1) i) 11.10.2023

a) $2x + 3y = 8$ $\left(\begin{array}{cc|c} 2 & 3 & 8 \\ 1 & -1 & -1 \end{array}\right)$ $\mathbb{L} = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$

$x - y = -1$

$\Rightarrow \begin{array}{c} \\ II - \frac{1}{2}I \end{array} \left(\begin{array}{cc|c} 2 & 3 & 8 \\ 0 & -\frac{5}{2} & -5 \end{array}\right)$ $\Rightarrow -\frac{5}{2} \cdot y = -5 \quad | \cdot \left(-\frac{2}{5}\right)$

$\Rightarrow y = \cancel{-8} \left(\cdot \frac{2}{8}\right) = 2$

$2x + 3 \cdot 2 = 8 \quad | -6 \quad | : 2 \quad \Rightarrow x = 1$

b) $x - 2y = -7$ $\left(\begin{array}{cc|c} 1 & -2 & -7 \\ 2 & 3 & 0 \end{array}\right)$

$2x + 3y = 0$

$\Rightarrow \begin{array}{c} \\ II - 2I \end{array} \left(\begin{array}{cc|c} 1 & -2 & -7 \\ 0 & 7 & 14 \end{array}\right)$ $\Rightarrow 7 \cdot y = 14 \quad | : 7$

$\Rightarrow y = 2$

$\Rightarrow \mathbb{L} = \left\{ \begin{pmatrix} -3 \\ 2 \end{pmatrix} \right\}$

$\Rightarrow x - 2 \cdot 2 = -7 \quad | +4$

$\Rightarrow x = -3$

c) $5x + y + 2z = 3$ $\left(\begin{array}{ccc|c} 5 & 1 & 2 & 3 \\ -2 & 0 & 1 & -1 \\ 1 & 1 & 1 & 0 \end{array}\right)$

$-2x + z = -1$

$x + y + z = 0$

$\Rightarrow \begin{array}{c} I - 5III \\ II + 2III \\ III \end{array} \left(\begin{array}{ccc|c} 0 & -4 & -3 & 3 \\ 0 & 2 & 3 & -1 \\ 1 & 1 & 1 & 0 \end{array}\right) \Rightarrow \begin{array}{c} I + 2II \\ II \\ 2III - II \end{array} \left(\begin{array}{ccc|c} 0 & 0 & 3 & 1 \\ 0 & 2 & 3 & -1 \\ 2 & 0 & -1 & 1 \end{array}\right)$

$\Rightarrow \begin{array}{c} I \\ II - I \\ 3III + I \end{array} \left(\begin{array}{ccc|c} 0 & 0 & 3 & 1 \\ 0 & 2 & 0 & -2 \\ 6 & 0 & 0 & 4 \end{array}\right) \Rightarrow \begin{array}{c} I/3 \\ II/2 \\ III/6 \end{array} \left(\begin{array}{ccc|c} 0 & 0 & 1 & \frac{1}{3} \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & \frac{2}{3} \end{array}\right)$

(1) $x = \frac{1}{3}$ (3) $z = \frac{2}{3}$ $\mathbb{L} = \left\{ \begin{pmatrix} \frac{1}{3} \\ -1 \\ \frac{2}{3} \end{pmatrix} \right\}$

(2) $y = -1$

ii)

a) $2x + 3y = b$ $\left(\begin{array}{cc|c} 2 & 3 & b \\ 1 & a & 4 \end{array}\right)$

$x + ay = 4$

$\rightsquigarrow \begin{array}{c} I - 2II \\ \\ \end{array} \left(\begin{array}{cc|c} 0 & 3-2a & b-8 \\ 1 & a & 4 \end{array}\right) \rightsquigarrow \begin{array}{c} \\ 2II \end{array} \left(\begin{array}{cc|c} 0 & 3-2a & b-8 \\ 2 & 2a & 8 \end{array}\right)$

$$\rightsquigarrow \begin{array}{c} \mathrm{I} \\ \mathrm{II}+\mathrm{I} \end{array} \left(\begin{array}{cc|c} 0 & 3-2a & b-8 \\ 2 & 3 & b \end{array}\right) \rightsquigarrow$$

$$\rightsquigarrow \begin{array}{c} \mathrm{I} \\ -3\mathrm{I}+(3-2a)\mathrm{II} \end{array} \left(\begin{array}{cc|c} 0 & 3-2a & b-8 \\ (3-2a)2 & 0 & \underbrace{(3-2a)b-3\cdot(b-8)}_{\cancel{3b}-2ab\cancel{-3b}+24} \end{array}\right)$$

$$\Rightarrow \begin{array}{c} \mathrm{I} \\ \mathrm{II}/2 \end{array} \left(\begin{array}{cc|c} 0 & 3-2a & b-8 \\ 3-2a & 0 & -ab+12 \end{array}\right) \qquad \underbrace{A}_{0}\vec{x}=\vec{b} \quad 0 \to 0$$

$3-2a=0 \quad |+2a \quad |:2$

$\Rightarrow a=\frac{3}{2}$ $\qquad$ <u>1. Fall</u> $a=\frac{3}{2} \Rightarrow 3-2a=0$

$$\left(\begin{array}{cc|c} 0 & 0 & b-8 \\ 0 & 0 & -\frac{3}{2}b+12 \end{array}\right)$$

(1) $0\cdot x+0\cdot y=b-8 \Rightarrow b=8$ in (2)

(2) $0\cdot x+0\cdot y=-\frac{3}{2}b+12$

$\Rightarrow 0=-\frac{3}{2}\cdot 8+12=-\frac{24}{2}+12=0$

$\Rightarrow a=\frac{3}{2},\ b=8$

<u>2. Fall</u> $a\neq\frac{3}{2} \Rightarrow 3-2a\neq 0$

$$\left(\begin{array}{cc|c} 0 & 3-2a & b-8 \\ 3-2a & 0 & -ab+12 \end{array}\right) \Rightarrow \begin{array}{c} \mathrm{I}/(3-2a) \\ \mathrm{II}/(3-2a) \end{array} \left(\begin{array}{cc|c} 0 & 1 & \frac{b-8}{3-2a} \\ 1 & 0 & \frac{-ab+12}{3-2a} \end{array}\right)$$

$1\cdot y=\frac{b-8}{3-2a}$ $\qquad \frac{y}{x}=\frac{b-8}{\cancel{3-2a}}\cdot\frac{\cancel{3-2a}}{12-ab}=\frac{b-8}{12-ab} \quad |\cdot x$

$1\cdot x=\frac{-ab+12}{3-2a}$

$\Rightarrow y=\frac{b-8}{12-ab}x \qquad \mathbb{L}=\left\{\begin{pmatrix} x \\ \frac{b-8}{12-ab}x \end{pmatrix}\right\}$

b) $$\left.\begin{array}{l} x+2y-z=5 \\ x+y=1 \\ y-z=2 \end{array}\right\} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{array}\right)$$

$$\Rightarrow \begin{array}{c} \mathrm{I}-\mathrm{II} \\ \\ \\ \end{array} \left(\begin{array}{ccc|c} 0 & 1 & -1 & 5-1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{array}\right) \Rightarrow \begin{array}{c} \mathrm{I}-\mathrm{III} \\ \\ \\ \end{array} \left(\begin{array}{ccc|c} 0 & 0 & 0 & 5-3 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{array}\right)$$

(1) $0 \cdot x + 0 \cdot y + 0 \cdot z = s - 3 \quad \Rightarrow s = 3$

(2) $x + y = 1 \quad \Rightarrow y = 1 - x$

(3) $y - z = 2 \quad \Rightarrow z = y - 2 = 1 - x - 2 = -x - 1$

$$\vec{r} = \begin{pmatrix} x \\ 1-x \\ -x-1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + x \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \Rightarrow \text{Gerade } g \text{ :D}$$

es existiert nur dann ein lösendes $\vec{x}$, $s = 3$

$\vec{x}$ liegt dann auf g

2)

i)

Modell	Schubladen	Böden	Türen
x	6	12	2
y	4	12	3
z	6	14	4

Produktion: $x = 15$, $y = 9$, $z = 6$ $\Rightarrow \vec{p} = \begin{pmatrix} 15 \\ 9 \\ 6 \end{pmatrix}$

$f(\vec{p})$: $P \rightarrow B$

$\vec{p} \rightarrow \vec{b} = V \cdot \vec{p}$

$$\vec{b} = \begin{pmatrix} s \\ b \\ t \end{pmatrix}$$

$$V \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} s \\ b \\ t \end{pmatrix}$$

$$V = \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{pmatrix} 6 & 4 & 6 \\ 12 & 12 & 14 \\ 2 & 3 & 4 \\ 4 & 8 & 10 \end{pmatrix} \end{array} \begin{array}{c} s \\ b \\ t \\ f \end{array}$$

$$V\vec{p} = \begin{pmatrix} 6 & 4 & 6 \\ 12 & 12 & 14 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 15 \\ 9 \\ 6 \end{pmatrix} = \begin{pmatrix} 6 \cdot 15 + 4 \cdot 9 + 6 \cdot 6 \\ 12 \cdot 15 + 12 \cdot 9 + 14 \cdot 6 \\ 2 \cdot 15 + 3 \cdot 9 + 4 \cdot 6 \end{pmatrix}$$

$$= \begin{pmatrix} 162 \\ 372 \\ 87 \end{pmatrix}$$

ii) a)
$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 5 & -1 \end{pmatrix}\begin{pmatrix} 2 & 1 & 1 \\ 4 & -3 & -2 \end{pmatrix} = \begin{pmatrix} 1\cdot 2+1\cdot 4 & 1\cdot 1+1\cdot(-3) & 1\cdot 1+1\cdot(-2) \\ 0\cdot 2+2\cdot 4 & 0\cdot 1+2\cdot(-3) & 0\cdot 1+2\cdot(-2) \\ 5\cdot 2+(-1)\cdot 4 & 5\cdot 1+(-1)(-3) & 5\cdot 1+(-1)(-2) \end{pmatrix}$$
$$= \begin{pmatrix} 6 & -2 & -1 \\ 8 & -6 & -4 \\ 6 & 8 & 7 \end{pmatrix}$$

b)
$$\begin{pmatrix} -2 & 3 & 1 \\ 1 & 1 & 1 \\ 0 & -2 & 1 \end{pmatrix}\begin{pmatrix} -3 & 5 & -2 \\ 1 & 2 & -3 \\ 2 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix} = 11\cdot \mathbb{1}$$

iii) a)
$$A = \begin{pmatrix} 2 & 3 \\ 4 & -3 \end{pmatrix} \Rightarrow A^{-1} = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} \\ \frac{2}{9} & -\frac{1}{9} \end{pmatrix}$$

b)
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix} \Rightarrow A^{-1} = \begin{pmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{pmatrix}$$

c)
$$A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 0 & 2 \\ -1 & 2 & 1 \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{4}\begin{pmatrix} -2 & 2 & -2 \\ -2 & 1 & 0 \\ 2 & 0 & 2 \end{pmatrix}$$

d)
$$A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 5 \\ -1 & 1 & -5 \end{pmatrix} \Rightarrow A^{-1} = ↯$$

$$(A \mid \mathbb{1}) \xRightarrow[\text{Zeilenumformungen}]{} (\mathbb{1} \mid A^{-1})$$

iv) $A\vec{x} = \vec{b}$

d)
$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 2 & 0 & 5 & 1 \\ -1 & 1 & -5 & -2 \end{array}\right) \qquad \vec{b} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} \to \text{nicht Lösung}$$

$$\Rightarrow \begin{array}{l} \mathrm{I} \\ \mathrm{II} \\ \mathrm{III}-\mathrm{I}+\mathrm{II}\end{array} \left(\begin{array}{ccc|c} 1&1&0&3\\ 2&0&5&1\\ 0&0&0&-4\end{array}\right) \Rightarrow 0\cdot x+0\cdot y+0\cdot z=-4$$

$$\Rightarrow 0=-4$$

$$\vec{s}=\begin{pmatrix}3\\1\\2\end{pmatrix} \quad \left(\begin{array}{ccc|c} 1&1&0&3\\ 2&0&5&1\\ -1&1&-5&2\end{array}\right)$$

↳ liegt auf Gerade

$$\Rightarrow \begin{array}{l} \mathrm{I} \\ \mathrm{II} \\ \mathrm{III}-\mathrm{I}+\mathrm{II}\end{array} \left(\begin{array}{ccc|c} 1&1&0&3\\ 2&0&5&1\\ 0&0&0&\underbrace{2-3+1}_{0}\end{array}\right) \Rightarrow 0\cdot x+0\cdot y+0\cdot z=0$$

$$\Rightarrow 0=0 \checkmark \quad :D$$

(1) $x+y=3 \Rightarrow x=3-y$

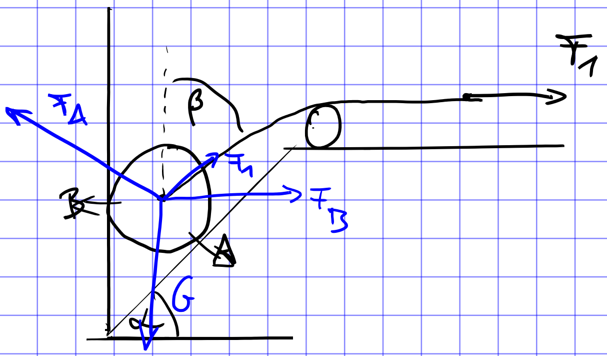
(2) $2x+5z=1 \Rightarrow 6-2y+5z=1 \Rightarrow 5z=-5+2y$

$$\Rightarrow z=-1+\frac{2}{5}y$$

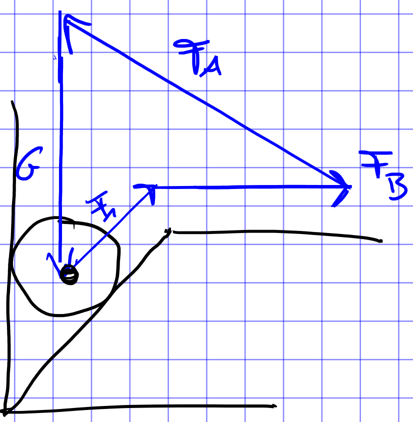
$$\vec{x}=\begin{pmatrix}3-y\\ y\\ -1+\frac{2}{5}y\end{pmatrix}=\begin{pmatrix}3\\0\\-1\end{pmatrix}+y\cdot\begin{pmatrix}-1\\1\\\frac{2}{5}\end{pmatrix} \Rightarrow \text{Gerade}$$

Nachtrag:

Aufgabe 7:



$\beta=60°$; $\alpha:=50°$; $F_G=500N=G$



3. Newton: actio = reactio

$$F_{AB}=-F_{BA}$$

$$\vec{F} = \begin{pmatrix} F_x \\ F_y \end{pmatrix}$$

$$F_{1x} = F_1 \sin(\beta) \qquad F_{1y} = F_1 \cdot \cos(\beta)$$

$$F_{Ax} = -F_A \sin(\alpha) \qquad F_{Ay} = F_A \cos(\alpha)$$

$$\sum F_x \stackrel{!}{=} 0 \qquad F_1 \cdot \sin(\beta) - F_A \sin(\alpha) + F_B = 0$$

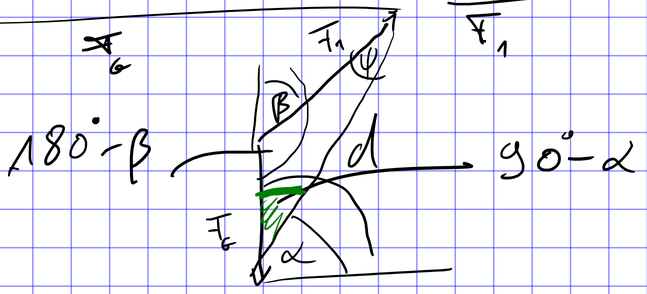
$$\sum F_y \stackrel{!}{=} 0 \qquad F_1 \cos(\beta) + F_A \cdot \cos(\alpha) - G = 0$$

$$F_A = \frac{G - F_1 \cdot \cos(\beta)}{\cos(\alpha)} = 622,3N$$

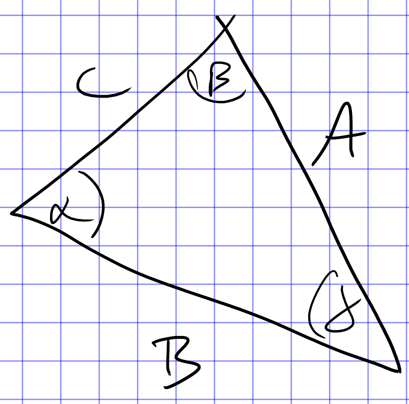
$$F_B = F_A \sin(\alpha) - F_1 \cdot \sin(\beta) = 303,5N$$

d.) $$F_A = \frac{500N - 200N \cos(60°)}{\cos(50°)} = \varphi$$

$$\frac{\sin(180° - (180° - \beta) - (90° - \alpha))}{F_G} = \frac{\sin(90° - \alpha)}{F_1}$$



$$F_1 = \frac{F_G \cdot \sin(90° - \alpha)}{\sin(180° - (180° - \beta) - (90° - \alpha))}$$



$$\frac{\sin(\alpha)}{A} = \frac{\sin(\beta)}{B}$$

$$= \frac{\sin(\gamma)}{C}$$