogisms in the other figures to be incomplete (*Prior Analytics* 24b23–27). This means that such syllogisms require proof. Proof of an imperfect syllogism can be provided by "reducing" it to a perfect syllogism by applying certain rules to the premises in order to change them in such a way that the result is a first figure syllogism. These rules are called rules of *immediate inference*. They simply amount to the principles of conversion, obversion, subalternation, and mutation (i.e., altering the order of the premises). As it turns out, all the imperfect syllogisms are reducible to the first figure perfect syllogisms. There is a feature that is common to the perfect syllogism and is consequently shared by all syllogisms.

Traditional logicians usually cited *Prior Analytics* 24b26–30 or 25b31–35 in formulating what was called *dictum de omni et nullo* (the principle of all and none), or simply the *dictum*. The claim is that the *dictum* is the underlying principle governing all syllogistic inference, "the foundation of the whole of syllogistic theory" (Leibniz 1966, 116). A typical version of the *dictum* is 'What is predicated (affirmed/denied) of all/some of X is likewise predicated of what X is predicated of'. Sometimes it is formulated in terms of parts and wholes: 'What is predicated of any whole is predicated of any part of that whole' (see Kneale and Kneale 1962, 79). Leibniz formulated the *dictum* as a rule of substitution: "To be a predicate in a universal affirmative proposition is the same as to be capable of being substituted without loss of truth for the subject in every other affirmative proposition where that subject plays the part of predicate" (Leibniz 1966, 88). Even Boole, it seems, took his basic rule of deduction ('Equals can be substituted for equals') as a version of the *dictum*, *qua* rule of substitution (Corcoran and Wood 1980, 615–616; also Green 2009). TFL also takes the *dictum* to be a rule of substitution (see Sommers and Englebretsen 2000, 133–135). In this case, it says that the predicate term, say P, of a universal premise can be substituted for the subject term, say M, of that premise for any instance or occurrence of that term, M, in another premise whenever the two occurrences of the term, M, have different distribution values (Englebretsen 2010, 54–55 and 2012, 74–75). In effect, this simply means that the occurrences of a pair of middle terms algebraically cancel out, in which case, the sum of the premises algebraically equals the conclusion (whenever the number of particular conclusions equals the number of particular premises). The *dictum*, as Leibniz rightly saw, is the foundational rule that governs mediate inference. It governs not only classical syllogisms but, as will be seen, all kinds of inferences, including those involving singular terms and relational terms. It governs, as well, the inferences of propositional logic.

It is possible to illustrate the *dictum* for the classical syllogisms by using the system of linear diagrams, ED. Consider Barbara, Celarent, Darii, and Ferio (the four perfect first figure syllogistic forms).

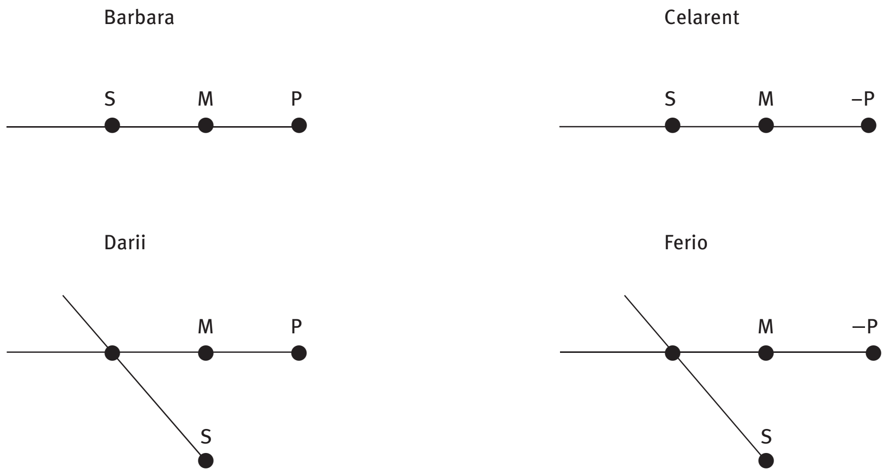


Figure 3.29: Line Diagrams for the Perfect Syllogisms

Notice that the charge (positive or negative) on the major term is immaterial. Each of the negative forms is simply a version of their positive counterparts.

Figure 3.30: Line Diagrams for Universal and Particular Syllogisms

Consider how the diagram for Barbara would have been constructed. Step 1: the major premise is depicted.

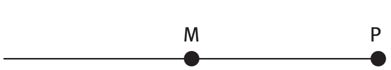


Figure 3.31: ‘Every M is P’

Then the minor premise is diagrammed by incorporating it into this diagram and making use of the already represented M. The result is the full diagram for Barbara:

Figure 3.32: Barbara

Cancellation of the middle term per the *dictum* simply amounts to ignoring M, reading the conclusion directly, immediately from the diagram. Here are some examples of diagrammed imperfect syllogisms.

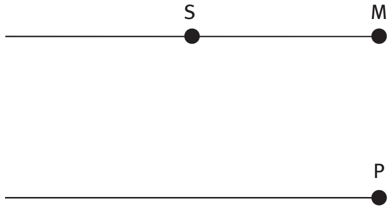


Figure 3.33: Cesare 1

Figure 3.34: Cesare 2

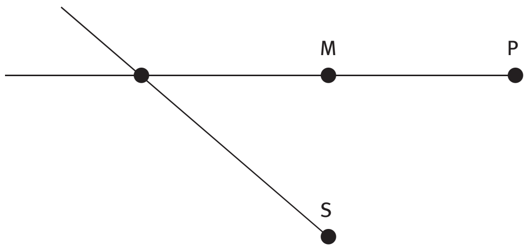


Figure 3.35: Datisi

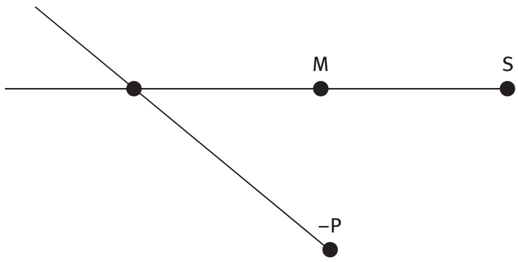


Figure 3.36: Bocardo

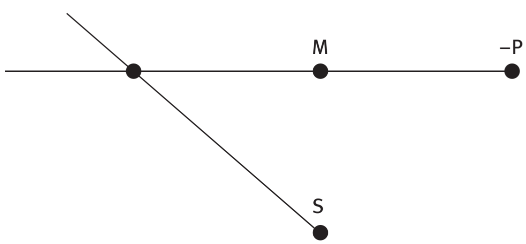


Figure 3.37: Ferison

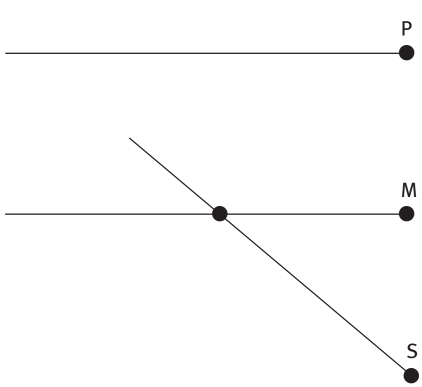


Figure 3.38: Fresison

Note that in a syllogism such as this the ED diagram indicates nothing about whether the S line extends all the way to the P line, thus indicating our lack of information. We know that at least one S is not P but we have no knowledge about any S that is P. A Venn diagram would represent the same with an x in the SM—P cell but nothing in the SMP cell.

Such line diagrams are easily constructed for every one of the twenty-four classic categorical syllogisms. Classical categorical syllogistic is thus complete. It is also sound. An exhaustive (and exhausting) check of each of the 232 classic invalid syllogistic forms shows that all are revealed to be invalid by means of this diagram system. The system is just as effective when it comes to the weakened syllogisms, valid syllogisms with a particular conclusion but no *explicit* particular premise. As with TFL, ED takes such syllogism to have a hidden premise that, when diagrammed makes the existence of the minor term explicit (as in Figure 3.27 above). For example, consider Barbarip, which has two universal premises that are the same as those of Barbara but has a particular conclusion. The present system makes the existential import of the minor term, say S, explicit by depicting the hidden premise that some S is S:

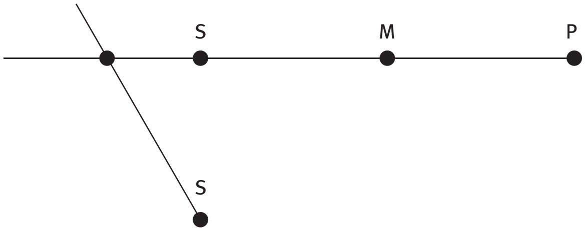


Figure 3.39: Barbarip

This system of line diagrams can be applied to arguments well beyond just classic syllogisms. One of the most crucial limitations on plane figure diagrams for logic is that they become less perspicuous and soon quite impractical as a usable tool for logic once the number of terms in an argument get much beyond four or five. No such limit applies to ED diagrams. Consider the following five-term sorites argument: Every A is B, every B is C, no C is D, some D is E; so some E is not A. Diagramming the universal premises together (in any order) and then adding the particular premise immediately reveals the conclusion:

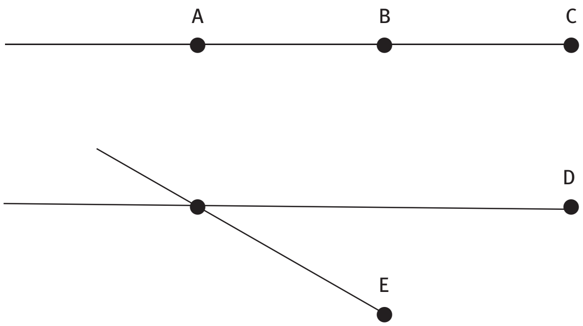


Figure 3.40: A Five-term Diagram

One task given to the logician is to determine whether or not a given set of statements is logically consistent, whether or not they can all be true together. Once such a decision is made, the next task is to deduce an explicit contradiction, usually some simple contradiction from the set. For example, any set of statements of the form: Every A is B, some A is not C, every B is C, is inconsistent. This can be shown by diagramming the statements together:

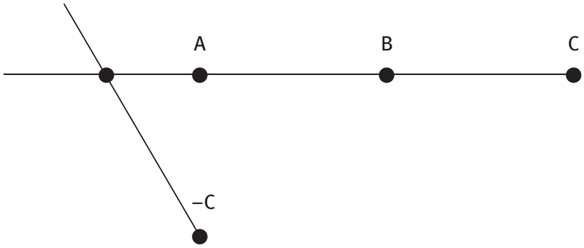


Figure 3.41: An Inconsistent Set

Note that this diagram reveals the contradictory statement that some C is nonC; it is a contrapiction. Now, since any valid argument can be proved *indirectly* by proving that the set of statements consisting of its premises plus the contradictory, or even the contrary, of its conclusion. Here is Baroco diagrammed indirectly:

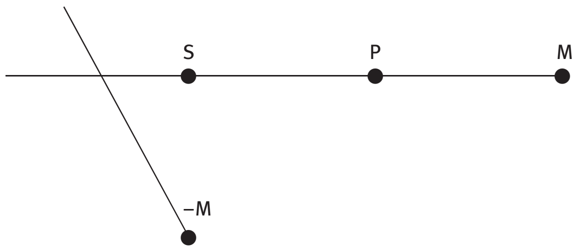


Figure 3.42: A Contrapiction of Baroco

In the next section a closer look at singular terms and how arguments in which they play a role are diagrammed.

3.4 The Point of Names

> Point exists only in line, which is in surface, which is in body, which is in matter.
>
> Avicenna

Aristotle held that the terms that were of interest to him in the building of syllogistic were, for the most part, terms of medium generality (*Prior Analytics* 43a41–42). According to some (Ross 1949, Łukasiewicz 1957, Bird 1964, Patzig 1968), for a variety of different reasons, Aristotle intended to exclude from syllogistic not only terms of highest generality (e.g., 'exists', 'substance', etc.) but also terms of lowest generality – singular terms (for counter arguments to each of their claims see Englebretsen 1980b). *Prior Analytics* is hardly bereft of examples of statements and entire syllogisms using singulars (e.g. 43a34–35, 47b24–25, 47b32–33, 67a33–37, 68b41–69a12, 70a16–29, 78b4–10). There is no prohibition of singulars from any term logic, Aristotle's syllogistic, traditional syllogistic, TFL. We have already seen how singular terms are represented (by labelled points) in ED. Moreover, one must keep in mind Leibniz's insight (Leibniz 1966, 115) that singular terms when used as subject terms accept arbitrarily either universal or particular quantity, *wild* quantity. It is because singular subject terms are wild in quantity (their quantity is arbitrary) that any sign of quantity (e.g., 'every', 'some') is ignored, suppressed in ordinary uses of a natural language. Nonetheless, from a strictly logical point of view, such singular terms *do* have a quantity. This suppression of quantity for singular subjects can be rendered explicit in ED. Consider the singular statement 'Socrates is a philosopher', which can be construed as either '(Every) Socrates is a philosopher' or '(Some) Socrates is a philosopher'. Diagrammatically:

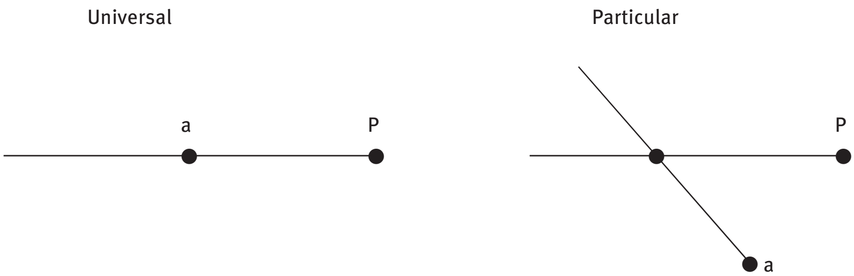


Figure 3.43: Wild Quantity

Since singular subject terms are wild in quantity, the two s-line segments in these diagrams are each reducible to the single s-point (as in Figure 3.18). Note that the distribution value of a singular subject term is likewise wild.

TFL takes all terms to be syntactically on a logical par, fit for any logical role in a statement. One consequence of this is that the semantic distinction between singular terms and general terms can be ignored (Englebretsen 1986b). Thus, singular terms, just like general terms, come in logically charged (positive/negative) pairs. Just like, for example, 'massive'/'massless', 'Kripke' has both a logically positive and a logically negative form. And, like 'married'/'unmarried', the positive charge on such terms tend to be suppressed: 'Kripke'/nonKripke'. The obvious objection at this point is that 'nonKripke' is not at name, even a negative name, of anyone or anything. More on this soon. A second consequence of this full integration of singular terms with general terms by TFL is that, just as general terms can be predicate terms, singular terms can be predicate terms as well.

We know that Frege took the ontological distinction between objects and concepts to be absolute and inviolable. But that distinction ultimately rests on a semantic distinction, the one between singular terms (especially names) and general terms (Sommers 1982, 37–40). While a general term is understood as denoting any number of individual things, a singular term is understood as denoting a single (exactly one) thing. It is just a matter of what is taken to be the size of sets of denotata, a matter of semantics. Most logicians following in Frege's footsteps presume that this semantic distinction is equally absolute and inviolable. The distinction between singular terms and general terms is also absolute and inviolable – the so-called *Asymmetry Thesis*. The thesis is often, oddly, formulated as claim that 'subjects' and 'predicates' are logically asymmetric; yet, even then, the target expressions are singular and general *terms*. Speaking of both traditional logic and modern logic, Frank Ramsey wrote:

> Both the disputed theories make an important assumption which to my mind, has only to be questioned to be doubted. They assume a fundamental antithesis between subject and predicate, that if a proposition consists of two terms copulated, these two terms must be functioning in different ways, one as subject, the other as predicate. (Ramsey 1925, 404)

Among the many who came to the defense of the asymmetry thesis, P.F. Strawson was one of the most careful and persistent. In defending the thesis, Strawson argued against both the possibility of negating singular terms and the possibility of predicating singular terms. In the former case, he argued that while a sentence could be negated simply by negating its predicate, the attempt to negate the subject does not yield the negation of the sentence. Indeed, the result of doing so just yields nonsense. This is so because general terms "come in incompatibility groups" but singular terms do not (Strawson 1970, 102–103; 1974, 19). For example, 'red', 'blue', green', etc. are mutually incompatible (contrary), but what is incompatible with 'Kripke'? Who or what is nonKripke? Kripke has a very large number of properties such as being from Omaha, being the author of *Naming and Necessity*, being a male, being American born, wearing a beard. He also lacks many properties such as being Canadian born, being Belgian born, being more than six feet tall. For Strawson, whatever nonKripke would be, he, she, it lacks all the properties Kripke has and has all the properties Kripke lacks. Consequently nonKripke would, *per impossible*, be both Canadian born and Belgian born (Strawson 1970, 111n). No individual can have incompatible properties at the same time; any such purported individual (like nonKripke) is impossible.

And Strawson was right. There can be no such individual as nonKripke. But that is because *the negation of a singular term is not a singular term* (Englebretsen 1985c, 1985d, and 2015, 19–20). The negation of a singular term is a general term. In ordinary discourse we often form the negation of a singular term. In English we do this with expressions such as 'other', 'besides', 'else', 'except', 'but', etc. Given a suitable determinable universe of discourse, the denotation of a negative singular term such as nonX would be everything in the domain other than X. For example, in the sentence 'Ed but not Tom came to the party' the 'but not Tom' does not denote an impossible individual (as Strawson thought). It denotes each of the party invitees with the exception of Tom. In the sentence 'No solar planets other than Earth are inhabited' the expression 'other than Earth' (formally 'nonEarth') denotes Mercury, Venus, Mars, Jupiter, Neptune, Uranus, and Saturn. Compare the diagrams for 'Earth is inhabited' and 'No solar planets other than earth are inhabited':

Figure 3.44: Diagrams for Singular and Negative Singular Subjects

Note that the unmarked singular term in the first diagram is nonetheless implicitly charged positively (and the quantity of the subject is implicitly wild). Note as well that in the second diagram the negative version of the singular term is not singular; it functions logically as a general term (and is universally quantified here). (For more on negated singular terms see Clark 1983 and Zemach 1981 and 1985.)

Strawson's dismissal of the second result of the asymmetry thesis, the claim that singular terms cannot be predicated, cannot be predicate terms, was brief – even blunt. He wrote that this asymmetry "seem[s] to be obvious and (nearly) as fundamental as anything in philosophy can be" (Strawson 1957, 446). *Prime facie*, the sentiment seems sound. And it seems to have a very long history. In *Categories*, Aristotle distinguished, among "things that are said," between those that are "said of" (predicated of) a subject and those that are "in" a subject. There are also those that are both and those that are neither. These latter are the subjects that the others are either said of or in or both. They are individual, numerically one. They are ontologically and logically basic. They are *primary substances*. "So if the primary substances did not exist it would be impossible for any of the other things to exist" (*Categories* 2b5 – 6). So individuals are not said of a subject, not predicated. In the most basic sense, they are the "things there are" (*Categories* 1a20). Aristotle was less than perfectly clear about the distinction between things that *are* and things that *are said*. He included primary substance in his survey of "things that are said" even though they are things that (fundamentally) are. So how is this to be treated in a term logic?

TFL treats all terms, singular or general, count or mass, concrete or abstract, the same. Thus, singular terms are just terms, and they can be subject terms or predicate terms. We often predicate singular terms. 'The only inhabited solar planet is Earth', 'The forty-fourth U.S. president was Obama', 'That woman is Eve', 'The creator of Harry Potter is J.K. Rowling', 'Twain is Clemens' (remember in this case that TFL requires no special treatment of identity). 'No solar planets other than Earth can be logically paraphrased as 'No nonEarth is an inhabited solar planet', which is logically equivalent to 'Every inhabited solar planet is Earth', another sentence with a singular predicate term.

As has been shown, TFL has no need of a special "theory of identity." There is no necessity for a so-called "*is* of identity." Sentences taken to be expressions of identity are logically construed as predications like any other. To say that x is identical to y is simply to say that x is y and y is x. This is due in part to the wild quantity thesis that derives from Leibniz. He also formulated the principle that two terms that denote the same thing can be substituted for one another anywhere without loss of truth (Leibniz 1966, 34, 43, 52–53, 122, 131). Consider, now the example of a syllogism offered by Leibniz (Leibniz 1966, 115):

> Should we say that a singular proposition is equivalent to a particular and to a universal proposition? Yes, we should. So also when it is objected that a singular proposition is equivalent to a particular proposition, since the conclusion in the third figure must be particular, and can nevertheless be singular; 'Every writer is a man, some writer is the Apostle Peter, therefore the Apostle Peter is a man'. I reply that here also the conclusion is really particular, and it is as if we had drawn the conclusion 'Some Apostle Peter is a man'. For 'some Apostle Peter' and 'every Apostle Peter' coincide, since the term is singular.

Note the form of this syllogism: 'Every W is M, Some W is A.P, so (some) A.P is M'. Here, the singular term occurs as a predicate term in the minor premise. It also appears as a subject term in the conclusion. The syllogism is easily diagrammed:

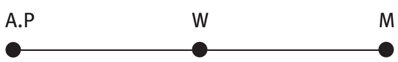


Figure 3.45: Leibniz's Syllogism with a Singular Predicate Term

Consider next a syllogism that requires a singular predicate term (twice) and a singular subject term that occurs once universally quantified and once particularly quantified: Some writer is Twain, Clemens is Twain, so Clemens is a writer' (formally, adopting the TFL convention of using lowercase letters to label individuals: Some W is t, c is t, so c is W). In this case, the particular quantity of the major premise demand the particular quantity of the conclusion as well as the universal quantity of the minor premise. Diagrammatically:

Figure 3.46: Diagrammed Syllogism with Two Singular Terms

An advantage claimed for the use of ‘=’ as a special binary relational term is that it shares the formal features of being reflexive, symmetric, and transitive, which are definitive of the mathematicians’ equal sign (also ‘=’). Yet these features are not lost when the ‘is’ of identity is abandoned. Let x, y and z be singulars (thus being fit at a subject term or as a predicate term and having wild quantity when used as a subject term). Thus ‘x is x’ has the tautological logical form ‘every x is x’. Also, the inference from ‘x is y’ to y is x’ has the valid logical form ‘some x is y, therefore some y is x’. Finally, the inference from ‘x is y’ and ‘y is z’ to ‘x is z’ has either of these two valid logical forms: ‘every x is y and every y is z, therefore every x is z’ or ‘some x is y and every y is z, therefore some x is z’. In other words, all three formal features of identity are preserved without requiring identity to be a special binary relation with special rules of inference and a special symbolization. They can be diagrammed very simply as long as the theory of wild quantity is observed:

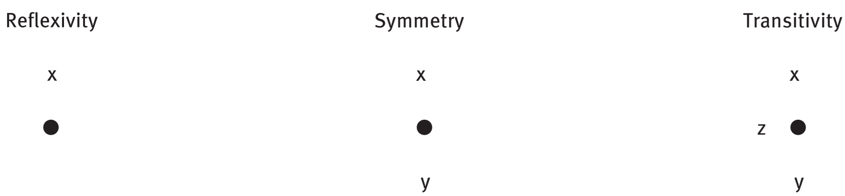


Figure 3.47: Reflexivity, Symmetry, and Transitivity in TFL

(For more on identity in TFL as well as the predication of singular terms see, for example, Sommers 1969 and 1982, Noah 1973, Lockwood 1975, Englebretsen 1985 f., Frederick 2013; also Moktefi 2015, 608 for the diagrammatic distinction between *is identical to* and *is*.)

Unquestionably, some singular terms are compound terms, so-called phrasal conjunctions and phrasal disjunctions. Advocates of the Asymmetry Thesis (especially Strawson) not only denied the possibility of negated singular terms and predicated singular terms but the possibility of conjoined or disjoined singular terms as well. Consider these two sentences: ‘Every Beatle sings’ and ‘Every logician sings’. In ordinary discourse contexts, the denotation of ‘Beatle’ consists of just four things but the denotation of ‘logician’ is much, much larger. In ‘Every prime number greater than 2 is odd’, the denotation of ‘prime number greater than 2’ is infinite. Now, when the denotation of a term is known to be relatively small, it is possible to dispense with the use of that term and use in its place another term that explicitly indicates each of the things in its denotation. In place of ‘Beatle’, for example, one could substitute ‘John, Paul, George, Ringo’. Call ‘⌞John, Paul, George, Ringo⌟’ a term of *explicit denotation* since it wears its denotation on its face. In fact, this is true of any term. In principle (though rarely practical) any term, T, whose denotation consists of a, b, c,…, is replaceable by ⌊a, b, c, …⌋.Thus, 'Every Beatle sings' can be replaced by 'Every ⌊John, Paul, George, Ringo⌋ sings'. This is clearly not an ordinary English sentence. Its ordinary version would be 'John, Paul, George, and Ringo sing'. In this case the logical quantifier 'every' is now indicated by the word 'and'. By contrast, the familiar sentence 'John, Paul, George, or Ringo plays drums' uses 'or' to indicate particular quantity. Its more formal version would be 'Some ⌊John, Paul, George, Ringo⌋ plays drums'. The use of such phrasal conjunctions and phrasal disjunctions is rare in ordinary discourse because the vast majority of terms have denotations that are too large or too indeterminate to be expressed by the use of corresponding terms of explicit denotation.

Consider, once more, 'Every Beatle sings'. It is a simple universal affirmation. It could be symbolized in TFL as: −B+S and, substituting for B its explicit denotation: − ⌊J, P, G, R⌋ +.S. But now consider 'The Beatles won the top quartet prize'. In this case, the expression 'the Beatles' could not reasonably be replaced by the explicit denotation version of 'Beatle'. In 'Every Beatle sings' the quantifier applied *distributively*, so that 'sings' can be predicated of each Beatle. That is why phrasal conjunctions are routinely replaced by sentential conjunctions in MPL ('Every (A and B) is C' becomes 'Every A is C and every B is C'). However, the expression 'the Beatles' is not a phrasal conjunction, not logically equivalent to 'John, Paul, George, and Ringo'. John didn't win the top quartet prize, nor did Paul, nor did George, nor did Ringo. Who won that prize? The quartet, consisting of John, Paul, George, and Ringo. A quartet always has four members, four individuals, *but is itself an individual*. Compare that with the sentence 'Russell and Whitehead wrote *Principia Mathematica*. Russell didn't write it. Whitehead didn't write it. The duo, the writing team whose members were Russell and Whitehead wrote it. Call terms like 'the Beatles', 'Bourbaki', 'the New York Yankees', 'the Vienna Philharmonic' *team terms*. On some occasions, 'Russell and Whitehead', 'Venus and Serena', 'John, Paul, George, and Ringo' are team terms (on other occasions they are simply phrasal conjunctions. The term 'Peter, Paul, and Mary' is the name of the trio, so it is simultaneously a team term and a term of explicit denotation. Team terms are singular terms. They denote one individual – a team (which happens to consist of individuals). Teams, being individuals, could themselves be members of teams of teams, and so forth. When a team term is quantified (plays the logical role of subject term or object term) its quantity is, of course, wild. Let a, b, and c constitute a team. The team may or may not have a name. In either case, let ⌜a, b, c⌝ denote the team, call it an *explicit team name*. For example, 'the Beatles' is a team name and ⌜John, Paul, George, Ringo⌝ is its explicit name. (For much more on terms of explicit denotation, team terms, etc., see Englebretsen 1996, 183–185, but especially Englebretsen 2015, 121–131).

Terms of explicit denotation can be treated just like any general term in TFL. In ED, they would be represented graphically just like any general term. For example, 'John, Paul, George, and Ringo sing' would be treated as 'Every ⌞John, Paul, George, Ringo⌟ sings (– ⌞J, P, G, R⌟ +S) and diagrammed as a normal A categorical. 'John, Paul, George, or Ringo plays drums would be treated as 'Some ⌞John, Paul, George, Ringo⌟ plays drums' (+ ⌞J, P, G, R⌟ +P) and diagrammed as a normal I categorical.

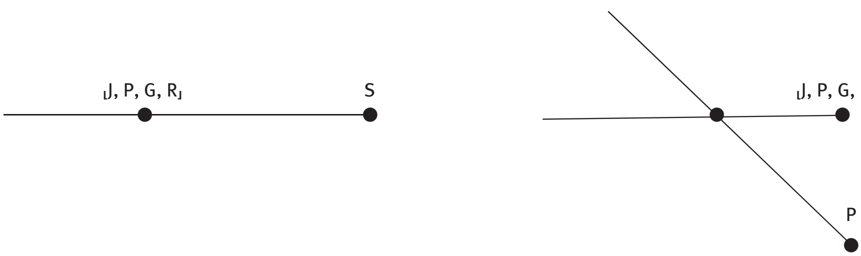


Figure 3.48: Diagrams With Terms of Explicit Denotation

A sentence such as 'The Beatles won the top quartet prize' could be construed as using a simple team name ('the Beatles') or an explicit team name (⌜John, Paul, George, Ringo⌝). In either case, they are treated as singulars (*B +W and *⌜J, P, G, R⌝+W) and diagrammed accordingly:

Figure 3.49: Diagrams With a Team Name and With an Explicit Team Name

3.5 Vectors of Relations

> It is well known that relations are more difficult to represent in a graphical system than are properties.
>
> S.-J. Shin

Modern logicians pride themselves in having at hand a system of formal logic that is adequate for the demands unmet by ancient and traditional logicians.

Prominent among such demands is that any adequate system of formal logic should be able to account for relational expressions, statements making use of such expressions, and deductions involving such statements. It is known that Aristotle, though unable to provide an adequate account of relationals, was fully aware of the importance of such a requirement, used and discussed examples of relational statements and syllogisms using such statements, and formulated rules for such deductions (see Bocheński 1951 and 1968, Sommers 1982, ch 7, Englebretsen 1982d). Traditional logicians also were mindful of the importance of accounting for relationals, with treatises on so-called *oblique* term cases. And, of course, Leibniz made an attempt to incorporate the logic of relationals into syllogistic. The problem with such efforts, according to the moderns, is that a categorical statement can have only one subject term, one quantified term, while relationals require two or more reference-making terms (terms that are either singular or quantified). Term Functor Logic illustrates just how a formal term logic can be adequate for the demand to account for the logic of relationals. Relational expressions are terms on par with all other terms. In fact, they are complex terms. Complex terms are always pairs of less complex terms that are logically copulated (by split or unsplit copulae). Moreover, the graphic system ED is able to provide a perspicuous representation of relational terms.

Consider this simple example, 'Romeo loves Juliet', symbolically: *R+(L*J). A relational term often has a "direction" (determining whether it is to be understood actively or passively). TFL indicates this by means of a system of subscribed numerals, which will be ignored for now. ED makes use of vector lines, arrows, to represent relational terms like 'loves'. Such vectors quite literally indicate direction. Thus:

Figure 3.50: Romeo loves Juliet

Not only does Romeo love Juliet, Juliet loves Romeo. So:

Figure 3.51: Juliet loves Romeo

Note that the use of vectors allows one to deduce from these two diagrams that Juliet is *loved by* Romeo and that Romeo is *loved by* Juliet. It all depends upon reading the vector from tail to head (active voice) or from head to tail (passive voice).

Consider now 'Some officer is giving a ticket to every speeder'. Its overall form is categorical. Its subject is the quantified simple general term 'some officer' and its predicate is the qualified complex term 'is giving a ticket to every speeder'. Those two terms, then, are copulated by the split copula 'some…is'. The predicate term, being complex, is itself the copulation of two less complex terms, 'giving a ticket to' and 'speeder', the copula of which is the unsplit 'every'. As well, the complex term 'giving a ticket to' is the copulation of two less complex terms, 'giving… to' and 'ticket' with the copula here being the unsplit 'a' (i.e., 'some'). It can be rendered symbolically as: +O+((G+T)−S). This could be diagrammed as an I categorical whose predicate term happens to be complex. In some logical context that would be enough. However, in most logical contexts, ones in which one of the constituents of the complex predicate term occurs without the rest, it is necessary to *analyze* such complex terms. One could analyze the complex predicate either partly or completely. In the former case, 'giving a ticket to' is left unanalyzed; in the latter case it is analyzed. Here is how our example is diagrammed with no such analysis:

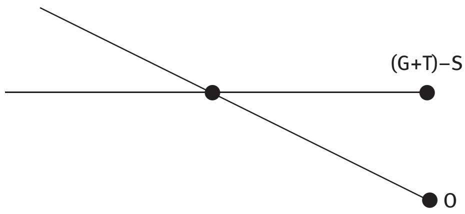


Figure 3.52: Unanalyzed Complex Relational

In other contexts, it could be necessary to at least partially analyze this complex relational:

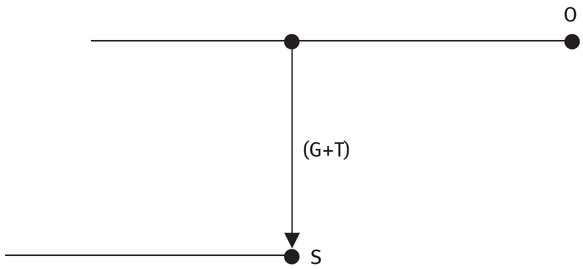


Figure 3.53: Partially Analyzed Complex Relational

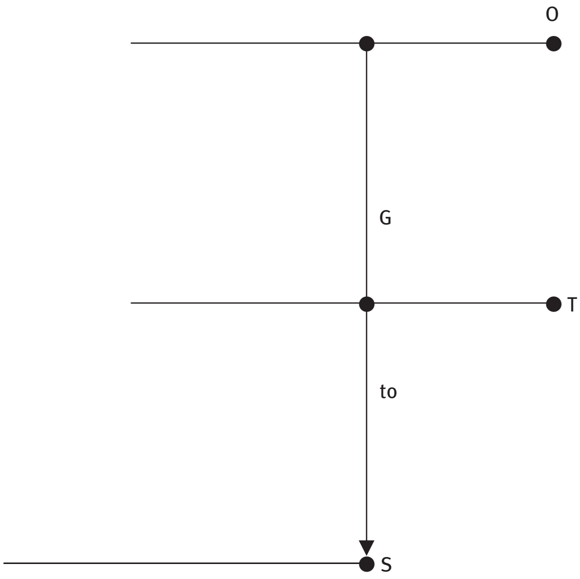


Figure 3.54: Fully Analyzed Complex Relational

It is important to notice that the locations of any points through which such vectors pass, including arrow tail points and arrow head points, indicate the quantities of the relata. The quantity is universal whenever the vector intersects the right terminus of a line and particular whenever it intersects a point to the left of the terminal point.

Of course, one could diagram such an example with the complex relational both unanalyzed and analyzed. This would be a representation of a fundamental principle: *Whatever is related (R) to some/every A is related (R) to some/every A*. Thus these two tautologies:

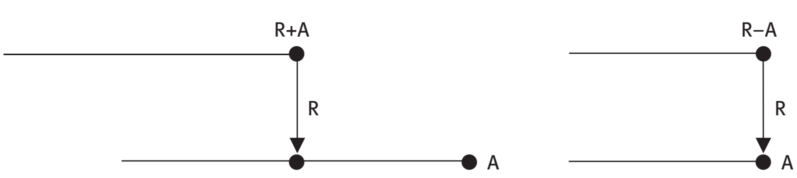


Figure 3.55: The Principle of Relational Analysis

So the corollary:

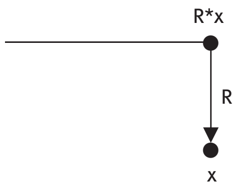


Figure 3.56: A Corollary

Consider 'Some A is R to some B'. It could be diagrammed as:

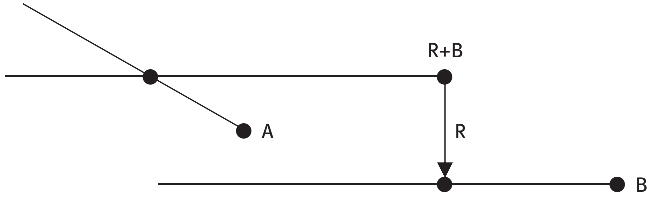


Figure 3.57: Unsimplified Relational

Since some of the information exhibited here is tautological and redundant (the inclusion of the unanalyzed relational term), the diagrammed can be simplified:

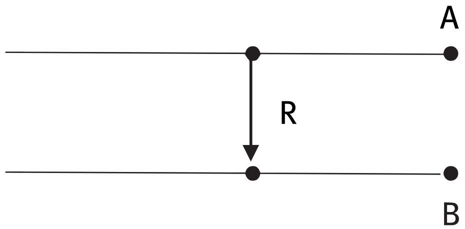


Figure 3.58: Simplified Relational

There are, however, cases requiring the inclusion of such tautological information. Consider the following argument: 'Some senator received a bribe, all bribes are illegal, whoever receives something illegal is a crook, therefore, some senator is a crook'. In this case the relational term 'received a bribe' must be represented diagrammatically as both analyzed and unanalyzed:

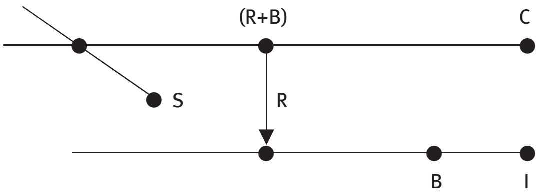


Figure 3.59: Valid Relational Inference

Such a strategy will always be required whenever a relational expression is used, as in this case, as a subject term in one occurrence and as a predicate term in another occurrence. The conclusion can be seen as already diagrammed just by diagramming the premises. But the diagram reveals much more. For example, it could also be concluded on the basis of the diagram that a bribe was received, that a bribe was received by a senator, that a bribe was received from a crook, something illegal was received, that something illegal was received from a senator, and that something illegal was received from a crook.

De Morgan's famous example of a relational inference ('Every horse is an animal, therefore, every head of a horse is a head of an animal') can provide some valuable insight into the logic of relationals. Letting C stand for 'head of', this is formalized in TFL as: −H+A ∴ −(C+H)+(H+A). TFL treats this as an enthymeme whose tacit premise is the tautological 'Every head of a horse is a head of a horse'. The full ED diagram for the inference is:

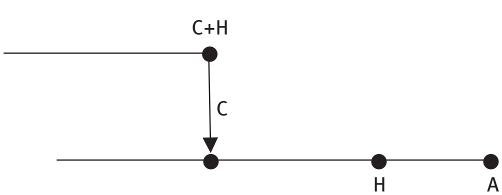


Figure 3.60: De Morgan's Inference

This example provides motivation for formulating a further fundamental principle governing the logic of relationals. Consider these four argument forms (followed by the appropriate diagrams):

1. Every X is Y, some S is R to some X, therefore, some S is R to some Y
2. Every X is Y, some S is R to every X, therefore, some S is R to some Y
3. Every X is Y, every S is R to some X, therefore, every S is R to some Y
4. Every X is Y, every S is R to every X, therefore, every S is R to some Y

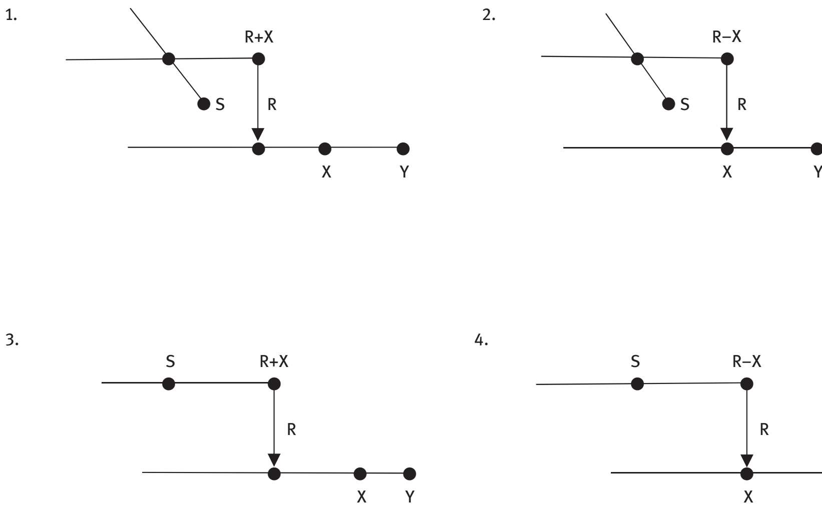


Figure 3.61: Four Special Inferences

As it happens, the conclusion of 2 and the conclusion of 4 can only be deduced by introducing a hidden premise of the form 'Some X is X', which is contingent. As the diagrams show, the only additional premise required here is a tautology found above in Figure 3.52. It is the Principle of Relational Analysis: whatever is R to some/every A is R to some/every A. Based on the four cases above, the following generalization can be formulated as an additional fundamental principle, the Principle of Relational Extension, governing the logic of relationals: *If every X is Y, then whatever is R to some/every X is R to some Y.* The diagrammatic version is:

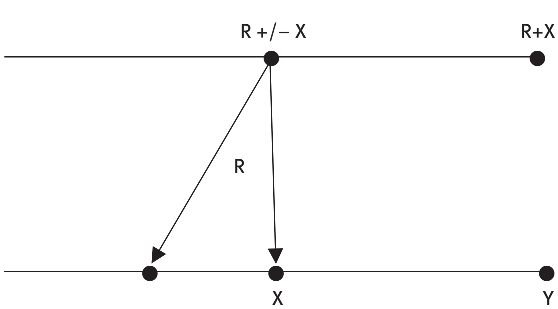


Figure 3.62: The Principle of Relational Extension

It is by virtue of the principle that a line segment representing a relational term ('R to some X' or 'R to every X') can be extended to the right so that it is a part of the line representing 'R to some Y' whenever every X is Y.

Binary relational predicates such as 'loves', 'picks up', 'next to' and the like are composed of a relational term and an object. An *object* is a quantified term. Syntactically, it is no different from any *subject*. A ternary (or more) relational predicate requires two (or more) objects. One such object will be a *direct* object while the other objects are *indirect* objects. In the example 'Some officer gave a ticket to every speeder', the relational predicate consists of the relational term ('gave…to'), an indirect object ('a ticket'), and a direct object ('every speeder'). Sometimes, *where* a particularly quantified term, whether a subject or an object, is used in a statement can determine how it is to be understood. A common illustration of this is provided by the (well-worn) example, 'Every boy loves some girl'. Here, the object ('some girl') is ambiguous. The sentence can be understood as saying of every boy (i) that he loves some girl or other, or (ii) that he loves some specific, certain girl. The two alternatives are given distinct logical formulations: (i) 'Every boy loves some girl (or other)', (ii) 'Some specific girl is loved by every boy'. The 'some girl' is taken to be *non-specific* in (i) but *specific* in (ii). Modern predicate logic resolves this by the order of quantifiers: (i) is a universally quantified existential, (ii) is an existentially quantified universal. TFL achieves the same resolution in terms of the order of positive and negative terms (see Sommers 1976b, 23 and Sommers 1982, 141–142 for brief but clear accounts). The two sentences would be diagrammed differently:

Figure 3.63: Specific vs Non-Specific Reference

The object in (ii) is specific; it refers to a specific girl, an individual. We might not know her name, but we can still refer to her. We might make use of a pronoun ('she'), so that we could paraphrase (ii) as 'Some girl is such that she is loved by every boy', and 'she' here is a singular term. Thus, when used as either a subject or an object it has wild quantity (*G+(L–B). That's why the diagram for (ii) represents her as merely a left endpoint of a line. More about pronouns later, but first, one more fundamental principle of the logic of relationals.

If Romeo loves Juliet, then Romeo loves (and as well, Juliet is loved). In natural language, relational terms have one or more objects. If Morgan offered a ride to Russell, then it follows that Morgan offered a ride. If he offered a ride to a friend, then he still offered a ride. In our ordinary use of our natural native language, we take relational terms and their non-relational partners on a par. Thus, for example, the 'loves' in 'Romeo loves Juliet' is the same 'loves' in 'Romeo loves' (likewise for 'lover of' in 'Romeo is a lover of Juliet' and 'Romeo is a lover'). We readily take a relational statement to entail the same statement without one or more of its direct or indirect objects. Thus, from 'Morgan offered a ride to a friend' one could deduce each of the following: 'Morgan offered a ride' and 'Morgan offered'. TFL takes this at face value, formulating the premise as: *M+((O+R)+F) and the conclusions as: *M+(O+R) and *M+O. Here, the term 'offered' can have any *adicity* (triadic, dyadic, monadic) depending on its sentential context. And that's natural. By contrast, MPL takes the adicity of any term as fixed, invariable, so that, for example, 'Romeo loves Juliet' and 'Romeo loves' are formulated as 'Lrj' and 'Lr', but the 'L' is ambiguous. It is a 2-place, binary relational term in the first, but it is a 1-place, monadic, non-relational predicate in the second. The inference from 'Romeo loves Juliet' to 'Romeo loves' ('Romeo is a lover', 'Romeo does love', etc.) is immediate for TFL and it is naturally so. By requiring that 'loves' be permanently binary, MPL can only deduce 'Romeo loves something' ('Romeo is a lover of something', 'Romeo does love something', etc.). At any rate, there is a general principle here that allows for the dropping of one or more objects from a relational: *Whatever is related R to every/some A_1 ...A_n is R to every/some A_{n-1}*. Diagrammatically then (using dotted line extensions to indicate quantity options):

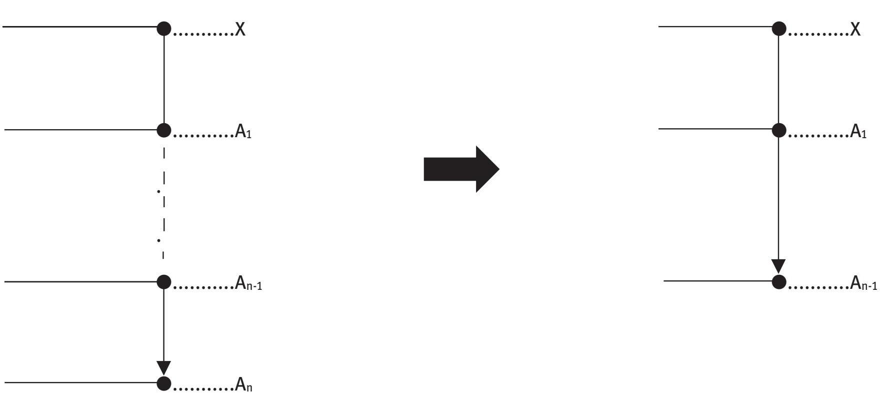


Figure 3.64: The Principle of Relational Reduction

Needless to say, other principles could be formulated to account for how converses of relationals are formed to allow for rearranging the order in which direct and indirect objects occur. For example, in English there are passive/active voice pairs such as 'loves'/'loved by' and 'gives … to'/'gives to …'/'given …by'. Nevertheless, we turn now to the task of incorporating into ED diagrams strategies for representing statements involving reflexive pronouns.

In English, reflexive pronouns are usually words such as 'itself', 'herself', 'himself', 'themselves', etc. They are always object terms. Compare 'Trump loves power' and 'Trump loves himself'. In each case, 'Trump' is the subject term. In the first case, the object of the relational is 'power', but in the second case, the reflexive pronoun 'himself' has 'Trump' as its referential antecedent. This allows the pronoun to be replaced by its antecedent, which yields 'Trump loves Trump'. We know how to diagram 'Trump loves power', but what of 'Trump loves Trump'? A first attempt might produce:

Figure 3.65: Attempted Singular Reflexive Diagram

The problem with this attempt is that it treats 'Trump' (T) as two individuals, one the subject of the relation and the other the object of the relation. 'Trump' denotes a single individual, and that individual is both the subject and object of the relation. So the diagram must represent that:

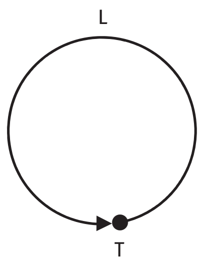


Figure 3.66: Singular Reflexive Diagram

Consider a sentence in which the subject/object of the relation is a general term: ‘Some senator nominated some senator’. One might attempt to diagram a sentence such as this in the same way the diagram for ‘Trump loves himself/ Trump’ was first attempted:

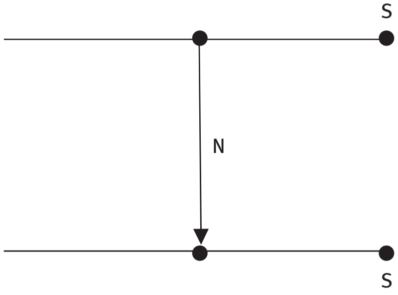


Figure 3.67: Attempted General Diagram

However, as in the first attempt, the denotations of the subject and the object are not distinct. There can only be one S line; otherwise a contradiction is depicted. The sentence can be diagrammed properly once this is recognized. Thus:

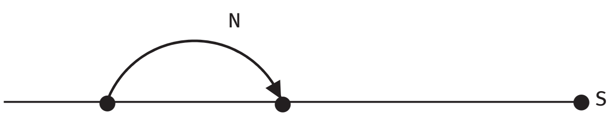


Figure 3.68: Proper Diagram

Here is an example of a valid argument involving a reflexive pronoun: ‘Some senator nominated herself. Every self-nominator is a fool. All fools deserve ridicule. Therefore, some senator deserves ridicule.” It can be diagrammed as follows:

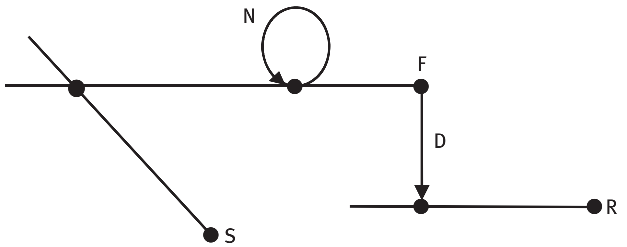


Figure 3.69: Argument With a Reflexive

As it happens, reflexive pronouns pose little challenge to the project of line diagrams. What now of (non-reflexive) personal pronouns in general? Again, we look to TFL for insight.

> It is after all a matter of historical fact that [traditional term logic] had no systematic treatment of pronouns and indeed that such a treatment is a justly celebrated accomplishment of MPL (wherein pronouns are represented as bound variables) … one may well wonder how this false charge has received such widespread and enduring acceptance by responsible logicians. That they were anxious to persuade students that the older logic was superseded is not by itself a sufficient explanation. A minimal attention to the methods of proof available to pre-Fregean logic would have given them pause. One doubts it could happen that two generations of modern geometers could falsely claim that Euclidean geometry could not prove certain theorems that are easily provable in some non-Euclidean systems. But then mathematicians who do modern geometry are not as tendentious as philosophers who do modern logic. (Sommers 1982, 146–147)

A brief note regarding some semantic matters is in order before continuing. A used term, one used relative to a given universe of discourse (a *domain*), *denotes* every thing in the domain of which it is true. If a speaker says, relative to the neighborhood in which he or she lives, ‘Every dog is barking’, then’ dog’ denotes, in this case, every dog in the neighborhood. A *quantified* term, used relative to a given domain, *refers* to either every or at least one thing in the domain that the term *per se* denotes. So, a quantified term (a logical subject or object) has a reference determined in part by the denotation of its term and its quantifier. Thus, the term ‘dog’ denotes every dog in the domain relative to which it is used; ‘every dog’ refers to every dog in the domain; ‘some dog’ refers to at least one dog in the domain. The late medieval logicians seem to have been relatively clear about this distinction. Whenever a used quantified term refers to just what it denotes it is said to be distributed; otherwise it is undistributed. Briefly: terms denote; quantified terms refer.

We have seen above examples of terms that are relational (‘loves some girl’ (L+G)). Relational terms are complex terms. However, some complex terms are not relational. For example, compound terms such as 'rich and famous' (R+F) are complex. Consider now the following pair: 'Some dog is barking' (+D+B) and 'It is annoying' (?+A). What is annoying? Clearly, the dog. Which dog? The dog that is barking. And that is just what the pronoun 'it' denotes. The pronoun's antecedent is 'some dog' in the first sentence. While it might be thought that 'it' refers to just what its antecedent refers to, the fact is that the pronoun denotes everything to which its antecedent refers (not just some dog, but rather, *the specific dog under consideration* – the one that is barking). Such pronouns are always implicitly quantified; they are logical subjects or objects. As such, they have a denotation and also refer. Again, reference is determined by both denotation and the relevant quantifier. In effect, such a pronoun has universal quantity in addition to the quantity of the antecedent subject. In effect, the pronoun for a definite subject 'some x' will always be an expression with 'wild' quantity. Pronouns that cross-refer to 'every x' are another matter" (Sommers 1976b, 26). TFL represents this tie that connects such pronouns to their antecedents by attaching to the antecedent term a superscripted letter that is then used as the pronoun. A pronoun inherits the quantity applied to its antecedent and also has implicitly its own universal quantity (since it is meant to denote all of what its antecedent refers to). In this case, that means it has wild quantity. Thus: (+D^i +B) and (*i+A). Of course, the first sentence alone, or in a different context, need not have any subsequent pronoun. It would then be treated as a simple particular affirmation whose subject is understood to be non-specific. But, once pronominalized, such a subject must be understood as making specific reference. In that case, the first sentence above could be diagrammed in two equivalent ways:

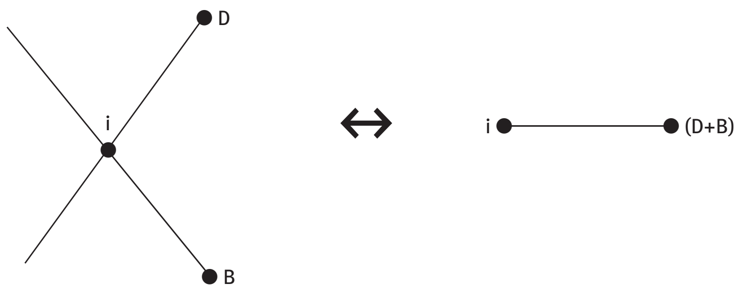


Figure 3.70: A Simple Pronominalization

Given the pronominal ties between the two original statements, they can now be diagrammed together as:

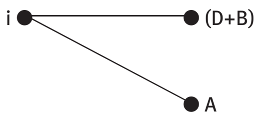
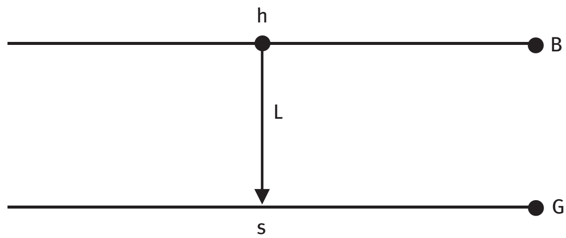


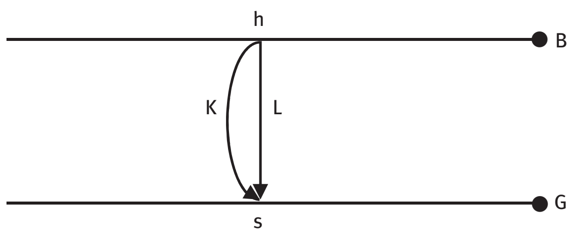
Figure 3.71: A Full Pronominalization

In MPL, pronouns, in the guise of bound variables, are ubiquitous. In natural languages, they seem to be used more sparingly. The antecedents of bound variable pronouns are always the quantifier expressions that bind them. Natural language personal pronouns usually follow their antecedents and are called ‘anaphoric pronouns’. Sometimes, however, they precede their “antecedents” and are then called ‘cataphoric pronouns’. Consider next an example that involves an interlocking pair of such pronominalizations: ‘A boy who loved her kissed a girl who slapped him’ $(+(B^h+(L*S))+(K*(G^s+(S+H)))$. Note the relations that hold between that boy and that girl. Among the boys and girls (in the domain), at least one of the boys kissed one of the girls, that boy (he, ‘h’) loved that girl, that girl (she, ‘s’) slapped that boy. Thus, the boy who loved the girl kissed the girl who slapped him. This might be diagrammed in three stages:

Step 1 (Some boy loves some girl)



Step 2 (He kissed her)



Step 3 (She slapped him)

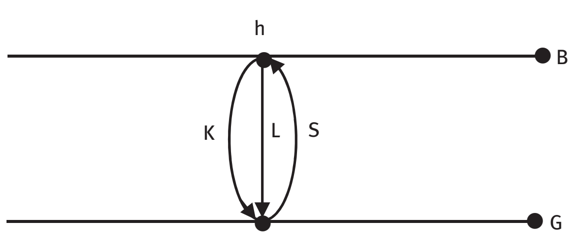


Figure 3.72: An Example of Interlocking Pronominalizations

Finally, here is an example of a diagrammed inference involving a pair of pronouns: ‘A cat is screeching; I own it; it annoys me; but all cats are lovable; so, something lovable annoys me’ $(+C^{f}+S)$, $(*i+(O*f))$, $(*f+(A*i)$,$(-C+L)$ $\therefore$ $+L$ $+(A*i)$.

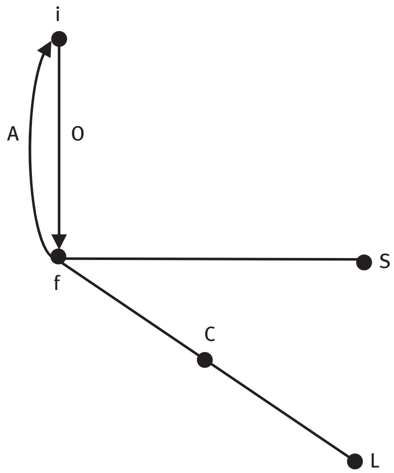


Figure 3.73: A Diagrammed Inference with Pronouns

We have claimed that all terms are positively or negatively charged. Consequently, the same is true of relational terms. In the absence of any contextual clues, the sentence 'Some critics don't like every Shakespeare play' is ambiguous between something like (i) 'Some critics fail to like every Shakespeare play'; perhaps they like the comedies but not the tragedies, or they like all of them except for Othello, and (ii) 'Some critics dislike every Shakespeare play'. This is shown in their TFL formulations: +C−(L−S) and +C+(−L−S). Note that the relational term ('likes') is positively charged in the first version (though it is part of the negative complex predicate term ('fails to like every Shakespeare play'). By contrast, the relational term in the second version is itself negative ('dislikes'), (while the complex predicate term of which it is a part is positive). Diagramming the two alternatives graphically represents their logical differences:

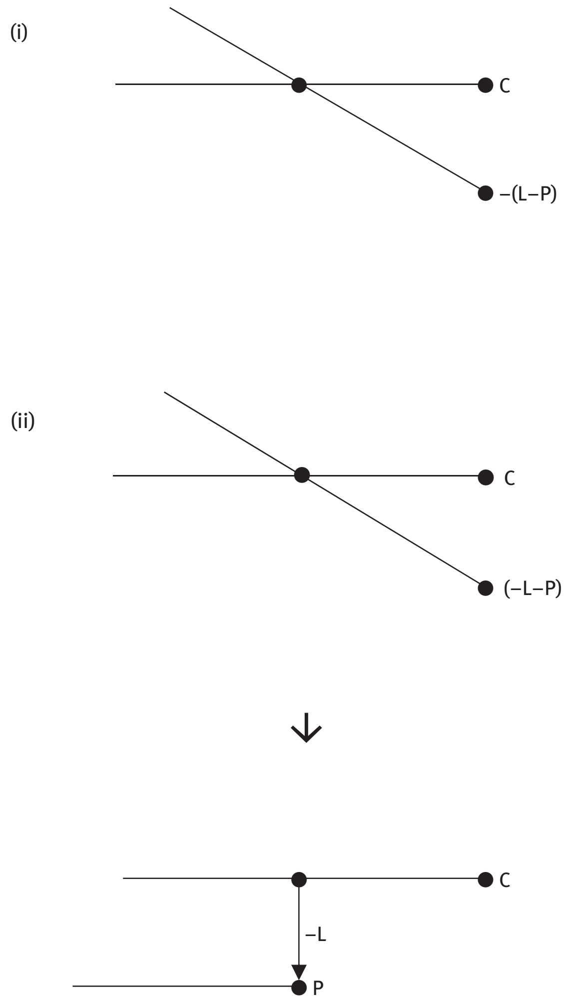


Figure 3.74: Negative Relationals

Note that the relational predicate term in the diagram of (ii) has then been analyzed. Note as well that, given the innocent tacit premise that there are Shakespeare plays, (ii) entails (i).

The real import of negative relationals is seen in cases of inferences in which they play a logically effective role. An example of such an inference is the following: Some critics dislike every Shakespeare play. *Othello* is a play by Shakespeare. All actors like *Othello*. So, no actor is a critic. It can be diagrammed thus:

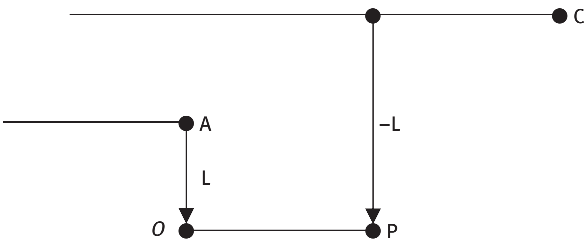


Figure 3.75: Inference with Negative and Positive Relationals

Note that in this case the A line cannot be raised to the level of the C line (which is why no A is C) since the L and –L arrows could never meet. Were the A line to be raised the result would express the contradictory proposition that all actors both like and dislike *Othello* – a contrapiction.

3.6 World Lines

> Between the [calculus of classes and the calculus of propositions] there is … a certain parallelism, which arises as follows: In any symbolic expression, the letters may be interpreted as classes or as propositions, and the relation of inclusion in the one case may be replaced by that of formal implication in the other.
>
> Russell

> It was Sommers' insight that propositional logic can be developed within syllogistic.
>
> John Bacon

Whether expressing it symbolically or representing it graphically, determining the logical form of a statement is always a matter of choosing the appropriate level of analysis (see, for example, Corcoran 1999a and 2008). To see this, consider an inference that involves a syntactically complex term (the relational term 'admires some logicians') requiring no further analysis of it into its non-complex constituents: 'Every philosopher admires some logicians, whoever admires any logician is misguided; so, every philosopher is misguided'. One could, of course, diagram this with the usual analysis of the relational, using an arrow for 'admires' with a tail on the right terminus of the line for 'philosopher' and a head on a non-terminal point on the line for 'logician'. The 'philosopher' line would then be extended to the right to accommodate the second premise. However, since the constituent terms of the relational term do not play any logical role in the argument, analysis of the relational term is unnecessary. It is simply the middle term of a simple Barbara syllogism and can be diagrammed as such.

Needless to say, there can be cases of inferences in which more than one complex term might be left unanalyzed. As well, there are many cases in which no such terms can be safely unanalyzed. Entire sentences are terms (sentential terms). Even with these kinds of terms, analysis into their constituent unanalyzable terms is not always required. After all, any sentence, of any complexity, can be treated as completely unanalyzed. Consider this argument: 'If every actor is a thespian then some actors are not comics, all actors are thespians; so, some actor is not a comics'. One could formalize this recognizing the categorical forms of its constituents but such an analysis is unnecessary. It is in fact just a simple modus ponens argument.

Peirce's Alpha graphs precede Beta graphs. Nonetheless, Dipert (Dipert 1981) showed that Peirce took the logic of propositions and the logic of terms to be isomorphic, equally primary, neither more basic than the other. Kant had claimed that there is an absolute difference between categoricals and compound statements, thus, between a logic of terms and a logic of propositions. Leibniz took the logic of terms to be primary. Boole's view on the matter of primacy foreshadowed Peirce's. He saw his "secondary" logic of propositions and his "primary" logic of terms to be nothing more than two of the interpretations of his algebra. Post-Fregean logicians rejected (mostly ignored) all of these views (isomorphism, exclusivity, and term priority).

Frege, and then Wittgenstein, famously claimed that a word has a meaning only in the context of a sentence (the "Context Principle"). Were this so, then the logician would be required, in the first instance, to provide a logic of sentences before going on to a logic of terms. So-called propositional logic, or sentential logic, or statement logic is a formal system for accounting for logical reckoning involving only unanalyzed sentences. In MPL, such a system is universally presumed to be *primary logic*. Nevertheless, it can be, and has been, argued that a logic of terms is prior to a logic of sentences, that a logic of terms is primary logic. In fact, the logic of unanalyzed sentences (sentential terms) is only a part of the logic of terms. The fullest expression of these claims was made by Sommers in his "The World, the Facts, and Primary Logic," where he argued that "propositional logic is a special branch of term logic" (Sommers 1993, 181). "The doctrine that term logic is primary logic has ancient roots. Historically, term logic came first, having been discovered and developed by Aristotle; propositional logic, primarily a Stoic innovation, came later" (Sommers 1993, 172). Much of this argument for the primacy of term logic was foreshadowed by some of Leibniz's insights. The most important of these was his recognition that propositions (viz., unanalyzed sentences) could be treated as terms. He said this many times and in many ways:

> However, the categorical proposition is the basis of the rest, and modal, hypothetical, disjunctive and all other propositions presuppose it. (Leibniz 1966, 17)
>
> … any proposition can be conceived as a term. (Leibniz 1966, 71)
>
> The proposition itself can be conceived as a term. (Leibniz 1966, 86).

He also argued that the notion of *containment*, which plays a key role in logic, applies to the relations among propositions in just the same way as it applies to relations among terms:

> … whatever is said of a term which contains a term can also be said of a proposition from which another proposition follows. (Leibniz 1966, 85)
>
> … that a proposition follows from a proposition is simply that a consequent is contained in an antecedent, as a term in a term. By this method we reduce inferences to propositions, and propositions to terms. (Leibniz 1966, 87).

Leibniz's aim in doing this was to produce a *unified* system of logic, one that did not treat the logic of terms and the logic of sentences differently. Once more, he wrote:

> If, as I hope, I can conceive all propositions as terms, and hypotheticals as categoricals, and if I can treat all propositions universally, this promises a wonderful ease in my symbolism and analysis of concepts, and will be a discovery of the greatest importance. (Leibniz 1966, 66).

And, as well, there is Leibniz's insight that singular subjects are arbitrarily universal or particular in quantity, the "wild" quantity thesis (Leibniz 1966, 115).

Sommers incorporated these Leibnizian insights, and much more, into his argument for construing the logic of propositions as a special branch of the logic of terms. In doing so, he went much farther than Leibniz. Leibniz had merely expressed a hope that he could achieve this kind of logical unity by treating sentences as terms, but he could not provide an adequate strategy for doing so. Sommers could. As we have seen, TFL treats sentences as nothing more than complex terms (pairs of logically copulated less complex terms). This idea reflects the obvious fact that conjunctive sentences and particular affirmations share the same formal features of being symmetric and associative (but not transitive), while conditional sentences and universal affirmations share the same formal features of being reflexive and transitive (but not symmetric and associative). So, if sentences are just complex terms, then the semantic relations that characterize terms must, per force, characterize sentences. As discussed earlier in this chapter (section 2), terms used relative to some specifiable domain of discourse, generally, stand in the three semantic relations of expressing concepts (senses), signifying properties, and denoting objects, Thus, for example, with respect to the actual world today, 'foolish' expresses the concept of foolishness, signifies the property of being foolish, and denotes whatever has that property. If nothing has that property, then the term is vacuous (has neither denotation nor signification). When a statement is made, it is made by means of a sentence used relative to a specifiable domain of discourse. The concept that a statement expresses (its sense) is construed as a *proposition* (in a strict sense of that word). What of the signification and denotation of such a sentence?

Just as used non-vacuous terms signify properties, used non-vacuous statements signify properties. In this case, the properties signified are properties of what are denoted. While terms denote objects (in the domain), statements denote the domain itself (Sommers calls a domain denoted by a statement a *world*. Indeed, worlds, domains, universes of discourse "constitute the ultimate subject-matter of the discussion" (Corcoran 1999b, 941, see also Corcoran 2004, 495 ff and Hodges 2009, 599). They are what a statement maker claims truth for. Consider the statement, 'Some politicians are foolish' asserted with respect to the world today. The condition for its truth is that there be at least one foolish politician. Suppose (suspend any contrary belief for now) that Trump is a foolish politician. On such a supposition, Trump has the properties of being a politician and being foolish. But as Hume and Kant have taught, *existing* is not an additional property of Trump. Frege (Frege 1950, section 53) had argued that existence was not a property of *objects* but rather a (secondary) property of *concepts*. For example, on such a view, to say that there are horses is to say that the concept of horse is not empty (applies to some object); to say that there are no unicorns is to say that the concept of unicorn is empty (applies to no object). Variations of the view were then offered by Russell (Russell 1918, Lecture V) and then Quine (Quine 1953, 13). In spite of such credentials, the view is rejected by Sommers in favour of the idea that to say of any object that it exists (does not exist) is just to say that it is (is not) a constituent of the domain of discourse at hand. "To be in a domain is to exist" (Sommers 1993, 175). To say that Trump exists (with respect to the world today) is to say that the world has Trump as one of its constituents. To say that there are horses is to say that horses are constituents of, present in, the world; to say that there are no unicorns is to say that unicorns are not constituents of, are absent from, the world. Thus existence and non-existence are always matters of such presence or absence (Sommers 1993, 174). The statement above about Trump *says* something about him (that he is a foolish politician); but it also *claims* something about the world (that it is characterized by the presence of Trump who is a foolish politician) (Sommers 1993, 179). While properties like being foolish or a politician are characteristics of objects, properties like having or lacking, presence or absence, are characteristics of domains/worlds. They are *constitutive characteristics*. Given a domain and a constitutive characteristic, the domain either does or does not have it. A constitutive characteristic (whether positive or negative) of a domain is a *fact*. A true statement expresses a proposition, signifies a fact (a constitutive characteristic), and denotes what has that fact. And, of course, what has that fact is the domain. A false statement fails to signify a fact and thus fails to denote the domain. It must be noted that a fact is not a constituent *in* the domain; it is a property, a constitutive characteristic *of* the domain. (For more on this account of truth see Englebretsen 2006).

Unanalyzed sentences can be construed as categoricals by taking them, then, as denoting worlds. If 'p and q' is construed as having the form 'some p is q', then the sentential variables here can be read as abbreviations for 'p-world' and 'q-world' (thus, 'some p-world is a q-world'). In the same way, 'if p then q' can be read as 'every p-world is a q-world'). Their symbolic expressions in TFL are '+p+q' and '−p+q'. A so-called atomic sentence, 'p' is simply read as 'some world is p' (+p); its contradictory negation would be 'some world is non-p' (−p). However, at this point, the doctrine that the logic of sentences is merely a special branch of the logic of terms (resulting in the unified system of formal logic envisaged by Leibniz and Sommers) faces what appears to be a serious challenge. Treating the conjunctions and conditionals of propositional logic as logically categoricals (I and A, respectively) reveals a pair of disanalogies (Sommers 1993, 172–173 and 179):

> 1st Disanalogy: 'p and q' ('+p+q') entails 'if p then q' ('−p+q'), but 'Some A is B' does not entail 'every A is B' ('+A+B' does not entail '−A+B').
>
> 2nd Disanalogy: 'p and q' (+p+q') is incompatible with 'p and not q' ('+p−q'), but 'Some A is B' ('+A+B') is compatible with 'Every A is B' ('−A+B').

It turns out that the resolution of these apparent challenges is due to the recognition of two important features of the Leibniz-Sommers logical program (Sommers 1993, 179–180). First, sentences used relative to a specifiable domain are such that, if they denote at all, they denote that domain. A domain, or world, is a totality of its constituents. While the sub-sentential terms that make up the sentence might denote things in that world, the sentence itself denotes just one thing, the world (not a thing in the world). Consequently, sentential terms like 'p', 'q', 'p-world', 'q-world', etc., are singular terms whose denotation is unique. This means that 'p and q' can just as well be read as 'the world is both p and q', 'if p then q' can be read as 'the world is not both p and not q', 'p' can be read as 'the world is p' and 'not p' can be read as 'the world is not p'. Singular statements (those having singular subject terms) can be taken to have wild quantity. Thus, conjunctions, conditionals (and any other truth-functional forms of statement) can be taken to have wild quantity. It is the wild quantity thesis that accounts for the two disanalogies, for it is the same thesis that resolves these two other disanalogies (let A denote artists, D denote Dutch residents, and V denote Vermeer:

> 3rd Disanalogy: 'Some A is D' ('+A+D') does not entail 'Every A is D' ('−A+D'), but 'Some V is D' ('+V+D') does entail 'Every V is D' ('−V+D'). Both of which are then formulated simply as '*V+D'.
>
> 4th Disanalogy: 'Some A is D' ('+A+D') is compatible with 'Some A is not D' ('+A−D'), but 'Some V is D' ('+V+D') is incompatible with 'Some V is not D' ('+V−D'), since '*V+D' and '*V−D' are incompatible.

In other words, the first disanalogy is simply an instance of the third and the second is an instance of the fourth, and the third and fourth are innocuous in light of the singularity (unique denotation) of sentential terms and the wild quantity thesis.

It is because sentential terms are singular that they are given wild quantity whenever they are quantified. "[A]ll propositional terms are uniquely denoting terms and all propositional statements are singular statements" (Sommers 1993, 179). Any sentence used to make a statement denotes the domain relative to which it is used. When an analyzed sentence (one whose constituent non-sentential elements are explicit) is used to make a statement, the sentence is simultaneously used (1) to *say* something about things in the domain (world) relative to which it is used and (2) to *claim* that what is being said (the proposition expressed) is true. When an unanalyzed sentence is used to make a statement, what it is used to *say* (assert, state) just is what it is used to *claim*. In other words, what such a statement states is not something about what is or is not in the domain – it is about the domain itself.

> The distinction between saying and claiming is idle in propositional logic since the statements we are there concerned with are represented by statement letters that give no clue as to the internal contents of the statements represented (nor is any needed for the purpose at hand). All statements of statement logic are understood as being about the world. Given 'p' we interpret it as *asserting its truth claim*, viz., that the world is a p-world. Given 'p&q' we interpret it to say that the world is both a p-world and a q-world and so on for other compound forms. (Sommers 1993, 179)
>
> Consider 'p&q'; in its categorical form 'some p-world is a q-world'. That this makes a claim about the one world is evident from its equivalence to '*The* world is both a p-world and a q-

> world'. … All statements of statement logic are understood as being about the world. … all sentential terms denote the *same* world. (Sommers 1993, 179 – 180)
>
> The doctrine that *all true statements denote one and the same domain* (though signifying different facts) is the key to understanding why all of the 'general categorical' statements of terminized propositional logic are semantically singular" (Sommers 1993, 181),

So, how can this account of statement logic as a special branch of term logic be incorporated into the visual logic of ED? How can "terminized propositional logic" be given a graphic treatment consonant with our system for diagramming term logic in general? There are three important features of TFL (and ED) to keep in mind. The first is that the unanalyzed statements of statement or propositional logic are understood as being singular terms. Consider a sentence such as 'Aristotle is a logician', which can be diagrammed as an I categorical with the line denoting Aristotle and the line denoting logicians intersecting. But, since 'Aristotle' is a singular, uniquely denoting subject term here, it can be treated arbitrarily as having either universal or particular quantity, i.e., wild quantity. So it could equally well be diagrammed as an A categorical with the line denoting logicians properly including the line denoting Aristotle. We have seen that this means that we can simply represent a singular subject, where quantity plays no important logical role, like this using just a point. This can be graphically illustrated as follows:

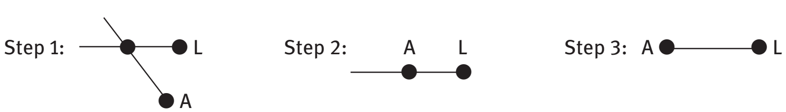


Figure 3.76: How to Diagram a Singular

Secondly, we saw that the existential import of a statement might be either *implicit* or *explicit*. In the latter case, the statement's diagram contains a specified, delineated point that is not the right terminus of any term line. Thus, the intersect points of particular categoricals (as well as conjunctions) are delineated points that are not right termini; no point of universal categoricals (as well as conditionals) is a delineated point that is not a right terminus. Explicit existence for universals results from the added assumption that subject term is not vacuous; and that is represented generally by diagramming that term as both the subject and predicate term of an added particular (these differences are illustrated in Figure 3.27).

Finally, as we have seen, there is a crucial distinction to be made between particularly quantified terms (logical subjects or logical objects) when their reference is taken to be *specific* or taken to be *non-specific*. Both MPL and TFL determine the difference on the bases of the order of quantifiers: a particularly quantified term is understood as having specific reference just when it precedes but does not follow a universally quantified term. The difference is graphically illustrated by the two diagrams shown in Figure 3.62. where it must be noted that the non-terminal (i. e., left-most) point on the G line represents a specific individual (a certain girl) in the second diagram (ii) but the leftmost point on the G line in the first diagram (i) represents a non-specific individual (some girl or other). That is because this point is actually the right terminus of a line representing the complex relational term 'every boy loves …' (−B+L), which appears analyzed (by the B line and the L arrow) in the diagram.

These three features of TFL and ED help shed light on just how propositional logic can be imbedded in (and thus seen as a special branch of) term logic. It can then be shown how ED preserves both the classical inference patterns of term logic in general but also those of propositional logic (such as *modus ponens, modus tollens,* conjunctive addition, hypothetical syllogism, etc.). In the end, it will be possible to account for the two apparent disanalogies that challenge such an incorporation. Given a statement that can be left completely unanalyzed, p, let @ denote the domain (world) "at hand" (the domain relative to which the statement is made. @ is of course singular since a statement is made relative to a *single*, specifiable, domain. While the intersection points for conjunctions are existentially explicit, conditionals have no such delineated points. Explicit "existential" import must be *made* explicit. Singular subjects always have such explicit import. Here are some simple diagrams for basic unanalyzed statement forms:

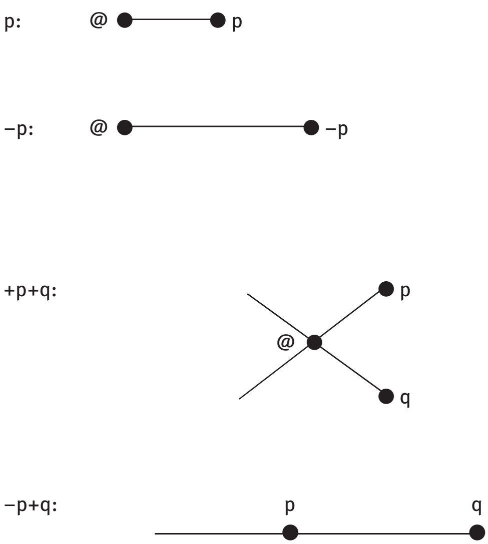


Figure 3.77: Diagrams for Sentential Logic

Note that a conditional is stated relative to a *domain of domains* (call it *D*), saying that every domain (or world) that is a p domain (a domain in which p is true) is a q domain (a domain in which q is true). We could just as well diagram it as:

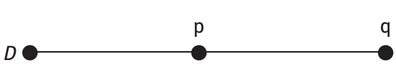


Figure 3.78: The Domain of Domains for Conditionals

Note also that a conjunction (+p+q) can equally well be understood as an attribution of a conjunctive sentential term to the domain (*@+(+p+q)). Thus the two alternative diagrams for a conjunctive statement:

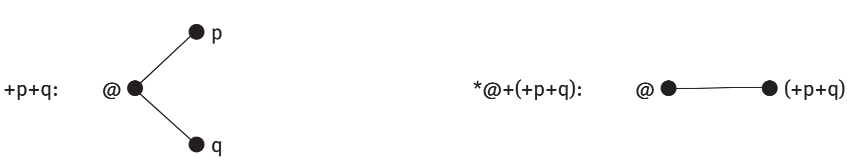


Figure 3.79: Alternative Diagrams for a Conjunctive Statement

Another important equivalence worth noting now is *contraposition:*

Figure 3.80: Diagrams for Contraposition

Recall that Frege had presented his system for logical notation (*Begriffsschrift*, concept-script) as a two-dimensional array of labelled lines. Moreover, he seems to have inaugurated at the same time the now-standard view that the logic of propositions is primary logic. His account of propositional logic took the functors of negation and conditionalization to be primitive. Importantly, he distinguished between the *content* of a proposition and the *judgment* (truth-claim) of it, and he distinguished them graphically with a horizontal line for the former and a small vertical line orthogonal to, and attached to, the left terminus of the horizontal line. While much of the logical theory advanced here is the result of rejecting a number of these Fregean views, it might be argued that his distinction between content and judgment can be seen in the ED representations, which indicate judgment by domain points at left termini while their absence indicates mere content. (For an insightful examination of Frege's notation for propositional logic see Schlimm, to appear.)

Since the TFL theory of logical syntax that accounts for the symbolic formulations of term logic applies as well to the logic of propositions, it is hardly surprising to see that the ED theory of graphic representation that applies to term logic applies as well to statement logic. Diagrams for some classical elementary valid argument forms (rules of inference) for propositional logic can now be constructed.

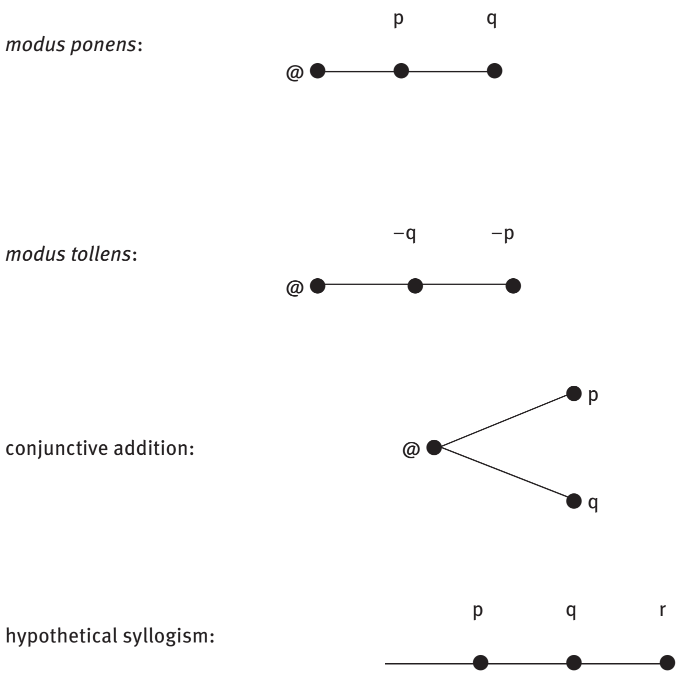


Figure 3.81: Some Elementary Rules of Propositional Logic

One can note as well that tautologies and contradictions can be diagrammed for propositional logic in just the ways they are diagrammed (Figures 3.19 and 3.20) in general (but taking the terms now to be sentential).

Just as any complex term either can be left unanalyzed or can be analyzed (both symbolically and diagrammatically, the same holds for complex sentential terms. For example, the proposition that if p then q can be left unanalyzed or it can be analyzed. The following figure represents the two results:

Figure 3.82: Unanalyzed and Analyzed Complex Terms

A few examples of deductions using this system are now in order. Consider the following argument: If p then if q then r, p, q; therefore r. We might diagram each of the three premises (step 1), then apply *modus ponens* to the first two premises (step 2), next analyze the complex term (step 3), finally, apply *modus ponens* to that result and the second premise (step 4):

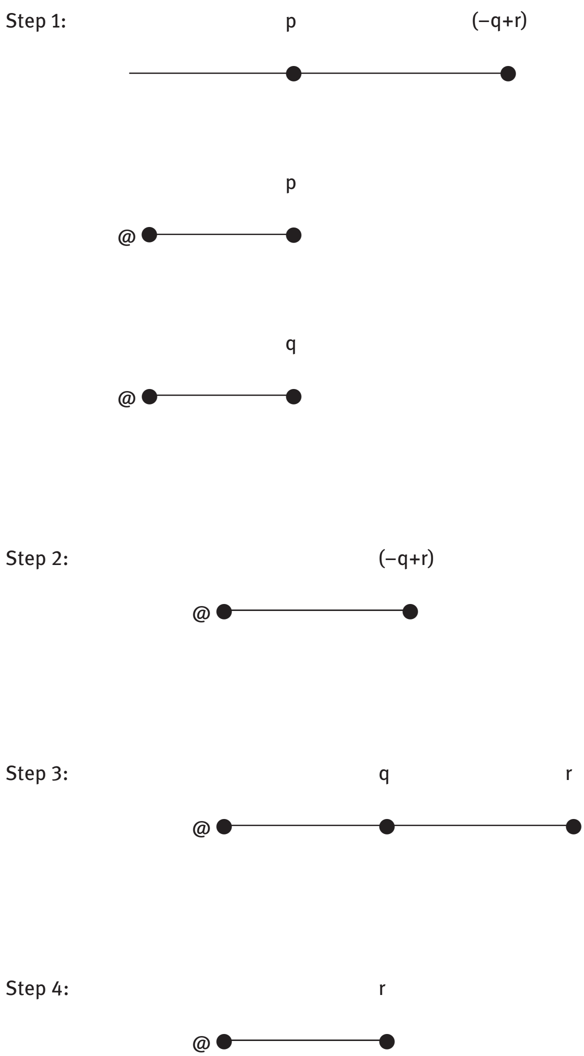


Figure 3.83: A Diagrammed Deduction

If a set of statements is consistent, its members can all be diagrammed together (i. e., no contrapiction is encountered). For example, statements of the following forms can be so diagrammed: if not r then s, if p then not r, p:

Figure 3.84: Diagram of a Consistent Set of Statements

Here is a proof that the set of four statements (if p then if not q then s, p and s, not q, not r) is not consistent by deducing a contrapiction:

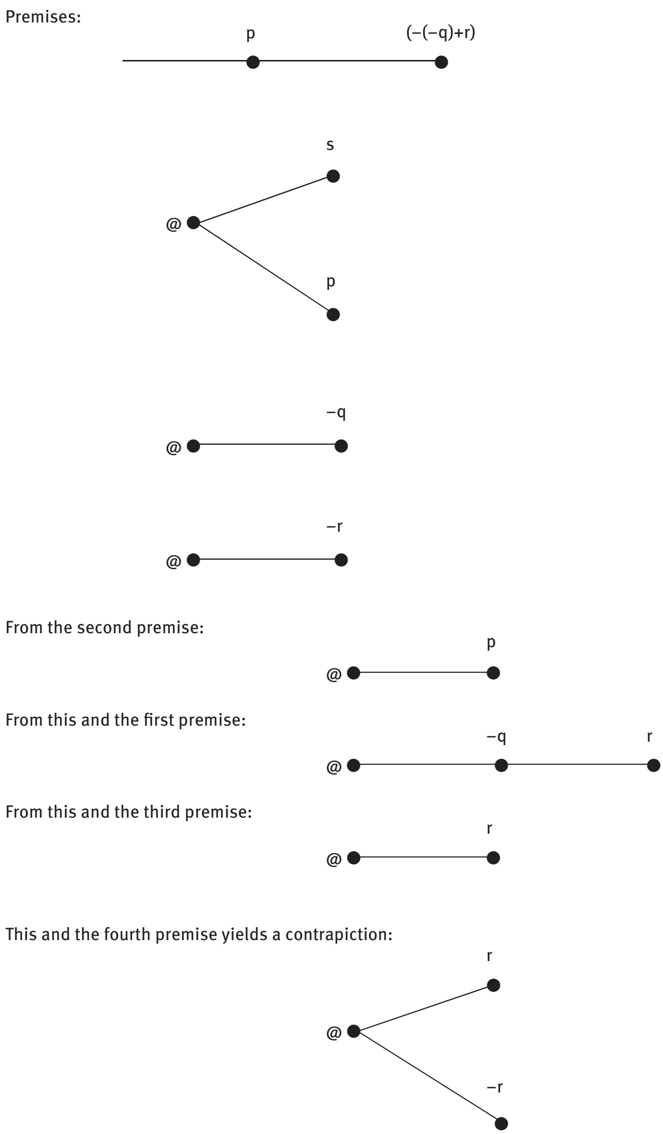


Figure 3.85: Proof of an Inconsistent Set of Statements

Note that in the first step a diagrammatic analogue of conjunctive simplification was used, viz., from an asserted conjunction any line branching from the domain point can be eliminated.

Earlier in this chapter (section 3), we have seen what the medieval logicians, Leibniz, the Port Royal logicians, the 19th century algebraic logicians, and many others have seen – the centrality of the *dictum de omni et nullo* for a logic of terms, both traditional syllogistic and TFL. It was shown there that the *dictum* amounts to a rule of substitution. In effect, the *dictum* permits the replacement of a distributed term in a sentence by the term constituting the remainder of that sentence in any other sentence in which that distributed term is now undistributed. For example, we saw that De Morgan's famous 'head of a horse' inference relies on the *dictum* in the following way. The stated premise ('Every horse is an animal') has the term 'horse' distributed. So the remaining term 'animal' can be substituted for 'horse' in another sentence which has 'horse' undistributed. The tacit, and logically innocent, premise, 'Every head of a horse is the head of a horse', has two tokens of 'horse' but only the second is undistributed. Substitution of 'animal' for 'horse' here yields the conclusion. A look at Figure 3.59 shows that H ('horse') can be ignored allowing A ('animal') to take over. The ED system of diagramming represents the application of the *dictum* whenever a middle term is ignored (see Figures 3.28 – 3.31). We see the *dictum* governing *modus ponens*, for example, when the consequent of the conditional is substituted for the antecedent (which must be a distributed sentential term) when that antecedent occurs in another sentence undistributed. And this is so even if the antecedent is a negative sentential term (e.g. 'if not p then q, not p; therefore, q', where the "middle" term, 'not p' is distributed in the first premise by the universal quantifier). In terms of ED, the *dictum* amounts to saying that a line segment that includes another line segment can replace that other line whenever it is not included as a proper part of another line. The fact that the *dictum*, which Leibniz called the foundational rule of mediate inference, is central to both the logic of terms and the logic of propositions, lends further weight to the conviction that the latter is a special branch of the former.

Earlier we encountered what appeared to be two disanalogies between the logic of terms and the logic of propositions. It was shown that by virtue of the fact that a sentential term denotes the unique domain of discourse relative to which it is used, the fact that this means that such terms are always singular (denoting just one thing), and the fact that when any singular term occurs quantified its quantity is wild (arbitrarily universal or particular), the disanalogies are disarmed. We will close this chapter with diagrammatic depictions of these resolutions.

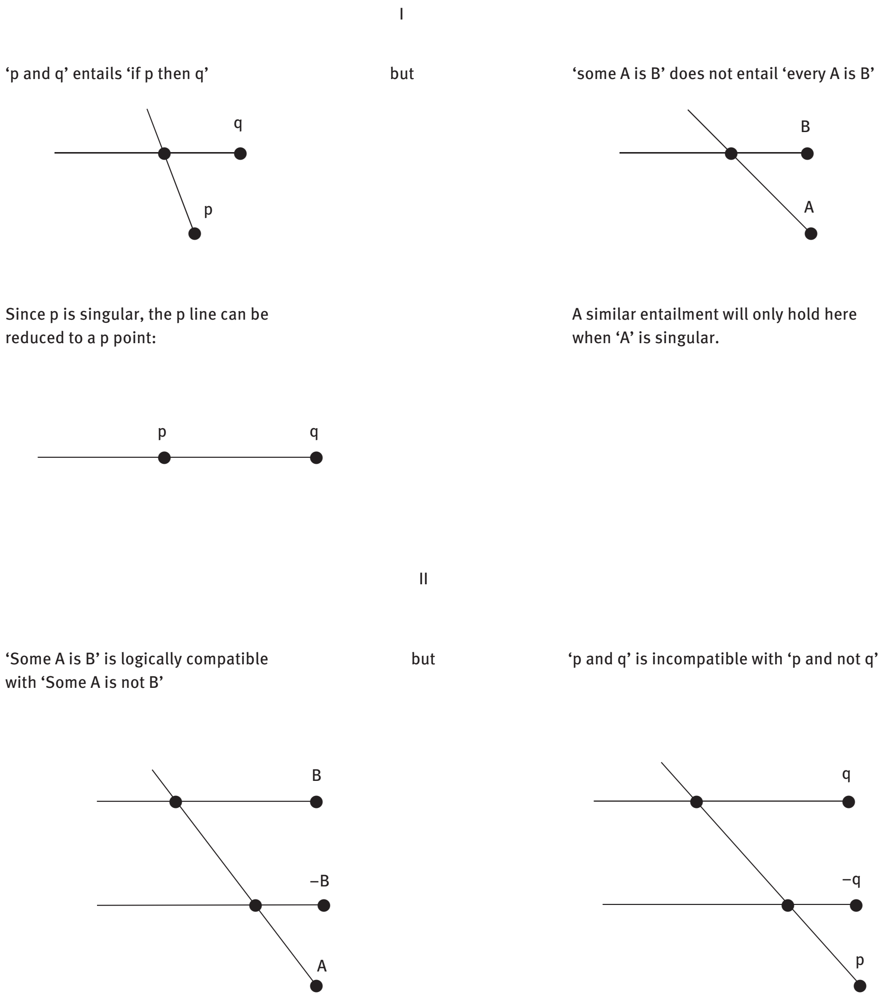


Figure 3.86: Diagrammatic Resolutions of the Two Disanalogies

In this case, since the terms are singular, the p line reduces to a p point, but there are two such points (on incompatible lines), so the diagram is a contrapiction.