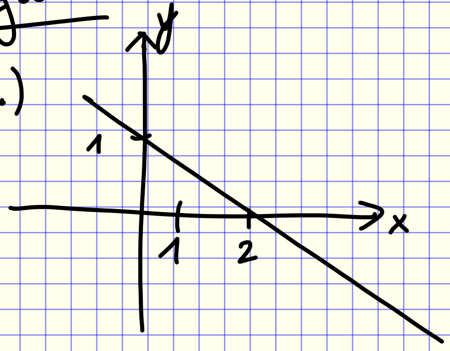
Aufgabe 1:

(I) a.)

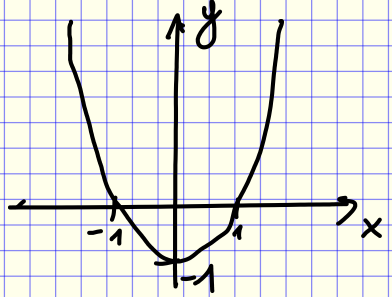


$f(x) = a + b \cdot x$

$f(x) = 1 - \frac{1}{2}x$

$f: \mathbb{R} \to \mathbb{R}$

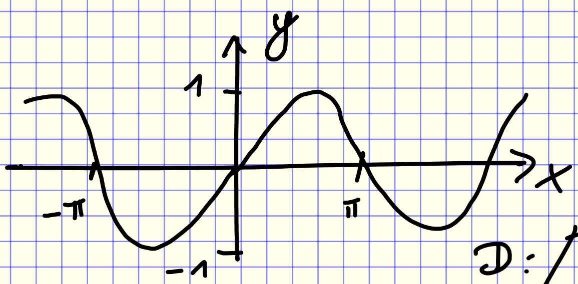
b.)



$f(x) = x^2 - 1$

$D: [0; \infty) \to [-1; \infty)$

c.)

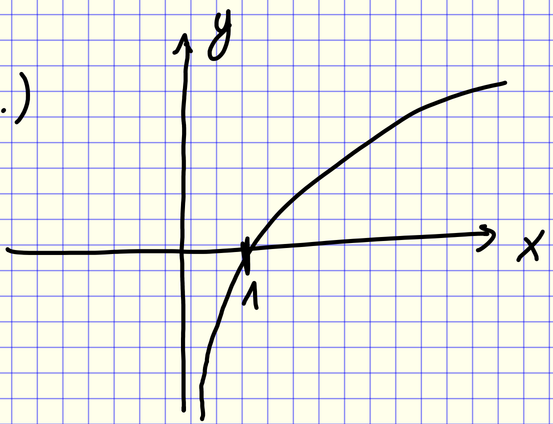


$f(x) = \sin(x) = \sin(x + 2\pi \cdot z)$

$z \in \mathbb{Z}$

$D: [-\frac{\pi}{2}; \frac{\pi}{2}] \to [-1; 1]$

d.)



$f(x) = \ln(x)$

$\mathbb{R}^+ \to \mathbb{R}$

(ii) a.) $f(x) = x^2 + 2x - 15$

pq - Formel ...

$x = -5 \vee 3$

a.) $D: \mathbb{R}$

$W: [-16; \infty)$

b.) $f(x) = (x-3) \cdot (x+5)$

c.) siehe Geogebra

d.) monoton fallend auf $x \le -1$

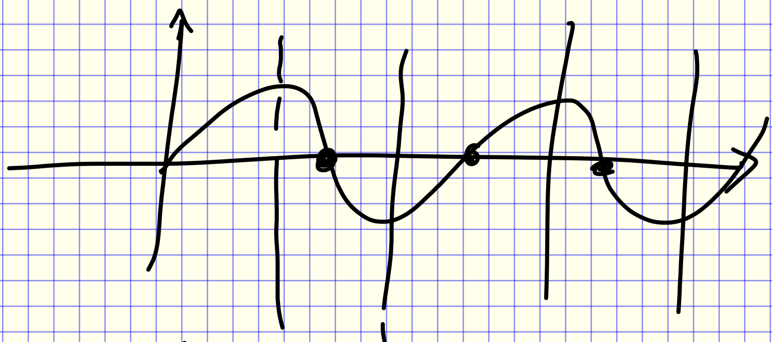
monoton wachsend auf $x \ge -1$

(iii)

a.) $\sin(x) = 0 \Rightarrow x = z \cdot \pi \quad , z \in \mathbb{Z}$

Maxima: $\sin(x) = 1 \Rightarrow x = \frac{\pi}{2} + 2\pi \cdot z \; ; z \in \mathbb{Z}$

Minima: $\sin(x) = -1 \Rightarrow x = \frac{3\pi}{2} + 2\pi \cdot z \; ; z \in \mathbb{Z}$



b.) $f(x) = \frac{1}{x+3}$ $\qquad D: \mathbb{R} \setminus \{-3\}$

$\lim_{x \to -3^+} f(x) = +\infty \quad ; \quad \lim_{x \to -3^-} f(x) = -\infty$

Punktsymmetrie bei $P(-3|0)$

c.) $f(x) = \begin{cases} 3x - 1, & x < 1 \\ x^2 + 1, & x \geq 1 \end{cases}$

(i.v)

a.) $f(x) = x \cdot \sin(x)$

$f(-x) = -x \cdot \underbrace{\sin(-x)}_{-\sin(x)} = x \cdot \sin(x) = f(x)$

$\Rightarrow$ Achsensymmetrie

Punktsymmetrie: $f(-x) = -f(x)$

Achsensymmetrie: $f(-x) = f(x)$

b.) $f(x) = \frac{e^x + e^{-x}}{2}$

$f(-x) = \frac{e^{-x} + e^x}{2} = f(x)$

$\Rightarrow$ Achsensymmetrie

c.) $f(-x) = \underbrace{((-x)^5 + 4 \cdot (-x)^3 - 2x)}_{-(x^5 + 4x^3 + 2x)} \cdot \underbrace{(-\sin(x))^2}_{\sin(x)^2} \cdot \frac{1}{\underbrace{|{-x}|}_{|x|} \cdot \underbrace{\cos(-x)}_{\cos(x)}}$

$= -\frac{(x^5 + 4x^3 + 2x)\sin^2(x)}{|x| \cdot \cos(x)} = -f(x)$ $\Rightarrow$ Punktsymm.

d.) $f(x) = (x+8)^3 - (x-8)^3$

$f(-x) = (-x+8)^3 - (-x-8)^3$

$= -(x-8)^3 + (x+8)^3 = f(x)$

$\Rightarrow$ Achsensymmetrie

e.) $f(x) = (x+8)^2 - (x-8)^2$

$= (-x+8)^2 - (-x-8)^2$

$= (-(x-8))^2 - (-(x+8))^2$

$= (x-8)^2 - (x+8)^2 = -\left[(x+8)^2 - (x-8)^2\right]$

$= -f(x)$

$\Rightarrow$ Punktsymmetrie

f.) $f(x) = \ln\left(\sqrt{x^2+1} + x\right)$

$f(x) + f(-x) = \ln\left(\sqrt{x^2+1} + x\right) + \ln\left(\sqrt{x^2+1} - x\right)$

$= \ln\left(\left(\sqrt{x^2+1} + x\right) \cdot \left(\sqrt{x^2+1} - x\right)\right)$

$= \ln\left(\cancel{x^2} + 1 \cancel{- x\sqrt{x^2+1}} \cancel{+ x\sqrt{x^2+1}} \cancel{- x^2}\right)$

$= \ln(1) = 0$

$\Rightarrow f(x) + f(-x) = 0$

$\Leftrightarrow f(-x) = -f(x) \quad \Rightarrow$ Punktsymmetrie

Aufgabe 2:

$f(x) = 4x^2 - 4x + 4 \quad , \; g(x) = x - 2$

$f(x) = 4x^2 - 4x + 4$

$= 4 \cdot (x^2 - x + 1)$

$x^2 - x + 1 = (x - \frac{1}{2})^2 - \frac{1}{4} + 1$

$= (x - \frac{1}{2})^2 + \frac{3}{4}$

$\rightarrow \quad f(x) = 4(x - \frac{1}{2})^2 + 3$

$g(x) = x - 2$

$$(f \circ g)(x) = f(g(x)) = 4(x-2)^2 - 4(x-2) + 4$$

$$= 4(x^2 - 4x + 4) - 4x + 8 + 4$$

$$= 4x^2 - 16x + 16 - 4x + 8 + 4$$

$$= 4x^2 - 20x + 28$$

$$= 4 \cdot (x^2 - 5x + 7)$$

$$x^2 - 5x + 7 = (x - \tfrac{5}{2})^2 - \tfrac{25}{4}$$

$$\Rightarrow 4 \cdot \left((x - \tfrac{5}{2})^2 \underbrace{- \tfrac{25}{4} + 7}_{\frac{3}{4}} \right)$$

$\mathbb{R} \to [3; \infty)$

$$\boxed{= 4 \cdot (x - \tfrac{5}{2})^2 + 3}$$

$$(g \circ f)(x) = 4x^2 - 4x + 4 - 2$$

$$= 4 \cdot (x - \tfrac{1}{2})^2 + \underbrace{3 - 2}_{1} = 4 \cdot (x - \tfrac{1}{2})^2 + 1$$

$\mathbb{R} \to [1; \infty)$