Aufgabe 1:

a.) $\sqrt[24]{x^3 / (x^3 (x^3)^2)} = \sqrt[24]{x^{27} / (x^9 \cdot x^6)}$

$= \sqrt[24]{x^{27} / x^{15}}$

$= \sqrt[24]{x^{12}} = x^{\frac{12}{24}} = x^{\frac{1}{2}} = \sqrt{x}$

$D = [0, \infty)$

b.) $\frac{1}{x} + \frac{2x}{x-2} - \frac{5}{x+3} - \underbrace{\frac{20}{x^2+x-6}}_{(x-2)\cdot(x+3)} - \underbrace{\frac{3}{x^2+3x}}_{x(x+3)}$

$\Leftrightarrow \frac{(x-2)\cdot(x+3) + 2x^2 \cdot (x+3) - 5x(x-2) - 20x - 3x + 6}{x \cdot (x-2)(x+3)}$

$\Leftrightarrow \frac{2x^3 + 2x^2 - 12x}{x(x-2)\cdot(x+3)} = \frac{2x \cdot (x^2 + x - 6)}{x \cdot (x^2 + x - 6)} = \frac{2x}{x} = 2$

c.) $\frac{\frac{a}{a-b} - \frac{b}{a+b}}{\frac{a}{a+b} + \frac{b}{a-b}} = \frac{\frac{a(a+b) - b(a-b)}{(a-b)\cdot(a+b)}}{\frac{a(a-b) + b\cdot(a+b)}{(a+b)(a-b)}}$

$= \frac{a(a+b) - b(a-b)}{\cancel{(a-b)(a+b)}} \cdot \frac{\cancel{(a+b)(a-b)}}{a\cdot(a-b) + b\cdot(a+b)}$

$= \frac{a(a+b) - b(a-b)}{a\cdot(a-b) + b\cdot(a+b)} = \frac{a^2 + ab - ba + b^2}{a^2 - ab + ba + b^2}$

$= \frac{a^2 + b^2}{a^2 + b^2} = 1$ //

$\sin(\varphi) \equiv \sin\varphi$

$\ln(e^2) \equiv \ln e^2$

d.) $\ln \sqrt{e^{3(\ln e^2 + \ln e^6)}}$ $(\ln = \log_e)$

$\ln e^2 = 2$

$\ln e^6 = 6$

$\Leftrightarrow \ln \sqrt{e^{3(2+6)}} = \ln \sqrt{e^{3\cdot 8}} = \ln e^{\frac{24}{2}} = \ln e^{12}$

$= 12$

e.) $\sqrt{x+16} - \sqrt{x-12} = 2 \quad / + \sqrt{x+12}$

$\sqrt{x+16} = 2 + \sqrt{x-12} \quad /()^2$

$x + 16 = (2 + \sqrt{x-12})^2$

$x + 16 = 4 + 4\sqrt{x-12} + x - 12 \quad / -x$

$16 = 4 + 4\sqrt{x-12} - 12$

$24 = 4\sqrt{x-12} \quad |:4$

$6 = \sqrt{x-12} \quad |()^2$

$36 = x - 12 \quad | +12$

$x = 48$

f.) $8x^2 - 14x = 9 \quad |:8 \; | -9$

$x^2 - \frac{7}{4}x - \frac{9}{8} = 0$

$x = \frac{7}{8} \pm \sqrt{\frac{49}{64} + \frac{72}{64}} = \frac{7}{8} \pm \frac{11}{8}$

$x \in \{\frac{9}{4}; -\frac{1}{2}\}$

g.) $x^4 - \frac{7}{4}x^2 - \frac{9}{8} = 0$

$y = x^2$

$y^2 - \frac{7}{4}y - \frac{9}{8} = 0$

$y \in \{\frac{9}{4}; -\frac{1}{2}\} \quad \wedge \quad y \geq 0$

$x^2 = \frac{9}{4} \Rightarrow x = \pm\frac{3}{2} \qquad x \in \{-\frac{3}{2}; \frac{3}{2}\}$

h.) $|x+1| + |x+2| \leq 2$

1. Fall: $x + 1 \geq 0 \Rightarrow x \geq -1$

(A) $x + 2 \geq 0 \Rightarrow x \geq -2$

$x + 1 + x + 2 \leq 2$

$2x + 3 \leq 2$

$2x \leq -1 \quad \Rightarrow x \leq -\frac{1}{2}$

$x \geq -1 \quad \wedge \; x \geq -2 \quad \wedge \quad x \leq -\frac{1}{2}$

$\Rightarrow \quad x \in [-1; -\frac{1}{2}]$

(B) $x + 2 < 0 \Rightarrow x < -2$

$x \in \{\}$

2. Fall: $x + 1 < 0 \Rightarrow x < -1$

(A) $x + 2 \geq 0 \Rightarrow x \geq -2$

$(-x - 1) + x + 2 \leq 2 \Rightarrow 1 \leq 2$

$\Rightarrow x \in [-2; -1$

(B) $x + 2 < 0 \Rightarrow x < -2$

$-x - 1 - x - 2 \leq 2$

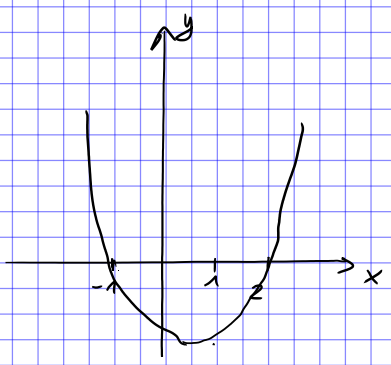
$-2x \leq 5 \qquad | \cdot (-\frac{1}{2})$

$x \geq -\frac{5}{2}$

$x \in [-\frac{5}{2}; -2)$

$x \in [-1; -\frac{1}{2}] \cup [-2; -1) \cup [-\frac{5}{2}; -2)$

$x \in [-\frac{5}{2}; -\frac{1}{2}]$



i.) $\frac{x+2}{x^2-x-2} < -1$

$x^2 - x - 2 = 0 \rightarrow x = \{-1; 2\}$

① $x^2 - x - 2 > 0 \quad ; x \in \mathbb{R} \setminus [-1; 2]$

$x + 2 < -x^2 + x + 2$

$x^2 < 0$

$x \in \{\}$

② $x^2 - x - 2 < 0 \quad x \in (-1; 2)$

$x^2 > 0$

$x \in \mathbb{R} \setminus \{0\}$

$\Rightarrow x \in (-1, 0) \cup (0, 2)$

Aufgabe 2:

$\frac{l}{a} = \frac{a}{b} \qquad \Phi = \frac{a}{b}$

$l = a + b$

$\frac{l}{a} = \frac{a+b}{a} = \frac{a}{b}$

$\Leftrightarrow \frac{a}{a} + \frac{b}{a} = \frac{a}{b}$

$\Leftrightarrow 1 + \frac{1}{\Phi} = \Phi \quad / -1$

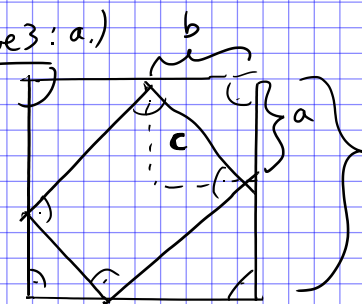
$\Leftrightarrow \frac{1}{\Phi} = \Phi - 1 \quad / \cdot \Phi$

$\Leftrightarrow 1 = \Phi^2 - \Phi \quad / -1$

$\Leftrightarrow \Phi^2 - \Phi - 1 = 0$

$\Phi = \frac{1+\sqrt{5}}{2} = \frac{a}{b}$

Aufgabe 3: a.)



$(a+b)^2 = A_{groß}$

$c^2 + \frac{1}{2} ab \cdot 4 = A_{groß}$

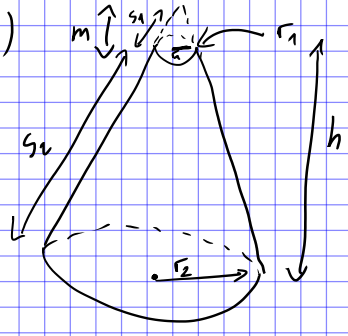
$\Leftrightarrow c^2 + 2ab = A_{groß}$

$(a+b)^2 = c^2 + 2ab$

$a^2 + 2ab + b^2 = c^2 + 2ab \quad / -2ab$

$a^2 + b^2 = c^2$

b.)



$$V = \frac{1}{3} \cdot h \cdot G$$

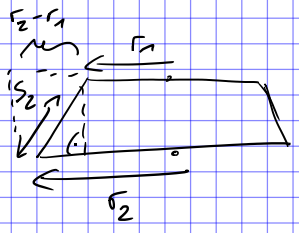
$$= \frac{h \cdot \pi}{3} \cdot (r_1^2 + r_1 r_2 + r_2^2)$$

$$G = \pi r^2$$

$$V \simeq 10{,}5\,\ell$$

$d_2 = 2 \cdot r_2$

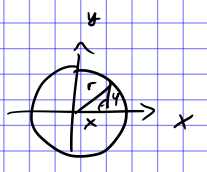
$d_1 = 2 \cdot r_1$



$$s = \sqrt{(r_2 - r_1)^2 + h^2}$$

$$M = (r_2 + r_1) \cdot \pi \cdot s = 0{,}1997\ m^2$$

c.)



$$r^2 = y^2 + x^2$$

$$\Rightarrow r = \sqrt{y^2 + x^2}$$

① $(1, 0, 0) \Rightarrow x = 1$

$y = 0$

$z = 0$

$$r = \sqrt{1^2 + 0^2} = 1$$

$x = r \cdot \cos(\varphi) \;/ \arccos(\,) \rightarrow \arccos\left(\frac{x}{r}\right) = \varphi$

$y = r \cdot \sin(\varphi) / \arcsin(\,) \rightarrow \arcsin\left(\frac{y}{r}\right) = \varphi$

$$\varphi = \arcsin\left(\frac{y}{r}\right) = \arcsin\left(\frac{0}{1}\right) = \arcsin(0) = 0$$

Zylinderkoordinaten: $(1\,|\,0\,|\,0)$

b.) $(1, 1, 0)$

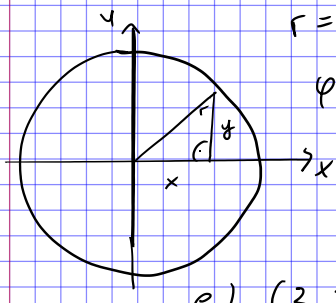
$z = 0 \rightarrow h = 0$

$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\varphi = \frac{\pi}{4} \Rightarrow \text{Z.K.} \left(\sqrt{2};\ \frac{\pi}{4};\ 0\right)$$

c.) $(1, 1, 1) \Rightarrow$ Z.K. $\left(\sqrt{2};\ \frac{\pi}{4};\ 1\right)$

d.) $(1, \sqrt{2}, 0)$



$$r = \sqrt{1^2 + \sqrt{2}^2} = \sqrt{1 + 2} = \sqrt{3}$$

$$\varphi = \arcsin\left(\frac{1}{\sqrt{3}}\right) = 54{,}74$$

Z.K. $:= (\sqrt{3};\ 54{,}74;\ 0)$

e.) $(2, 3, 4)$

$$r = \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13}$$

$$\varphi = \arcsin\left(\frac{3}{\sqrt{13}}\right) = 56{,}31$$

$z = h = 4$

Z.K. $(\sqrt{13};\ 56{,}31;\ 4)$

f.) $(0,0,0) \Rightarrow (0,0,0)$

Aufgabe 4:

a.) Newton 2: $\vec{F} = m \cdot \vec{a} = \frac{d\vec{p}}{dt} = \dot{\vec{p}}$

$[F] = 1\,kg \cdot \frac{m}{s^2}$

b.) $E_{kin} = \frac{1}{2} m v^2$

$[E] = 1\,kg \cdot \frac{m^2}{s^2}$

c.) $\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{1} \cdot \frac{\vec{r}}{r^3}$ $\qquad \varepsilon_0 = \frac{As}{Vm}$

$[E] = \frac{Vm}{As} \cdot C \cdot \frac{m}{m^3}$

$= \frac{V}{m}$

d.) ① $2\,35\,km^2 = 2{,}35 \cdot 10^{10}\,dm^2$

② $0{,}287\,m^2 = 28{,}7\,dm^2$

③ $342748\,mm^2 = 34{,}27\,dm^2$

④ $8{,}342 \cdot 10^4\,mm \cdot m = 8{,}342 \cdot 10^3\,dm^2$

⑤ $3{,}648 \cdot 10^{17}\,\mu m^2 = 3{,}648 \cdot 10^7\,dm^2$

⑥ $2\,\mu m \cdot m \cdot \frac{dm}{km} = 2 \cdot 10^{-8}\,dm^2$

① $9{,}837 \cdot 10^{19}\,nm^3 = 9{,}837 \cdot 10^{-2}\,cm^3$

② $5{,}32 \cdot 10^4\,ml = 5{,}32 \cdot 10^4\,cm^3$

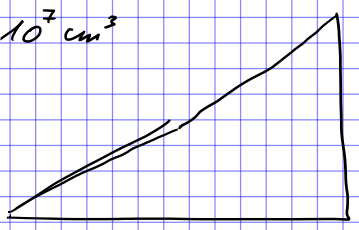
③ $0{,}0345\,m^3 = 3{,}45 \cdot 10^4\,cm^3$

④ $2\,\mu m \cdot dm \cdot km = 200\,cm^3$

⑤ $4370\,mm \cdot l/cm = 4{,}37 \cdot 10^5\,cm^3$

⑥ $0{,}45\,\mu m^2/dm \cdot km^2/nm \cdot cm = 4{,}5 \cdot 10^7\,cm^3$

Aufgabe 5:



keine richtigen Dreiecke:
Das große Unterdreieck hat den Anstieg $\frac{3}{8}$ und das kleine $\frac{2}{5}$. Die Hypothenusen müssen eine Gerade bilden, die Anstiege müssen also eigentlich gleich sein.