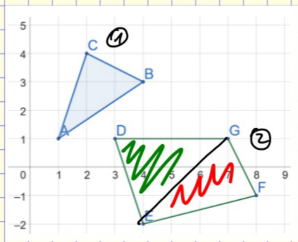
Aufgabe 1:



① $\overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \quad \overrightarrow{BC} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

② $\overrightarrow{DG} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \quad \overrightarrow{GF} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\overrightarrow{FE} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}, \quad \overrightarrow{ED} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

① $A = \frac{1}{2} \left| \vec{a} \times \vec{b} \right| = \frac{1}{2} \left| \begin{pmatrix} 3 \\ 2 \end{pmatrix} \times \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right| = \frac{1}{2} \left| \begin{pmatrix} 2 \cdot 0 - 3 \cdot 0 \\ 0 \cdot 1 - 0 \cdot 3 \\ 3 \cdot 3 - 2 \cdot 1 \end{pmatrix} \right| = \frac{1}{2} \left| \begin{pmatrix} 0 \\ 0 \\ 9 - 2 \end{pmatrix} \right| = \frac{1}{2} \sqrt{7^2} = \frac{1}{2} \cdot 7$

$= 3,5$

2) $\frac{1}{2} \left| \begin{pmatrix} 4 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ 3 \end{pmatrix} \right| = \frac{1}{2} \left\| \begin{pmatrix} 0 \\ 0 \\ 12 \end{pmatrix} \right\| = \frac{1}{2} \cdot 12 = 6$

$\frac{1}{2} \left| \begin{pmatrix} -4 \\ -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ -2 \end{pmatrix} \right| = \frac{1}{2} \left\| \begin{pmatrix} 0 \\ 0 \\ 8 + 1 \end{pmatrix} \right\| = \frac{1}{2} \cdot 9 = 4,5$

$\rightarrow 6 + 4,5 = 10,5$

Aufgabe 2:

$\vec{a} = \begin{pmatrix} 200 \\ 50 \\ -20 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$|\vec{a}| = \sqrt{40000 + 2500 + 400} = \sqrt{42900} = 207,12$

$|\vec{b}| = 1$

$\cos \varphi = \frac{200}{207,12} \Rightarrow \varphi = 15,07°$

Aufgabe 3)

a) z.z $\underbrace{(\vec{a} \times \vec{b})}_{①} \perp \vec{a}$, $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

① $\vec{a} \times \vec{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$

$\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \end{matrix}$

Wenn Aussage wahr (orthogonalität)

$$\Rightarrow (\vec{a} \times \vec{b}) \cdot \vec{a} = 0$$

$$\begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = 0$$

$\Leftrightarrow a_1 \cdot (a_2 b_3 - a_3 b_2) + a_2 (a_3 b_1 - a_1 b_3) + a_3 (a_1 b_2 - a_2 b_1)$

$\Leftrightarrow a_1 a_2 b_3 - a_1 a_3 b_2 + a_2 a_3 b_1 - a_2 a_1 b_3 + a_3 a_1 b_2 - a_3 a_2 b_1 = 0$

quod erat demonstrandum