Aufgabe 1: Ausdrücke vereinfachen

$$\frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + c \cdot b}{b \cdot d}$$

j) $\frac{2-x}{4-x^2} + \frac{x+1}{x} + (-1)\frac{x+4}{x+2} + (+1)\frac{2}{x^2+2x}$

$= \frac{(2-x)\cdot x + (x+1)\cdot(4-x^2)}{(4-x^2)x} - \left(\frac{(x+4)(x^2+2x) + 2\cdot(x+2)}{(x+2)(x^2+2x)}\right)$

$= \frac{2x - x^2 + 4x - x^3 + 4 - x^2}{4x - x^3} - \frac{x^3 + 2x^2 + 4x^2 + 8x + 2x + 4}{x^3 + 4x^2 + 4x}$

$= \frac{(2x - x^2 + 4x - x^3 + 4 - x^2)(x^3 + 4x^2 + 4x) - (x^3 + 2x^2 + 4x^2 + 8x + 2x + 4)(4x - x^2)}{(4x - x^3)(x^3 + 4x^2 + 4x)}$

$= \frac{2x^4 - x^5 + 4x^4 - x^6 + 4x^3 - x^5 + 8x^3 - 4x^4 + 16x^3 - 4x^5 + 16x^2 - 4x^4}{(4x - x^3)(x^3 + 4x^2 + 4x)}$

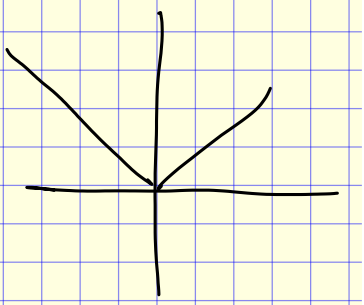
$+ \frac{8x^2 - 4x^3 + 16x^2 - 4x^4 + 16x - 4x^3 - 4x^4 - 8x^3 - 16x^3 - 32x^2 - 8x^2 - 16x}{(4x - x^3)(x^3 + 4x^2 + 4x)}$

$- \frac{-x^6 - 2x^5 - 4x^5 - 8x^4 - 2x^4 - 4x^3}{(4x - x^3)(x^3 + 4x^2 + 4x)}$

$= 0$

Aufgabe 2: $x \in \mathbb{R}$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



a) $\frac{|1-x|}{x+3} \geq -2 \quad | \cdot (x+3)$

Fall 1: $x+3$ positiv $\Leftrightarrow x+3 > 0 \Leftrightarrow x > -3$

$\Rightarrow |1-x| \geq -2 \cdot (x+3) = -2x - 6$

$$|1-x| = \begin{cases} 1-x, & x \leq 1 \\ -1+x, & x > 1 \end{cases}$$

Fall 1.1. $x \leq 1$

$1 - x \geq -2x - 6 \quad | + x | + 6$

$\Leftrightarrow 7 \geq -x \quad | \cdot (-1)$

$\Leftrightarrow x \geq -7$

Fall 1.2. $x > 1$

$x - 1 \geq -2x - 6 \quad | + 1 | + 2x$

$\Leftrightarrow 3x \geq -5 \quad | : 3$

$\Leftrightarrow x \geq -\frac{5}{3}$

Fall 2: $x+3$ negativ $\Leftrightarrow x + 3 < 0$

$|1-x| \leq -2x - 6$

Fall 2.1. $x \leq 1$

$1 - x \leq -2x - 6 \quad | + 2x | - 1$

$\Leftrightarrow x \leq -7$

Fall 2.2.: $x > 1$

$x - 1 \leq -2x - 6 \quad | + 1 | + 2x$

$\Leftrightarrow 3x \leq -5 \quad | : 3 \Leftrightarrow x \leq -\frac{5}{3}$

insgesamt:

1: 1.1.: $x > -3 \wedge x \leq 1 \wedge x \geq -7 \Rightarrow x \in (-3; 1]$

1.2.: $x > -3 \wedge x > 1 \wedge x \geq -\frac{5}{3} \Rightarrow x \in (1, \infty)$

2. 2.1.: $x < -3 \wedge x \leq 1 \wedge x \leq -7 \Rightarrow x \in (-\infty, -7]$

2.2.: $x < -3 \wedge x > 1 \wedge x \leq -\frac{5}{3} \Rightarrow x \in \{\}$

Zusammen:

$x \in (-\infty; -7] \cup (-3; 1] \cup (1, \infty)$

$= (-\infty; -7) \cup (-3, \infty) = \mathbb{R} \setminus (-7; -3]$ //

(1) $\frac{2 \cdot |x|}{x+3} \leq 1 \quad | \cdot (x+3)$

Fall 1: $x+3 > 0 \Leftrightarrow x > -3$

$2|x| \leq x+3$

Fall 1.1. $x \geq 0$

$2x \leq x+3 \quad | -x$

$\Leftrightarrow x \leq 3$

Fall 1.2. $x < 0$

$-2x \leq x+3 \quad | -x$

$\Leftrightarrow -3x \leq 3 \quad | : (-3)$

$\Leftrightarrow x \geq -1$

Fall 2: $x+3 < 0 \Leftrightarrow x < -3$

$2|x| \geq x+3$

Fall 2.1: $x \geq 0$

$2x \geq x+3 \quad | -x$

$\Leftrightarrow x \geq 3$

Fall 2.2. $x < 0$

$-2x \geq x+3 \quad | -x$

$\Leftrightarrow -3x \geq 3 \quad | : (-3)$

$\Leftrightarrow x \leq -1$

insgesamt:

1. 1.1: $x \geq -3 \wedge x \leq 3 \wedge x \geq 0 \Rightarrow x \in [0, 3]$ (=)

1.2.: $x > -3 \wedge x < 0 \wedge x \geq -1 \Rightarrow x \in [-1, 0)$

2. 2.1.: $x < -3 \wedge x \geq 0 \wedge x \geq 3 \Rightarrow x \in \{\}$

2.2.: $x < -3 \wedge x < 0 \wedge x \leq -1 \Rightarrow x \in (-\infty; -3)$

Zusammen:

$\Rightarrow x \in (-\infty; -3) \cup [-1; 0) \cup [0, 3] = (-\infty; -3) \cup [-1; 3]$ //

m) $|2x+4| \leq x+5$

Fall 1: $2x+4 \geq 0 \Leftrightarrow 2x \geq -4 \mid :2 \Leftrightarrow x \geq -2$

$2x+4 \leq x+5 \mid -x \mid -4$

$\Leftrightarrow x \leq 1$

Fall 2: $2x+4 < 0 \Leftrightarrow 2x < -4 \mid :2 \Leftrightarrow x < -2$

$-2x-4 \leq x+5 \mid -x \mid +4$

$\Leftrightarrow -3x \leq 9 \mid :-3$

$\Leftrightarrow x \geq -3$

insgesamt:

Fall 1: $x \geq -2 \wedge x \leq 1 \Rightarrow x \in [-2; 1]$

Fall 2: $x < -2 \wedge x \geq -3 \Rightarrow x \in [-3, -2)$

$\Rightarrow x \in [-3; 1]$

Aufgabe 1

i) $\frac{\mu - \nu}{\nu - \mu} = \frac{\mu - \nu}{(-1)(\mu - \nu)} \left(= \frac{\mu - \nu}{(-1)\cdot\mu - (-1)\cdot\nu} = \frac{\mu - \nu}{-\mu + \nu} = \frac{\mu - \nu}{\nu - \mu} \right)$

$= \frac{1}{-1} = -1$

a) $20x^2 \cdot \frac{3\alpha}{5x} - \frac{\alpha(x+6)}{3} = \alpha \cdot \left(20x^{\not 2} \cdot \frac{3}{5\not x} - \frac{x+6}{3} \right)$

$= \alpha \cdot (12x - \frac{x}{3} - 2)$

$= \alpha \cdot (\frac{36}{3}x - \frac{x}{3} - 2) = \alpha \cdot (\frac{35}{3}x - 2)$

b) $\frac{a^2 - b^2}{a+5} = \frac{(a+5)(a-b)}{a+5} = a - b$

c) $x^{3k+2} \cdot 3x^{4k+7} - 7x^{n-9-7k} = 21 \cdot x^{3k+2+4k+7+n-9-7k}$

$= 21 \cdot x^n$

d) $\left(\frac{x^2 y}{u^2 v^2}\right)^4 : \left(\frac{xy^3}{u^2 v}\right)^2 < \left(\frac{x^8 \cdot y^4}{u^8 v^8}\right) : \left(\frac{x y^6}{u^4 v^2}\right) = \frac{x^8 \cdot y^4 \cdot u^4 \cdot v^2}{u^8 \cdot v^8 \cdot x^2 \cdot y^6} = x^6 y^{-2} u^{-4} v^{-6}$

e) $(-a)^{-2}\, a = \frac{a}{(-a)^2} = \frac{a}{a^2} = \frac{1}{a}$

$\sqrt[n]{x} = x^{\frac{1}{n}}$

$a^{x^y} = a^{x \cdot y}$

f) $-a^{-2}\, a = -\frac{a}{a^2} = -\frac{1}{a}$

g) $\sqrt[5]{32 y^{10}} = (32 y^{10})^{\frac{1}{5}} = 2^{5 \cdot \frac{1}{5}}\, y^{10 \cdot \frac{1}{5}} = 2y^2$

h) $\sqrt[3]{\sqrt[4]{x^{24}}} = \left((x^{24})^{\frac{1}{4}}\right)^{\frac{1}{3}} = x^{24 \cdot \frac{1}{4} \cdot \frac{1}{3}} = x^2$

Aufgabe 2:

f) $\log_{10}(3x+4) = 3 \quad | 10^{(\,)}$

$10^{\log_{10}(3x+4)} = 10^3$

$3x+4 = 1000 \quad |-4 \quad :3$

$\Leftrightarrow x = 332$

Aufgabe 1:

k) $\frac{(x^2)^4 - x^{(2^4)}}{x^8} + x^8 = \frac{x^8 - x^{16}}{x^8} + x^8 = 1 \frac{x^8}{x^8} - \frac{x^{16}}{x^8} + x^8 = 1$

l) $\frac{x^{6n+2}\, x^{3-n}}{(x^2)^n (x^{n+3})^2} = \frac{x^{6n+2+3-n}}{x^{2n+2n+6}} = \frac{x^{5n+5}}{x^{4n+6}} = x^{5n+5-4n-6} = x^{n-1}$

Aufgabe 2:

a) $\frac{2x-1}{2-x} = \frac{7}{3x+4} \quad | \cdot (3x+4) \,|\cdot (2-x)$

$\Leftrightarrow (2x-1)(3x+4) = 7 \cdot (2-x)$

$\Leftrightarrow 6x^2 + 8x - 3x - 4 = 14 - 7x$

$\Leftrightarrow 6x^2 + 5x - 4 = 14 - 7x \quad | + 7x \quad | -14$

$\Leftrightarrow 6x^2 + 12x - 18 = 0 \quad | :6$

$\Leftrightarrow x^2 + 2x - 3 = 0$

$\Leftrightarrow x^2 + 2x + 1 - 1 - 3 = 0 \quad \Leftrightarrow (x+1)^2 - 4 = 0 \quad | +4 \quad | \sqrt{\ }$

$\Leftrightarrow x+1 = 2 \quad | -1 \quad \vee \quad x+1 = -2 \quad | -1$

$\Leftrightarrow x = 1 \qquad \Leftrightarrow x = -3$

$\Rightarrow \mathbb{L} = \{1, -3\}$

b) $\frac{x+1}{2x-4} = \frac{x+2}{x-2} \quad | \cdot (x-2) \quad | \cdot (2x-4)$

$\Leftrightarrow (x+1)(x-2) = (x+2)(2x-4)$

$\Leftrightarrow x^2 - 2x + x - 2 = 2x^2 \cancel{-4x + 4x} - 8 \quad | -2x^2 \; | +8$

$\Leftrightarrow -x^2 - x + 6 = 0 \quad | \cdot (-1)$

$\Leftrightarrow x^2 + x - 6 = 0$

$\Leftrightarrow x^2 + x + \frac{1}{4} - \frac{1}{4} - 6 = 0$

$\Leftrightarrow (x + \frac{1}{2})^2 - \frac{25}{4} = 0 \; | + \frac{25}{4} \; | \sqrt{}$

$\Leftrightarrow x + \frac{1}{2} = \frac{5}{2} \; \vee \; x + \frac{1}{2} = -\frac{5}{2} \quad | -\frac{1}{2}$

$\Leftrightarrow x = \cancel{\frac{4}{2}} \; \vee \; x = -\frac{6}{2}$

$= 2 \qquad = -3$

c) $2 - 3 \cdot (7 - 4x) = 5x - 7 + 2 \cdot (4x + 3)$

$\Leftrightarrow 2 - 21 + 12x = 5x - 7 + 8x + 6$

$12x - 19 = 13x - 1 \quad | -12x \; | +19$

$\Leftrightarrow 0 = x + 18 \quad | -18 \Leftrightarrow x = -18$

d) $x \cdot (x - 15)(x + 23) = 0 \Leftrightarrow x \in \{0, 15; -23\}$