Aufgabe 3: $Re = \frac{\rho_0 \cdot v \cdot L}{\eta}$

geg:

$d_R = 4mm \rightarrow r_R = 2 \cdot 10^{-3}\ m$

$d_N = 10\mu m \rightarrow r_N = 5 \cdot 10^{-6}\ m$

$\rho_W = 1 \frac{g}{cm^3} = 1.000 \frac{kg}{m^3}$

$\rho_L = 1{,}2 \frac{kg}{m^3}$

$\eta_L = 0{,}02\ mPa \cdot s = 0{,}02 \cdot 10^{-3}\ Pa \cdot s$

$\rho = \frac{m}{V} \Leftrightarrow m = \rho_W \cdot V$

$m = \rho_W \cdot \frac{4}{3} \pi r^3$

$m_N = 5{,}236 \cdot 10^{-13} kg$

$m_R = 3{,}351 \cdot 10^{-5} kg$

(1) Newton

$$F_G = F_N$$

$$m \cdot g = \frac{1}{2} \rho_L \cdot v^2 \cdot c_w \cdot A$$

$$\Leftrightarrow \quad v = \sqrt{\frac{2mg}{\rho_L \cdot c_w \cdot A}} \qquad A = \pi r^2$$

$$\Leftrightarrow \quad v = \sqrt{\frac{2 \cdot \frac{4}{3} \pi \cdot r^3 \cdot \rho_w \cdot g}{\rho_L \cdot c_w \cdot \pi r^2}}$$

$$\Leftrightarrow \quad v = \sqrt{\frac{\frac{8}{3} \cdot r \cdot \rho_w \cdot g}{\rho_L \cdot c_w}}$$

a.) Regentropfen: $v = \sqrt{\dfrac{\frac{8}{3} \cdot 2 \cdot 10^{-3} m \cdot 1.000 \frac{kg}{m^3} \cdot 9{,}81 \frac{m}{s^2}}{1{,}2 \frac{kg}{m^3} \cdot 0{,}45}}$

$= \dfrac{2\sqrt{218}}{3} \approx 9{,}843 \frac{m}{s}$

$(= 35{,}435 \frac{km}{h})$

b.) Nebel: $v = \dfrac{\sqrt{218}}{30} \approx 0{,}492 \frac{m}{s}$

$(= 1{,}771 \frac{km}{h})$

② Stoke:

$F_G = F_S$

$m \cdot g = 6\pi \eta_L \cdot r \cdot v$

$v = \dfrac{mg}{6\pi \eta_2 \cdot r}$

$v = \dfrac{2}{9} \cdot \dfrac{r^2 \cdot \rho_W \cdot g}{\eta_L}$

a.) Regentropfen: $v = 436 \frac{m}{s}$ $(\approx 1.570 \frac{km}{h})$

b.) Nebel: $v \approx 2{,}725 \cdot 10^{-3} \frac{m}{s}$

$(= 9{,}81 \cdot 10^{-3} \frac{km}{h})$

Blatt 12

i) a) $z_1 = 1 + i$ $\quad z = \underset{\mathrm{Re}(z)}{a} + \underset{\mathrm{Im}(z)}{b}i$ $\quad |z| = \sqrt{a^2 + b^2}$

$\mathrm{Re}(z) = 1 = \mathrm{Im}(z)$ $\quad |z_1| = \sqrt{1^2 + 1^2} = \sqrt{2}$

b) $z_2 = 2 - 3i$

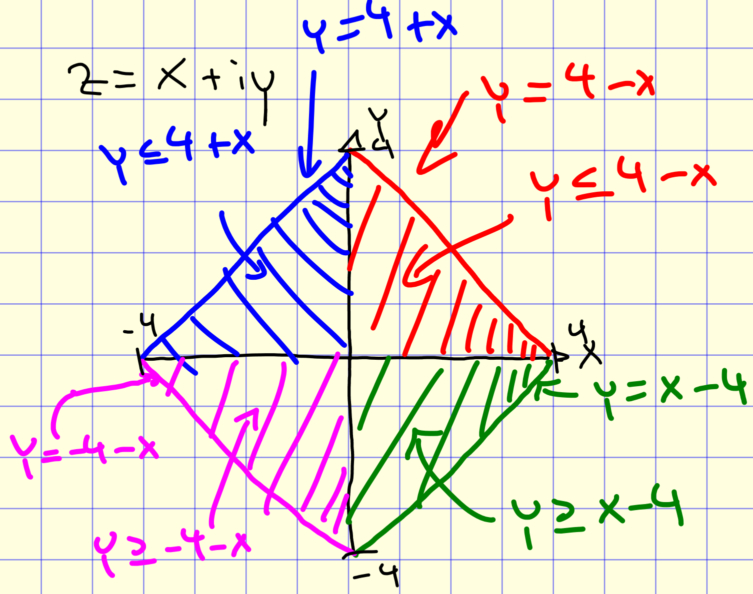
$\mathrm{Re}(z_2) = 2$ $\quad \mathrm{Im}(z_2) = -3$ $\quad |z_2| = \sqrt{2^2 + (-3)^2} = \sqrt{4 + 9} = \sqrt{13}$

c) $z_3 = \sqrt{3} + i$

$|z_3| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$

$Re(z_3) = \sqrt{3}$ $\quad Im(z_3) = 1$

ii) a) $|Re(z)| + |Im(z)| \le 4$ $\quad z = x + iy$



$|x| + |y| \le 4 \quad | -|x|$

$\Rightarrow |y| \le 4 - |x|$

Fall 1: $x \ge 0 \Rightarrow |x| = x \Rightarrow |y| \le 4 - x$

a) $y \ge 0 \Rightarrow |y| = y \Rightarrow y \le 4 - x$

$\hookrightarrow x \ge 0 \wedge y \ge 0 \Rightarrow y \le 4 - x$

b) $y < 0 \Rightarrow |y| = -y \Rightarrow -y \le 4 - x \quad | \cdot (-1)$

$\Rightarrow y \ge -4 + x = x - 4$

$\hookrightarrow x \ge 0 \wedge y < 0 \Rightarrow y \ge x - 4$

Fall 2: $x < 0 \Rightarrow |x| = -x \Rightarrow |y| \le 4 + x$

a) $y \ge 0 \Rightarrow |y| = y \Rightarrow y \le 4 + x$

$\hookrightarrow x < 0 \wedge y \ge 0 \Rightarrow y \le 4 + x$

b) $y < 0 \Rightarrow |y| = -y \Rightarrow -y \le 4 + x \quad | \cdot (-1)$

$\Rightarrow y \ge -4 - x$

$\hookrightarrow x < 0 \wedge y < 0 \Rightarrow y \ge -4 - x$

b) $|z| \leq 2 \cdot \mathrm{Re}(z)$ $\qquad z = x + iy$

$|z| = \sqrt{x^2+y^2}$

$\Rightarrow \sqrt{x^2+y^2} \leq 2 \cdot x \quad | ()^2$

$\Rightarrow x^2+y^2 \leq 4x^2 \quad | -x^2$

$\Rightarrow y^2 \leq 3x^2 \quad | \sqrt{\ }$

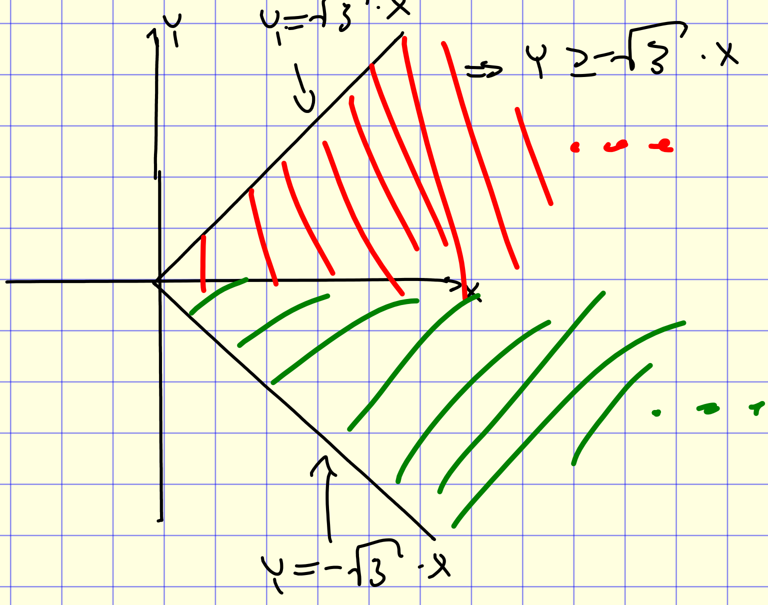
$\Rightarrow \sqrt{y^2} \leq \sqrt{3}\sqrt{x^2}$

$\Rightarrow |y| \leq \sqrt{3}\underbrace{|x|}_{=x} \Rightarrow |y| \leq \sqrt{3} \cdot x$

Fall 1: $y \geq 0 \Rightarrow |y| = y \Rightarrow y \leq \sqrt{3} \cdot x$

Fall 2: $y < 0 \Rightarrow |y| = -y \Rightarrow -y \leq \sqrt{3} \cdot x \quad | \cdot (-1)$

$\Rightarrow y \geq -\sqrt{3} \cdot x$



iii)

$x^2 - 10x + 34 = 0$

$\Rightarrow x^2 - 10x + 25 - 25 + 34 = 0$

$\Rightarrow (x-5)^2 + 9 = 0 \quad | -9 \quad | \sqrt{\ }$

$\Rightarrow x - 5 = \sqrt{-9} = \underbrace{\sqrt{-1}}_{i} \cdot \underbrace{\sqrt{9}}_{\pm 3}$

$= \pm 3 \cdot i \quad | +5 \quad \Rightarrow x = 5 \pm 3i$

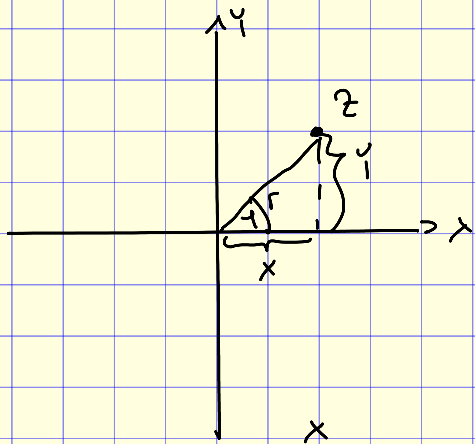
iv) a) $(1+i)^5$

NR: $(1+i)^2 = \cancel{1}^2 + 2\cdot i \cancel{-1} = 2i$

$= (1+i)^2(1+i)^2(1+i)$

$= 2i \cdot 2i\,(1+i)$

$= -4(1+i) = -4-4i$



$|1+i| = \sqrt{2}$

$x = 1 = y \qquad e^{i\varphi} = \cos(\varphi) + i\sin(\varphi) \quad | \cdot r$

$\Rightarrow re^{i\varphi} = \underbrace{r\cos(\varphi)}_{x} + \underbrace{i\cdot r\cdot\sin(\varphi)}_{y}$

$\Rightarrow \underbrace{re^{i\varphi}}_{\text{Polardarstellung}} = x + iy = z$

$\cos(\varphi) = \frac{x}{r} \quad | \cdot r$

$\Rightarrow x = \cos(\varphi)\cdot r$

$1+i$ in Polardarstellung

$\sin(\varphi) = \frac{y}{r} \quad | \cdot r$

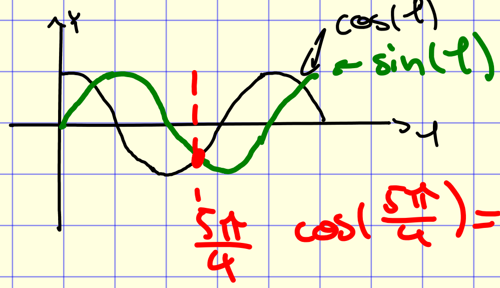
$r = \sqrt{x^2+y^2} = \sqrt{2}$

$\tan(\varphi) = \frac{y}{x} \quad | \arctan()$

$\Rightarrow y = \sin(\varphi)\cdot r$

$\Rightarrow \varphi = \arctan\left(\frac{1}{1}\right) = \frac{\pi}{4} = 45°$

$(1+i)^5 = \left(\sqrt{2}\cdot e^{i\cdot\frac{\pi}{4}}\right)^5 = 2^{\frac{5}{2}} \cdot \underbrace{e^{i\frac{5\pi}{4}}}_{\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}} - i\cdot\frac{1}{\sqrt{2}}}$



$\cos\left(\frac{5\pi}{4}\right) = \sin\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

$(1+i)^5 = 2^{\frac{5}{2}} \cdot \left(-\frac{1}{\sqrt{2}} - i\cdot\frac{1}{\sqrt{2}}\right)$

$= \underbrace{\frac{2^{\frac{5}{2}}}{2^{\frac{1}{2}}}}_{2^{\frac{5}{2}-\frac{1}{2}} = 2^{\frac{4}{2}} = 2^2 = 4} \cdot (-1-i) = -4-4i$

b) $\frac{1}{2+3i}$

NR: $(2+3i)(2-3i) = 4-6i+6i+9$
$= 4+9$
$= 13$

$= \frac{1}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{2-3i}{(2+3i)(2-3i)} = \frac{2-3i}{13}$

$= \frac{2}{13} - \frac{3}{13} \cdot i$

c) $\frac{(-i+1)^3}{i+2}$

$(-i+1)^3 = \underbrace{(-i+1)(-i+1)}(-i+1) = -2i(-i+1) = -2-2i$

$\cancel{i^2} - 2i \cancel{+1} = -2i$

$\Rightarrow \frac{-2-2i}{i+2} = \frac{-2-2i}{2+i} \cdot \frac{(2-i)}{(2-i)} = \frac{(-2-2i)(2-i)}{4+1} = \frac{-4+2i-4i-2}{5}$

$(-2-2i)(2-i) = -4+2i-4i\underbrace{+2i^2}_{-2}$ $\qquad = -\frac{6}{5} - \frac{2}{5}i$

v) $x_n = a_n + ib_n \Rightarrow x = a+ib$

$x^3+27=0 \quad |-27$

$\Rightarrow x^3 = -27$

$\Rightarrow (a+ib)^3 = -27$

$\Rightarrow (a+ib)^2 (a+ib) = -27$

$\Leftrightarrow (a^2 + 2abi - b^2)(a+ib) = -27$

$\Leftrightarrow a^3 + a^2bi + 2a^2bi - 2ab^2 - b^2a - ib^3 = -27$

$\Rightarrow a^3 + 3a^2bi - 3ab^2 - ib^3 = -27$

$\Rightarrow \underbrace{a^3-3ab^2}_{a} + i\underbrace{(3a^2b - b^3)}_{b} = \underbrace{-27}_{a}$ "$+ib$"

$\Rightarrow$ ① $a^3 - 3ab^2 = -27 \Rightarrow a^3 - 3a\,3a^2 = -27 \Rightarrow -8a^3 = -27 \quad |:(-8) \quad |\sqrt[3]{\ }$

② $3a^2b - b^3 = 0 \quad |+b^3 \quad |:b$ $\qquad \Rightarrow a = \sqrt[3]{\frac{27}{8}}$

$\Rightarrow 3a^2 = b^2 \quad |\sqrt{\ }$ $\qquad z = \sqrt[3]{\frac{27}{8}} + i\sqrt{3} \cdot \sqrt[3]{\frac{27}{8}}$

$\Rightarrow b = \sqrt{3} \cdot a = \sqrt{3} \cdot \sqrt[3]{\frac{27}{8}}$

vi) $e^{i\varphi} = \cos(\varphi) + i\sin(\varphi)$

$z = a + ib = r \cdot e^{i\varphi}$, $r = \sqrt{a^2+b^2}$

$\ln(z)$ mit $z = 4+3i$?

$\ln(z) = \ln(r \cdot e^{i\varphi}) = \ln(r) + \ln(e^{i\varphi})$ $\qquad r = \sqrt{a^2+b^2} = \sqrt{4^2+3^2} = \sqrt{16+9}$

$= \ln(r) + i\varphi$ $\qquad = 5$

$= \ln(5) + i \cdot 0{,}644$ $\qquad \tan(\varphi) = \frac{b}{a} \quad | \arctan(\,)$

$\Rightarrow \varphi = \arctan\left(\frac{3}{4}\right) \approx 36{,}87°$

$\approx 0{,}644$

$\ln(4+3i) = \ln(5) + i \cdot 0{,}644$

vii) $z = 1+i$ in Polardarstellung

$+ z^5$? siehe iv) a)

<u>Aufgabe 2:</u>

$$\int \frac{\frac{\sin^2(x)}{\cos^4(x)} - \frac{4}{\cos^2(x)}}{\tan^3(x) - \tan^2(x) - 4\tan(x) + 4} \cdot dx$$

$u = \tan(x)$

$\frac{du}{dx} = \frac{1}{\cos^2(x)} \quad | \cdot dx \quad | \cdot \cos^2(x)$

$\Rightarrow dx = \cos^2(x) \cdot du$

$$= \int \frac{\left(\frac{\sin^2(x)}{\cos^{\cancel{4}\,2}(x)} - \frac{4}{\cancel{\cos^2(x)}}\right) \cdot \cancel{\cos^2(x)}}{\tan^3(x) - \tan^2(x) - 4\tan(x) + 4} \cdot du$$

$\frac{\sin^2(x)}{\cos^2(x)} = \tan^2(x)$

$$= \int \frac{u^2 - 4}{u^3 - u^2 - 4u + 4} du$$

Nullstellen von Nenner

$u^3 - u^2 - 4u + 4 = 0 \qquad u = 1$

$1^3 - 1^2 - 4 \cdot 1 + 4 = 0$

$0 = 0 \quad \Rightarrow \quad LF: (u-1)$

$$(u^3-u^2-4u+4):(u-1)=u^2-4 \quad \Rightarrow (u-1)(u^2-4)$$
$$\begin{array}{l} -(u^3-u^2) \\ \hline \quad -4u+4 \\ \quad -(-4u+4) \\ \hline \qquad 0 \end{array}$$

einsetzen

$$\Rightarrow \int \frac{\cancel{u^2-4}}{(u-1)\cancel{(u^2-4)}}\,du = \int \frac{1}{u-1}\,du = \ln(|u-1|)$$
$$\overset{RS}{=} \ln(|\tan(x)-1|)+C$$

$$b)\ \int_{-1}^{1} \frac{x^2}{\sqrt{1-x}}\,dx = \int_{-1}^{1} \underset{f(x)}{x^2} \cdot \underset{g(x)}{\frac{1}{\sqrt{1-x}}}\,dx \qquad \int f(x)g(x)\,dx = f\cdot G - \int f'G$$

P.I.

$$= -2\cdot\sqrt{1-x}\cdot x^2\Big|_{-1}^{1} + 4\cdot\int_{-1}^{1} \underset{h(x)}{\sqrt{1-x}}\cdot \underset{b(x)}{x}\,dx \qquad -\int f'(x)\cdot G(x)\,dx$$
$$-\int 2x\cdot(-2)\cdot\sqrt{1-x}$$

$$NR:\ \int_{-1}^{1}\sqrt{1-x}\cdot x\,dx = -\frac{2}{3}(1-x)^{\frac{3}{2}}\cdot x + \frac{2}{3}\underbrace{\int (1-x)^{\frac{3}{2}}dx}_{-\frac{2}{5}(1-x)^{\frac{5}{2}}} \qquad -\cdot(-4)\int x\sqrt{1-x}\,dx = +4\int x\sqrt{1-x}\,dx$$

$$= -2\cdot\sqrt{1-x}\cdot x^2 - \frac{8}{3}(1-x)^{\frac{3}{2}} - \frac{16}{15}(1-x)^{\frac{5}{2}}\Big|_{-1}^{1}$$

$$= \frac{\sqrt{2}\cdot 14}{15} \approx 1{,}319$$