Vorrechnen Selbsteinschätzung II

Friday, 10 October 2025 11:00

Aufgabe 1:

a) $\frac{(t^2-4t+4)(4+t^2+4t)}{t^2-4} = \frac{(t-2)^{\cancel{2}}(t+2)^{\cancel{2}}}{\cancel{(t+2)}\cancel{(t-2)}} = (t-2)(t+2)$

b) $\frac{13}{9} - \frac{1}{63} - \frac{21}{49} = \frac{13\ 7}{63} - \frac{1}{63} - \frac{3\cdot 9}{63} = \frac{91-1-27}{63}$

$= 1$

c) $\frac{3a}{b} : \frac{a}{2b} \cdot \frac{2a}{3b} = \frac{3a}{b}\cdot\frac{2b}{a}\cdot\frac{2a}{3b} = \frac{12a^2\cancel{b}}{3\cancel{a}\ b^{\cancel{2}}} = 4\frac{a}{b}$

d) $\left(\frac{x^3\cdot y}{n^2\ m^3}\right)^5 : \left(\frac{x\cdot y^2}{n\cdot m^5}\right)^2 = \frac{x^{15}\cdot y^5}{n^{10}\cdot m^{15}} : \frac{x^2\cdot y^4}{n^2\ m^{10}}$

$= \frac{x^{\cancel{15}\,13}\ y^{\cancel{5}\,1}}{n^{\cancel{10}\,8}\cdot m^{\cancel{15}\,5}}\ \frac{n^{\cancel{2}}\cdot m^{\cancel{10}}}{x^{\cancel{2}}\cdot y^{\cancel{4}}}$

$= \frac{x^{13}y}{n^8 m^5}$

e) $\sqrt{\left(\sqrt{\sqrt[5]{a^2}}\right)^4\sqrt{b^3}} = ((((a^2)^{\frac{1}{5}})^{\frac{1}{2}})^4)^{\frac{1}{2}}\ ((b^3)^{\frac{1}{2}})^{\frac{1}{2}}$

$= a^{(2\ \frac{1}{5}\ \frac{1}{2}\ 4\ \frac{1}{2})}\cdot b^{(3\cdot\frac{1}{2}\ \frac{1}{2})}$

$= a^{\frac{2}{5}}\cdot b^{\frac{3}{4}} = \sqrt[5]{a^2}\ \sqrt[4]{b^3}$

f) $\left(\frac{\sqrt[5]{\sqrt{7}x}}{\sqrt[4]{x^3}}\right)^{-2} = (((7^{\frac{1}{2}})^{\frac{1}{5}})^{-2}\cdot((x^{\frac{1}{5}})^{-2}\cdot(((x^3)^{\frac{1}{4}})^{-1})^{-2}$

$= 7^{\frac{1}{2}\cdot\frac{1}{5}\cdot(-2)}\cdot x^{\frac{1}{5}\,(-2)}\cdot x^{3\cdot\frac{1}{4}\cdot(-1)\,(-2)}$

$= \frac{x^{\frac{11}{10}}}{7^{\frac{1}{5}}}$

Aufgabe 2:

a) $8x-(5x+2) = 3-(5-2x)$

$\Leftrightarrow 8x-5x-2 = 3-5+2x$

$\Leftrightarrow 8x-7x = -2+2 \Leftrightarrow x=0$

b) $x^4-2x^2+1 = 0$

$\Leftrightarrow (x^2-1)^2 = 0$

$\Leftrightarrow (x^2-1)^2 = 0$

$\Leftrightarrow x^2 - 1 = 0 \Leftrightarrow x = \pm 1$

c) $x^2 - 2x + 10 \leq 5$

$\Leftrightarrow x^2 - 2x - 15 \leq 0$

<u>NR</u>: $x^2 - 2x - 15 = 0$

$\overset{PQ}{\Leftrightarrow} x = -1 \pm 4$

$x_1 = 3, \; x_2 = -5$

$\Rightarrow (x-3)(x+5) \leq 0$

<u>Fallunterscheidung</u>

<u>Fall 1</u>: $(x-3) \geq 0 \wedge (x+5) \leq 0$

$\Leftrightarrow x \geq 3 \wedge x \leq -5$

$\Leftrightarrow x \in \mathbb{R} \setminus (-5; 3)$

<u>Fall 2</u>: $(x-3) \leq 0 \wedge (x+5) \geq 0$

$\Leftrightarrow x \leq 3 \wedge x \geq -5$

$\Leftrightarrow x \in [-5; 3]$

d) $-2x^4 + 2x^3 - 14x + 10 > 4 - 7x + x^3(1-2x)$

$\Leftrightarrow x^3 - 7x + 6 > 0$

NR $(x^3 - 7x + 6) : (x-1) = x^2 + x - 6$

$$\begin{array}{l} -(x^3 - x^2) \\ \hline \quad x^2 - 7x + 6 \\ \quad -(x^2 - x) \\ \hline \qquad -6x + 6 \\ \qquad -(-6x + 6) \\ \hline \qquad\qquad 0 \end{array}$$

$(x-1)(x^2 + x - 6) > 0$

$\Leftrightarrow (x-1) > 0 \wedge x^2 + x - 6 > 0$

<u>NR</u> $x^2 + x - 6 = 0$

$\Leftrightarrow x_1 = 2, \; x_2 = -3$

$\Rightarrow (x-1)(x-2)(x+3) > 0$

$\Leftrightarrow (x-1) > 0 \wedge (x-2) > 0 \wedge (x+3) > 0$

$\Leftrightarrow x > 1 \wedge x > 2 \wedge x > -3$

$\Leftrightarrow (x-1) > 0 \wedge (x-2) > 0 \wedge (x+3) > 0$

$\Leftrightarrow x > 1 \wedge x > 2 \wedge x > -3$

$\Rightarrow x > 2$

weitere Fallunterscheidungen ...

$\Rightarrow \mathbb{L} = (-3, 1) \cup (2, \infty)$

e) $\frac{x-1}{|x+4|} < 1$

<u>Fallunterscheidung:</u>

<u>Fall 1:</u> $x+4 \geq 0 \Rightarrow |x+4| = x+4 \Rightarrow x \geq -4$

$\Rightarrow \frac{x-1}{|x+4|} = \frac{x-1}{x+4} < 1$

$\Leftrightarrow x-1 < x+4 \quad \forall x \in \mathbb{R}$

<u>Fall 2:</u> $x+4 < 0 \Rightarrow |x+4| = -x-4 \Rightarrow$

$\frac{x-1}{|x+4|} = \frac{x-1}{-x-4} < 1$ $\qquad x+4 < 0 \Leftrightarrow x < -4$

$\Leftrightarrow x-1 < -x-4$

$\Leftrightarrow 2x < -3 \Leftrightarrow x < -\frac{2}{3} \Rightarrow x \in (-\infty, -4)$

$\Rightarrow x \in \mathbb{R} \setminus \{-4\}$

<u>Aufgabe 3:</u> sechs Pyramiden $\Rightarrow$ 1 Würfel

$\Rightarrow$ Würfel umschließt innere Kugel

$V_{Würfel} = a^3$ $\qquad 6 \cdot V_{Körper} = a^3 - \frac{4}{3}\pi \left(\frac{a}{2}\right)^3 = \left(1 - \frac{\pi}{6}\right)a^3$

$\Rightarrow V_{Körper} \approx 0{,}079\, a^3$

Fläche: Grundfläche a^2

Seitenflächen: Dreiecksflächen − Kreissegment

$\tan\left(\frac{\varphi}{2}\right) = \frac{\frac{a}{2}}{\frac{a}{\sqrt{2}}} = \frac{1}{\sqrt{2}} \Rightarrow \varphi = 70{,}53°$

$A_{Segment} = \frac{\varphi}{360°} \cdot \pi \left(\frac{a}{2}\right)^2 \approx 0{,}154 \cdot a^2$

$A_{Segment} = \frac{\varphi}{360°} \cdot \pi \left(\frac{a}{2}\right)^2 \approx 0{,}154 \cdot a^2$

$\Rightarrow A_{seite} = \frac{a}{2} \cdot \frac{a}{\sqrt{2}} - A_{segment} \approx 0{,}2 a^2$

gekrümmte Fläche: $\frac{1}{6} \cdot A_{Kugel} = \frac{1}{6} \cdot 4\pi \cdot \left(\frac{a}{2}\right)^2 = \frac{\pi a^3}{6}$

$A_{ges} = 0{,}8 a^2 + a^2 + \frac{\pi}{6} a^2 \approx 2{,}32 a^2$

Aufgabe 4

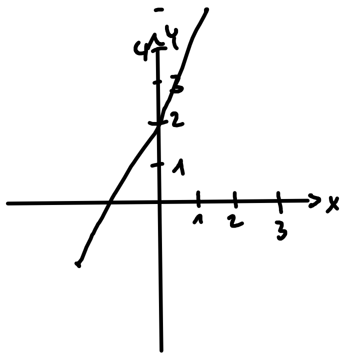
a) $\sin\left(-\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

b) $\cos\left(\frac{11}{2}\pi\right) = \cos\left(\frac{1}{2}\pi\right) = 0$

c) $\sin\left(\frac{104}{13}\pi\right) = \sin\left(\frac{8 \cancel{13}}{\cancel{13}}\pi\right) = \sin(8\pi) = 0$

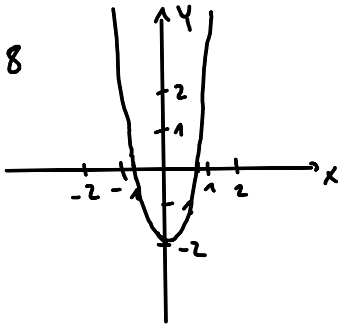
Aufgabe 5.

a) $y(x) = 3x + 2$



b) $y(x) = 3x^2 - 2$

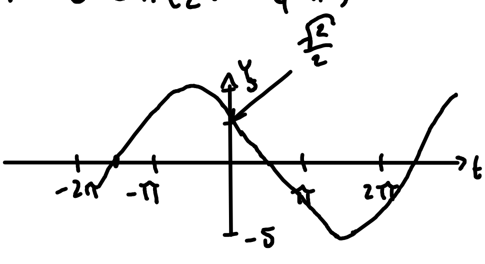
Nst. $\pm\sqrt{\frac{2}{3}} \approx \pm 0{,}8$



$\frac{1}{2}t + \frac{3}{4}\pi = 0$

$t = -\frac{3}{2}\pi$

c) $A(t) = 5 \sin\left(\frac{1}{2}t + \frac{3}{4}\pi\right)$



Aufgabe 6:

(I) $x - 2y - z = 0 \quad \Rightarrow \quad (1 \quad -2 \quad -1 \mid 0)$

Aufgabe 6:

(I) $x - 2y - z = 0$

(II) $x + 3y + z = -1$

(III) $-x + 2y + 2z = 1$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 1 & 3 & 1 & -1 \\ -1 & 2 & 2 & 1 \end{array}\right)$$

$$\begin{array}{c} \rightsquigarrow \\ \text{II} - \text{I} \\ \text{III} + \text{I} \end{array} \left(\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & 5 & 2 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$\Rightarrow$ III $z = 1$

II $5y + 2 \cdot 1 = -1 \Leftrightarrow y = -\frac{3}{5}$

I $x + \frac{6}{5} - 1 = 0 \Leftrightarrow x = -\frac{1}{5}$

$$\Rightarrow \mathbb{L} = \left\{ \begin{pmatrix} -\frac{1}{5} \\ -\frac{3}{5} \\ 1 \end{pmatrix} \right\}$$

Aufgabe 7:

a) $f(x) = x^{-3}$, $f'(x) = -3x^{-4} = -\frac{3}{x^4}$

b) $f(x) = x^{\frac{5}{2}}$, $f'(x) = \frac{5}{2} x^{\frac{3}{2}} = \frac{5}{2}\sqrt{x^3}$

c) $f(x) = \cos\left(\frac{1-x^2}{x}\right)$

$f'(x) = -\sin\left(\frac{1-x^2}{x}\right) \cdot \left(\frac{-2x^2 + 1 - x^2}{x^2}\right) = -\sin\left(\frac{1-x^2}{x^2}\right)\left(\frac{1-3x^2}{x^2}\right)$

Aufgabe 8:

a) $\int (x^3 - 3x + 4)\,dx = \frac{1}{4}x^4 - \frac{3}{2}x^2 + 4x + C$

b) $\int \sin\left(\frac{x}{4} + 3\right) dx = -4\cos\left(\frac{x}{4} + 3\right) + C$

c) $\int_1^e \frac{\cos(\ln(x))}{x}\,dx = \left[\sin(\ln(x))\right]_1^e = \sin(\ln(e)) - \sin(0)$

$= \sin(1)$

Aufgabe 9.

$A = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$, $B = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$, $C = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, $D = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$\rightarrow$ Begründung, warum Parallelogramm

$B - A = \begin{pmatrix} 4 \\ -1 \end{pmatrix} = C - D$, $C - A = \begin{pmatrix} 6 \\ 2 \end{pmatrix} \neq 0$

$$B - A = \begin{pmatrix} 4 \\ -1 \end{pmatrix} = C - D \,, \quad C - A = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \neq 0$$

$\blacksquare$