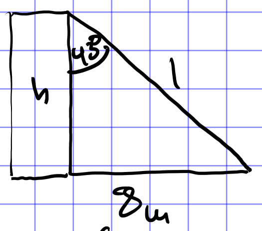
# Vorrechnen 05

09.10.23

1)



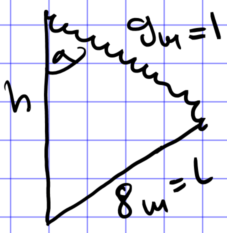

$$\tan(43^\circ) = \frac{G}{A} = \frac{8m}{h} \quad |\cdot h \quad |:\tan(43^\circ)$$

$$\Rightarrow h = \frac{8m}{\tan(43^\circ)} = 8{,}58m$$

a) $$\sin(43^\circ) = \frac{8m}{l} \quad |\cdot l \quad |:\sin(43^\circ)$$

$$\Rightarrow l = \frac{8m}{\sin(43^\circ)} = 11{,}7m$$

b)




$$L^2 = l^2 + h^2 - 2hl\cos(\alpha)$$

$$\Rightarrow \cos(\alpha) = \frac{l^2+h^2-L^2}{2hl}$$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{l^2+h^2-L^2}{2hl}\right)$$

$$= 54^\circ$$

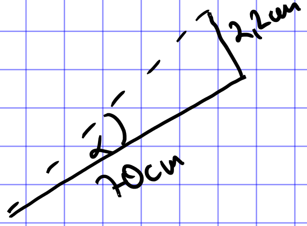
2) $d_{Mond} = 3480\,km$

$r_{max} = 405.500\,km$ $\quad r_{min} = 363.000\,km$

$r_{mittel} = \frac{1}{2}(r_{max} + r_{min}) = 384.000\,km$

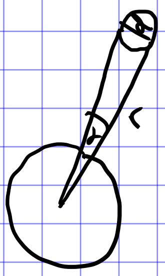
$d_{Daumen} = 2{,}2cm$ $\quad r_{ich} = 70cm$

a)




$$\tan(\alpha) = \frac{2{,}2}{70} \quad |\tan^{-1}()$$

$$\Rightarrow \alpha = 1{,}8^\circ$$




$r = 384.000\,km$

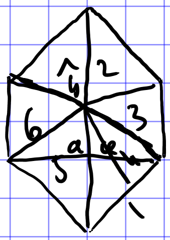
$$\frac{d}{2\pi r} = \frac{\varphi}{360^\circ} \quad |\cdot 360^\circ$$

$$\Rightarrow \varphi = \frac{3480\,km}{2\pi \cdot 384.000\,km} \cdot 360^\circ = 0{,}5^\circ$$

b) $1^\circ = 60$ Bogenmin $\quad |\cdot \frac{1}{2}$

$\Rightarrow \frac{1}{2} \cdot 1^\circ = 30$ Bogenmin.

3)





$h^2 + \left(\frac{1}{2}\right)^2 = 1^2 \quad \Big| -\left(\frac{1}{2}\right)^2$

$= \frac{3}{4} l^2 \quad \Big| \sqrt{\phantom{x}}$

$\Rightarrow h = \frac{\sqrt{3}}{2} l \quad \Big| \cdot \frac{2}{\sqrt{3}}$

$\Rightarrow l = \frac{2}{\sqrt{3}} h$

$a = 2 \cdot h = \sqrt{3} \cdot l$

$b = 2 \cdot l \qquad \Rightarrow 2l > \sqrt{3}\, l$

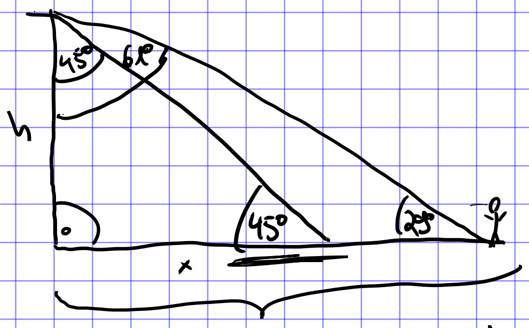
$2l \qquad \sqrt{3}\, l \approx 1{,}73\, l$

$\delta l = 0{,}3\, l$

$\frac{2l}{\sqrt{3}\, l} = \underbrace{1{,}15}_{15\%}$

$d_{Außen} = 1{,}15 \cdot d_{Innen}$

4)




$\tan(45°) = \frac{x}{h} \quad | \cdot h \Rightarrow x = \tan(45°) \cdot h$

$\tan(61°) = \frac{x + 35m}{h}$

$\Rightarrow \tan(61°) = \frac{\tan(45°) \cdot h + 35m}{h} \quad | \cdot h$

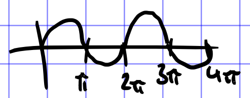
$\Rightarrow \tan(61°) \cdot h = \tan(45°) \cdot h + 35m$

$\Rightarrow h \cdot (\tan(61°) - \tan(45°)) = 35m$

$\Rightarrow h = \frac{35m}{\tan(61°) - \tan(45°)} = 43{,}52m$

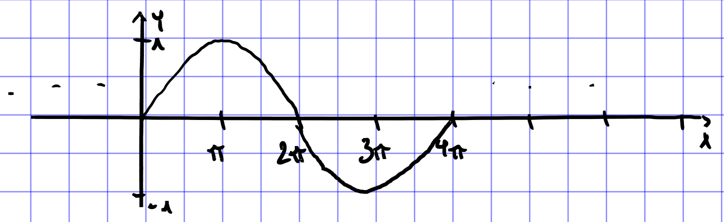
5) a) $f(x) = \sin\left(\underbrace{\frac{x}{2}}\right)$ $\qquad f(x) = \sin(x) \quad x = z \cdot \pi$

$\Rightarrow$ Streckung in x-Richtung um Faktor 2




$\frac{x}{2} = z\pi \quad | \cdot 2 \quad \Rightarrow x = 2 \cdot z\pi \quad \Rightarrow$ Nullstellen $\qquad z \in \mathbb{Z}$

$\frac{x}{2} = \frac{\pi}{2} + z \cdot 2\pi \quad \Rightarrow x = \pi + 4z\pi = (4z+1) \cdot \pi \quad \Rightarrow$ Maxima

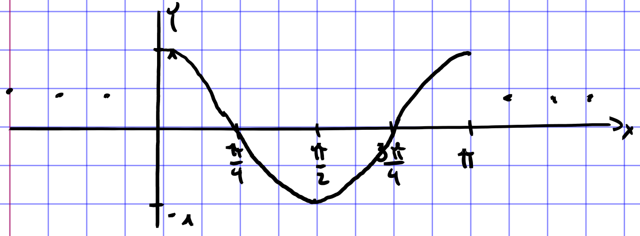


b) $f(x)=\cos(2x)$ ⇒ Stauchung um den Faktor 2 (in x-Richtung)

⇒ Nullstellen: $2x = \frac{\pi}{2} + z\cdot\pi \quad |:2$ $\qquad z \in \mathbb{Z}$

$\Rightarrow x = \frac{\pi}{4} + \frac{z}{2}\cdot\pi = \frac{\pi}{2}\left(\frac{1}{2} + z\right)$

⇒ Maxima: $2x = 2\pi z \quad |:2 \qquad \Rightarrow x = \pi z$




c) $f(x) = \sin\left(\frac{x}{3} - 3\right)$

$\frac{x}{3} - 3 = z\cdot\pi \quad |+3 \,|\cdot 3$

$\Rightarrow x = 3\cdot(z\pi + 3) = 3\pi z + 9$ ⇒ Nullstellen

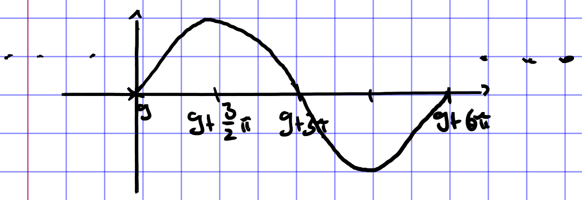
$\frac{x}{3} - 3 = \frac{\pi}{2} + z\cdot 2\pi \quad |+3 \quad |\cdot 3$

$\Rightarrow x = \left(6z + \frac{3}{2}\right)\pi + 9$

$f(x) = \sin\left(\frac{x}{3} - 3\right)$

$= \sin\left(\frac{1}{3}(x - 9)\right)$

↑ Streck. ↑ Verschiebung um 9 in pos. x-Richtung



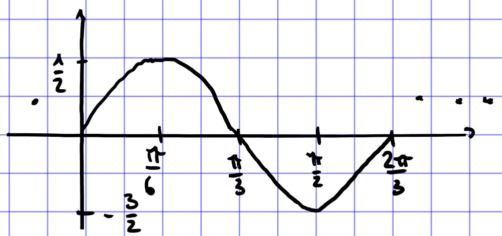

d) $f(x) = \sin(3x) - \frac{1}{2}$ ← $-\frac{1}{2}$ in y-Richtung verschoben

Maxima: $\frac{\pi}{2} + z\cdot 2\pi = 3x \Rightarrow x = \frac{\pi}{6} + z\cdot\frac{2}{3}\pi \qquad z \in \mathbb{Z}$

$= \frac{\pi}{3}\left(\frac{1}{2} + 2z\right)$

$-\frac{1}{2}$-Stellen: $3x = z\cdot\pi$

$x = z\cdot\frac{\pi}{3}$

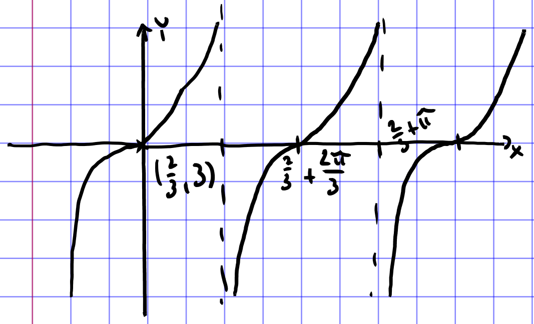


e) $f(x) = \tan\left(\frac{3}{2}x - 1\right) + 3$

Nullstellen: („3-Stellen") $\frac{3}{2}x - 1 = z \cdot \pi \quad \Rightarrow x = \frac{2}{3}(z\pi + 1)$

Polstellen: $\frac{3}{2}x - 1 = \frac{\pi}{2} + \pi \cdot z \quad \Rightarrow x = \frac{2}{3}\left(z\pi + 1 + \frac{\pi}{2}\right) = \frac{2}{3} + \frac{\pi}{3}(2z + 1)$

$3 + \tan\left(\frac{3}{2}x - 1\right) = \tan\left(\frac{3}{2}\left(x - \frac{2}{3}\right)\right) + 3$




f) $f(x) = \cos^2(2x) \qquad \cos^2(2x) = \frac{1}{2} + \frac{1}{2}\cos(4x)$

$\cos(4x) = \cos(2x + 2x)$

$= \cos(2x) \cdot \cos(2x) - \sin(2x)\sin(2x)$

$= \cos^2(2x) - \underbrace{\sin^2(2x)}_{1 - \cos^2(2x)}$

$\sin^2(x) + \cos^2(x) = 1$

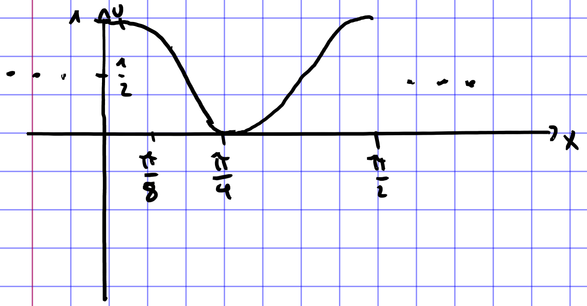
$= 2 \cdot \cos^2(2x) - 1$

$\cos(4x) = 2 \cdot \cos^2(2x) - 1 \quad |+1$

$\Rightarrow \cos(4x) + 1 = 2\cos^2(2x) \quad |:2$

$\Rightarrow \cos^2(2x) = \frac{\cos(4x) + 1}{2}$

$\cos^2(2x) = \frac{\cos(4x) + 1}{2} = \frac{1}{2}\cos(4x) + \frac{1}{2}$




6) i) a) $\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$

$2\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

$\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) = \frac{1}{2}$

b) $\cos(2a) = \cos(a)\cos(a) - \underbrace{\sin(a)\sin(a)}_{\sin^2(a) = 1 - \cos^2(a)}$

$= 2\cos^2(a) - 1$

c) $\cos^2\left(\frac{3}{8}\pi\right)$ $\quad \cos^2(a) = \frac{\cos(2a)+1}{2}$

$$\cos^2\left(\frac{3}{8}\pi\right) = \frac{\cos\left(\frac{3\pi}{4}\right)+1}{2} \qquad \cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$= \frac{1-\frac{1}{\sqrt{2}}}{2} = \frac{1}{2} - \frac{1}{\sqrt{8}}$$

d) $1 - 2\sin^2\left(\frac{\pi}{8}\right)$

$$\frac{1}{\sqrt{2}} = \cos\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{8}+\frac{\pi}{8}\right) = \underbrace{\cos\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)}_{\cos^2\left(\frac{\pi}{8}\right) = 1-\sin^2\left(\frac{\pi}{8}\right)} - \sin\left(\frac{\pi}{8}\right)\sin\left(\frac{\pi}{8}\right)$$

$$= 1 - 2\sin^2\left(\frac{\pi}{8}\right) = \frac{1}{\sqrt{2}}$$

e) $\frac{1}{\tan^2\left(\frac{\pi}{8}\right)+1}$ $\quad \tan^2(x)+1 = \frac{1}{\cos^2(x)}$

$$\Rightarrow \frac{1}{\frac{1}{\cos^2\left(\frac{\pi}{8}\right)}} = \cos^2\left(\frac{\pi}{8}\right)$$

$$\cos\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{8}+\frac{\pi}{8}\right)$$

$$= \cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right)$$

$$= 2\cos^2\left(\frac{\pi}{8}\right) - 1$$

$$\Rightarrow \cos^2\left(\frac{\pi}{8}\right) = \frac{\cos\left(\frac{\pi}{4}\right)+1}{2} = \frac{1+\frac{1}{\sqrt{2}}}{2} = \frac{1}{2} + \frac{1}{\sqrt{8}}$$

f) $\cot^2\left(\frac{3\pi}{8}\right) - 1$ $\quad \cos^2\left(\frac{3\pi}{8}\right) = \frac{1}{2} - \frac{1}{\sqrt{8}}$

$$= \frac{\cos^2\left(\frac{3\pi}{8}\right)}{\sin^2\left(\frac{3\pi}{8}\right)} - 1 \qquad \sin^2\left(\frac{3\pi}{8}\right) = 1 - \cos^2\left(\frac{3\pi}{8}\right) = 1 - \left(\frac{1}{2} - \frac{1}{\sqrt{8}}\right)$$

$$\Rightarrow \sin^2\left(\frac{3\pi}{8}\right) = \frac{1}{2} + \frac{1}{\sqrt{8}} \qquad = 1 - \frac{1}{2} + \frac{1}{\sqrt{8}}$$

$$\cot^2\left(\frac{3\pi}{8}\right) - 1 = \frac{\frac{1}{2}-\frac{1}{\sqrt{8}}}{\frac{1}{2}+\frac{1}{\sqrt{8}}} - 1 = \frac{2-\sqrt{2}}{2+\sqrt{2}} - 1 \qquad = \frac{1}{2} + \frac{1}{\sqrt{8}}$$

$$= \frac{2-\sqrt{2}-2-\sqrt{2}}{2+\sqrt{2}} = \frac{-2\cdot\sqrt{2}}{2+\sqrt{2}} \qquad \frac{1\cdot(2-\sqrt{2})}{1\cdot(2-\sqrt{2})}$$

$$= \frac{-2\cdot\sqrt{2}(2-\sqrt{2})}{4-2} = -\sqrt{2}(2-\sqrt{2}) = -2\cdot\sqrt{2}+2$$

ii) a) $\tan(x \pm y) = \dfrac{\tan(x) \pm \tan(y)}{1 \mp \tan(x)\tan(y)}$ $\qquad \tan(x) = \dfrac{\sin(x)}{\cos(x)}$

$$\frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x)\tan(y)} = \frac{\frac{\sin(x)}{\cos(x)} \pm \frac{\sin(y)}{\cos(y)}}{1 \mp \frac{\sin(x)\sin(y)}{\cos(x)\cos(y)}}$$

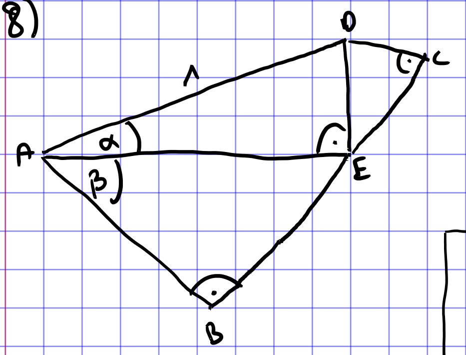
$$= \frac{\sin(x)\cos(y) \pm \sin(y)\cos(x)}{\cos(x)\cos(y) \mp \sin(x)\sin(y)} \begin{matrix} \} \sin(x \pm y) \\ \} \cos(x \pm y) \end{matrix} = \tan(x \pm y)$$

b) $\cot(x \pm y) = \dfrac{\cot(x)\cot(y) \mp 1}{\cot(y) \pm \cot(x)}$ $\qquad \cot(x) = \dfrac{1}{\tan(x)} = \dfrac{\cos(x)}{\sin(x)}$

$$= \frac{1 \mp \overbrace{\frac{1}{\cot(x)}}^{\tan(x)} \frac{1}{\cot(y)}}{\frac{1}{\cot(x)} \pm \frac{1}{\cot(y)}} = \underbrace{\frac{1 \mp \tan(x)\tan(y)}{\tan(x) \pm \tan(y)}}$$

$$\frac{1}{\tan(x \pm y)} = \cot(x \pm y$$

8)



$\overline{AD} = 1$

$\underline{AED:}$ $AE = \cos(\alpha)$

$ED = \sin(\alpha)$

$\underline{ABE:}$ $AE$: Hypotenuse

$AB$: Ankathete

$\Rightarrow \dfrac{AB}{AE} = \cos(\beta) \quad \Rightarrow AB = \cos(\beta) \cdot AE$

$= \cos(\beta) \cdot \cos(\alpha)$

$\underline{ECD:}$ $EC = \cos(\beta) \cdot \sin(\alpha)$

$DC = \sin(\beta) \cdot \sin(\alpha)$

$BE$: Gegenkathete

$BE = \sin(\beta) \cdot \underbrace{\cos(\alpha)}_{AE}$

① $DF = BC = \cos(\alpha)\sin(\beta) + \cos(\beta)\sin(\alpha) = \sin(\alpha + \beta)$

② $AF = \cos(\alpha + \beta) = AB - \underbrace{FB}_{DC} = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$