Aufgabe 1:

a.) $\frac{1}{2} + \frac{2}{3} - \frac{3}{4} - \frac{1}{6}$ $\qquad \sqrt[n]{x^m} = x^{\frac{m}{n}}$

$= \frac{6}{12} + \frac{8}{12} - \frac{9}{12} - \frac{2}{12} = \frac{3}{12} = \frac{1}{4}$

b.) $\left(\sqrt[3]{2^2} \cdot \sqrt[4]{2^{-3}}\right)^{12} = \left(2^{\frac{2}{3}} \cdot 2^{-\frac{3}{4}}\right)^{12}$

$= \left(2^{\frac{2}{3} - \frac{3}{4}}\right)^{12} = \left(2^{\frac{8}{12} - \frac{9}{12}}\right)^{12}$

$= \left(2^{-\frac{1}{12}}\right)^{12} = 2^{-1} = \frac{1}{2}$

c.) $\log_7(49) = 2$

d.) $\lg\left(\frac{1}{10}\right) = \log_{10}\left(\frac{1}{10}\right) = -1 \quad \left(= \log_{10}\left(10^{-1}\right)\right)$

e.) $\log_{10}\left(\sqrt[3]{1000}\right) = \log_{10}(10) = 1$

f.) $2^x = \frac{1}{8} \quad / \log_2(\;)$

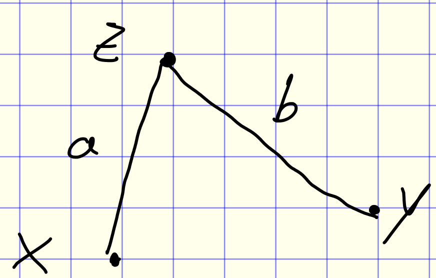
$x = \log_2\left(\frac{1}{8}\right)$

$x = -3$

Aufgabe 2:

Siehe Stream !?

Aufgabe 3:



$$\begin{cases} a+b=12 \\ b+2=a \end{cases} \longrightarrow 5+2=a=7$$

$$b+2+b=12$$

$$2b+2=12 \quad |-2$$

$$2b=10 \quad |:2$$

$$b=5$$

Aufgabe 4:

(i) a.) $f(x)=(x-2)$; $g(x)=(1-2x)$

- $(f+g)(x)$: $-x-1$ $\qquad D=\mathbb{R}$
- $(f-g)(x)$: $3x-3$ $\qquad D=\mathbb{R}$
- $(f\cdot g)(x)$: $-2x^2+5x-2$ $\qquad D=\mathbb{R}$
- $\frac{f}{g}(x)$: $\frac{x-2}{1-2x}$ $\qquad D=\mathbb{R}\setminus\{\frac{1}{2}\}$

b.) $f(x)=\sin(x)$; $g(x)=\cos(x)$

- $(f+g)(x)=\sin(x)+\cos(x)$ $\qquad D=\mathbb{R}$
- $(f-g)(x)=\sin(x)-\cos(x)$ $\qquad D=\mathbb{R}$
- $(f\cdot g(x)=\sin(x)\cdot\cos(x)$ $\qquad D=\mathbb{R}$
- $\frac{f}{g}(x)=\frac{\sin(x)}{\cos(x)}$ $\qquad D=\mathbb{R}\setminus\{\frac{\pi}{2}+k\cdot\pi\ ; k\in\mathbb{Z}\}$

(ii) a.) $h(x) = \ln(x+1)$

$g(x) = \ln(x)$

$f(x) = x+1$

b.) $h(x) = \left(\frac{x+2}{x+1}\right)^2$

$g(x) = x^2$

$f(x) = \frac{x+2}{x+1}$

c.) $h(x) = \cos^2(x)$

$g(x) = x^2$

$f(x) = \cos(x)$

(iii) $f(x) = x^2$; $g(x) = \sqrt{x}$; $h(x) = \frac{1}{x}$

a.) $(g \circ f)(x) = \sqrt{x^2} = |x|$ $\qquad D = \mathbb{R}$

$(f \circ g)(x) = \sqrt{x}^2 = x$ $\qquad D = [0; \infty)$

b.) $(f \circ (g+h))(x)$

$= \left(\sqrt{x} + \frac{1}{x}\right)^2 = x + \frac{2}{\sqrt{x}} + \frac{1}{x^2}$ $\qquad D = (0; \infty)$

c.) $(h \circ (f \cdot g))(x) = \frac{1}{x^2\sqrt{x}} = \frac{1}{x^{\frac{5}{2}}}$ $\qquad D = (0; \infty)$

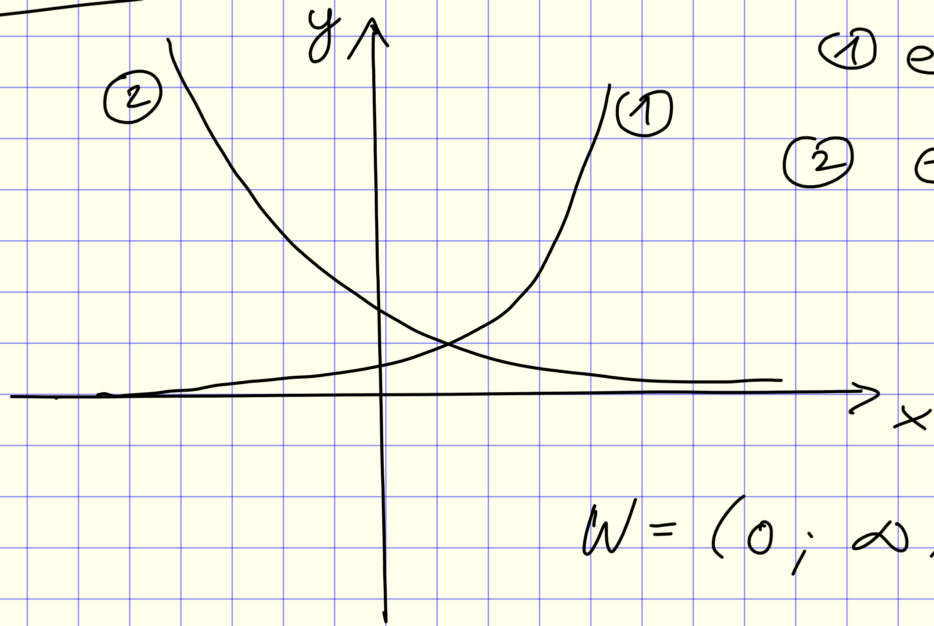
d.) $(f \circ (g \circ h))(x)$

$$= \sqrt{\frac{1}{x}}^{\,2} = \frac{1}{x} \qquad D = \mathbb{R} \setminus \{0\}$$

Aufgabe 5:

(i)

a.)



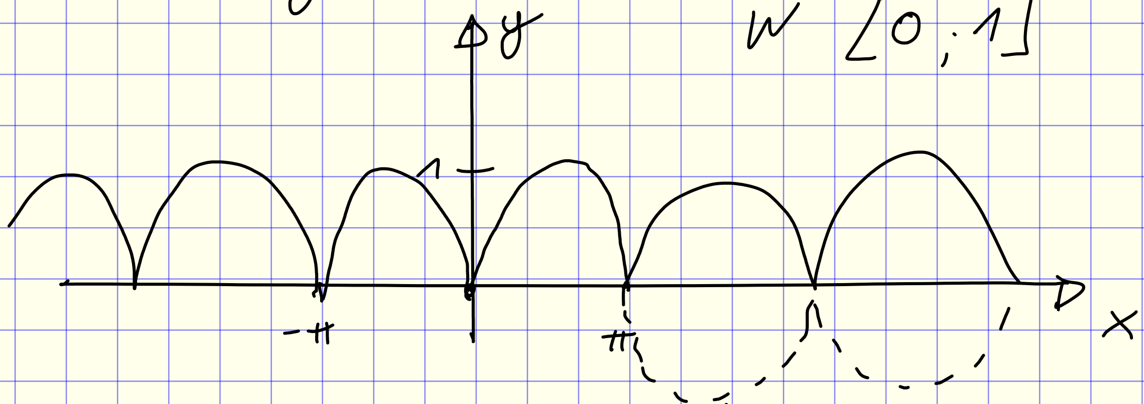
① e^{x}

② e^{-x}

$W = (0;\infty)$

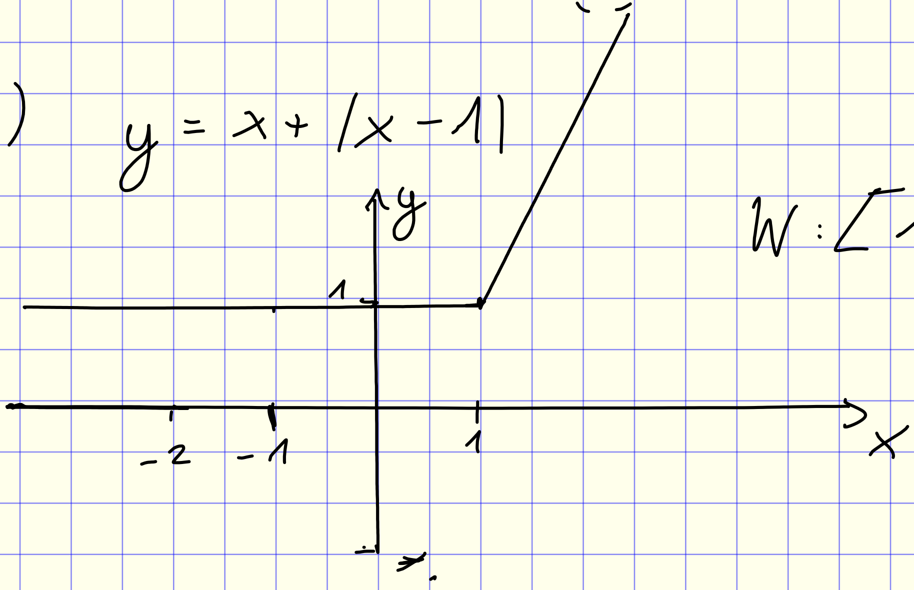
b.) $y = |\sin(x)|$

$W\ [0;1]$



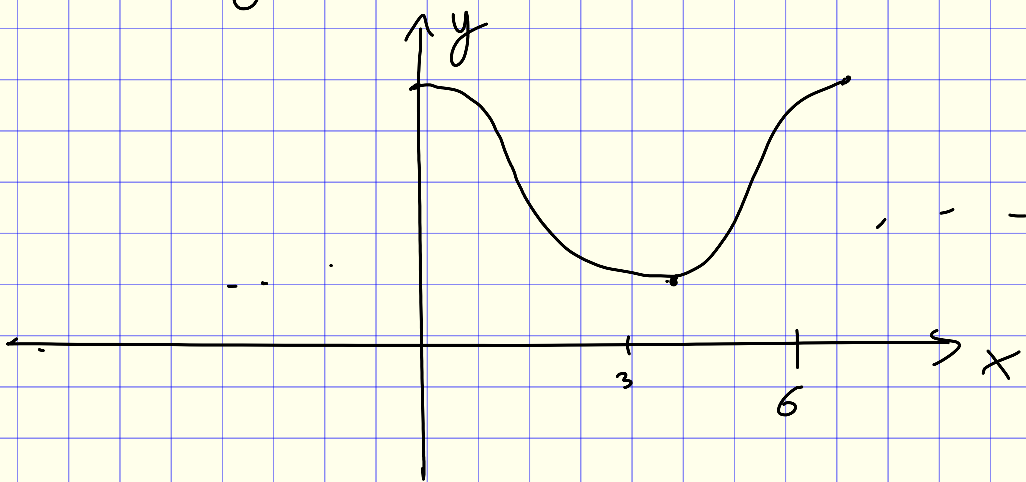
c.) $y = x + |x-1|$

$W: [1;\infty)$



d.)

$$y = e^{\cos(x)}$$



Achsensymmetrie

(ii)

Strecken/Stauchen x-Richtung

Phasenverschiebung

a.) $f(x) = \frac{5}{6} \sin(7x+8)$

Amplitude

$$f(x) = A \cdot \sin(\omega t + \varphi)$$

$$\sin(7x+8) = \sin(7x+8+2\pi)$$

$$= \sin(7(x+\tfrac{2}{7}\pi)+8)$$

$$f(x) = f(x+\tfrac{2}{7}\pi)$$

$$-1 \leq \sin(7x+8) \leq 1$$

$$-\frac{5}{6} \leq f(x) \leq \frac{5}{6}$$

$$|f(x)| \leq \frac{5}{6}$$

b.) $\frac{1}{4+\sin(x)} = \frac{1}{4+\sin(x+2\pi)} = f(x)$

$\to$ Periodizität: $2\pi = T$

$$-1 \leq \sin(x) \leq 1$$

$$3 \leq 4+\sin(x) \leq 5$$

$$\frac{1}{3} \geq \frac{1}{4+\sin(x)} \geq \frac{1}{5}$$

$|f(x)| \leq \frac{1}{3}$ kleinste Funktionswert: $\frac{1}{5}$

größte Funktionswert: $\frac{1}{3}$

(iii) $f(x) = x^3 - 39x - 70$

$x = -2 \Rightarrow (x+2) \cdot g(x) = f(x)$

$$
\begin{array}{l}
(x^3 - 39x - 70) : (x+2) = x^2 - 2x - 35 \\
\underline{-(x^3 + 2x^2)} \\
\quad -2x^2 - 39x - 70 \\
\quad \underline{-(-2x^2 - 4x)} \\
\qquad -35x - 70 \\
\qquad \underline{-(-35x - 70)} \\
\qquad\qquad 0
\end{array}
$$

$x^2 - 2x - 35 = 0$

$\Rightarrow 1 \pm \underbrace{\sqrt{1+35}}_{6}$

$= 7 \lor -5$

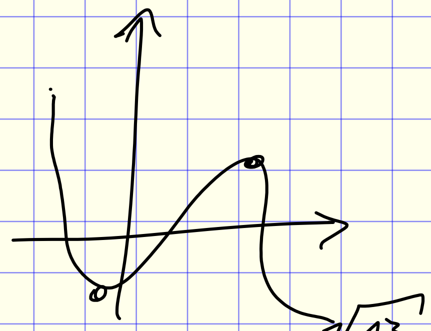
$f(x) = (x+2)(x+5)(x-7)$

$\Rightarrow$ Nullstellen: $x \in \{-2; -5; 7\}$

Maxima und Minima:

$$f'(x) = 3x^2 - 39 \overset{!}{=} 0$$

$$x = \pm\sqrt{13}$$

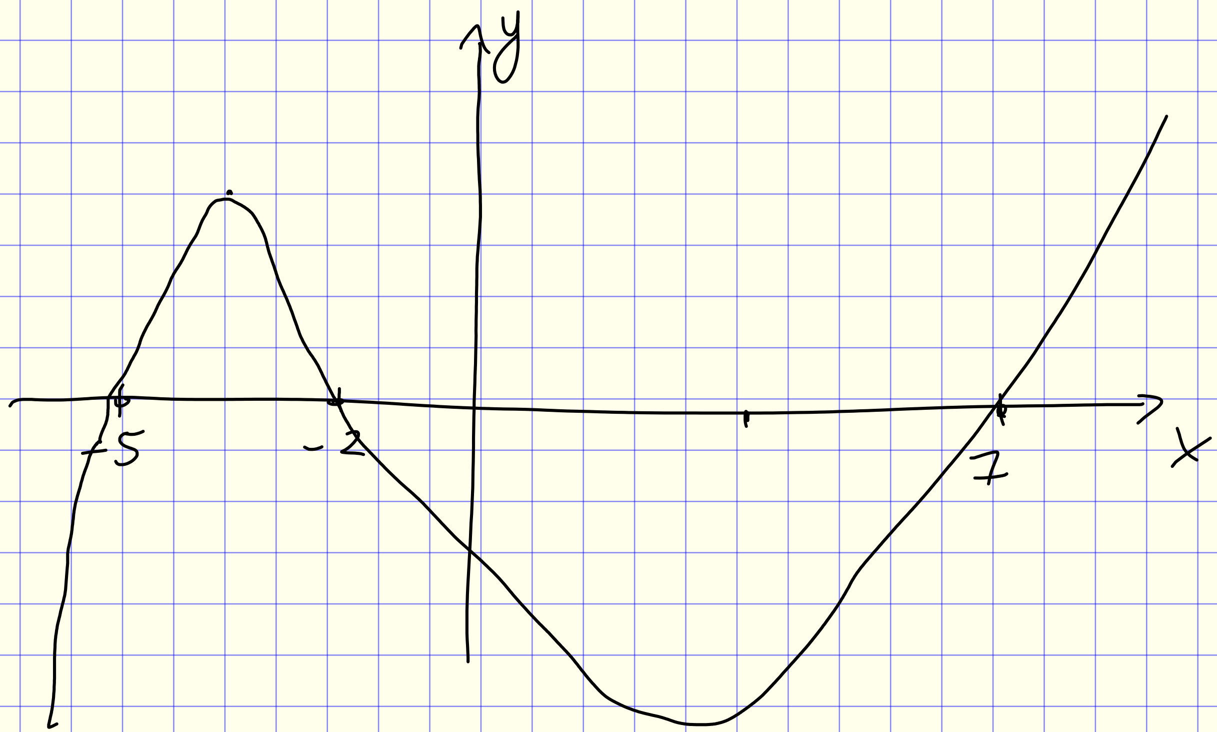


x	$-\infty$			$\sqrt{13}$
$f(x)$				
$f'(x)$	$+$	0	$-$	0
$f''(x)$				

$$f''(x) = 6x \Rightarrow x = 0$$

$$f''(\sqrt{13}) > 0$$

$$f''(-\sqrt{13}) < 0$$



<u>Aufgabe 6:</u>

$$T(t)$$

$$T_0 = T(t = 0\,s)$$

$$T_1$$

$$\Delta T(t) = \Delta T_0 \cdot e^{-kt} = (T_0 - T_1)\, e^{-kt}$$

$$\Delta T(t) = T(t) - T_1$$

$$\Delta T_0 = T_0 - T_1$$

<u>geg:</u> $T_1 = 20°C$; $T(t = \frac{1}{3}h) = 80°C$; $T(t = 3h) = 30°C$

$$\frac{\Delta T(3h)}{\Delta T(\frac{1}{3}h)} = \frac{30°C - 20°}{80°C - 20°C} = \frac{10}{60} = \frac{1}{6}$$

$$= \frac{e^{-k \cdot 3}}{e^{-k \cdot \frac{1}{3}}} = e^{k(\frac{1}{3} - 3)} = \frac{1}{6} \quad / \ln()$$

$$k \cdot \left(\frac{1}{3} - 3\right) = \ln\left(\frac{1}{6}\right)$$

$$-\frac{8}{3} \cdot k = \ln\left(\frac{1}{6}\right) \quad / \cdot \left(-\frac{3}{8}\right)$$

$$k = -\frac{3}{8} \cdot \ln\left(\frac{1}{6}\right) \quad ; [k] = \frac{1}{h}$$

$$\Delta T_0 = \Delta T(3h) \cdot e^{k \cdot 3h} = \Delta T(3h) \cdot e^{\ln(6^{\frac{3}{8}}) \cdot 3}$$

$$= \Delta T(3h) \cdot 6^{\frac{9}{8}}$$

$$T_0 - T_1 = \Delta T_0 = 10 \cdot 6^{\frac{9}{8}} °C = 75{,}06°C$$

$$T_0 = 95{,}06°C$$

$$\Delta T(t) = \underbrace{75{,}06°C}_{T_0 - T_1} \cdot e^{-0{,}672 \frac{t}{h}}$$

$$\Delta T(t) = 75{,}06\,°C \cdot \exp\left(-0{,}67\,t/h\right)$$

$$\Delta T(t) = 5\,°C \quad ; \quad \Delta T(t) \stackrel{!}{=} 25$$

$$t = \frac{\ln\left(\frac{5}{75}\right)}{-0{,}672}\,h \approx 4h$$