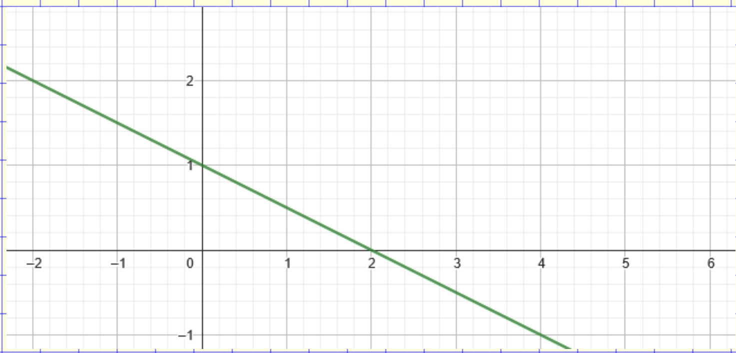
Aufgabe 1:

i.)

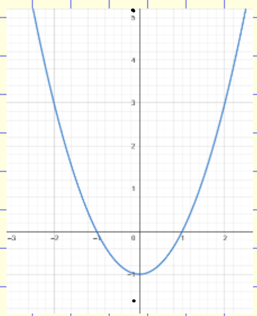


a.) $D = \mathbb{R}$

$W = \mathbb{R}$

bijektiv

$f(x) = -\frac{1}{2}x + 1$



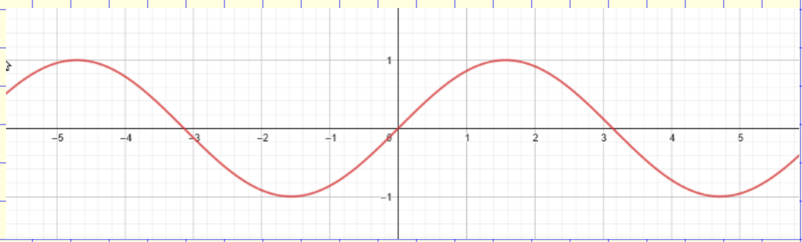
b.) $f(x) = x^2 - 1$

$D = [0;\infty)$

$W = [-1;\infty)$

bijektiv

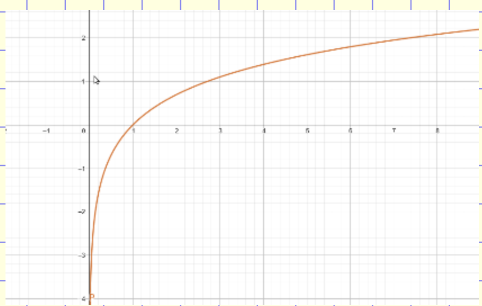
surjektiv



c.) $f(x) = \sin(x)$ surjektiv auf $\mathbb{R}$

$D = [-\frac{\pi}{2}; \frac{\pi}{2}]$

$W = [-1; 1]$



d.) $f(x) = \ln(x)$

$D = \mathbb{R}^+$

$W = \mathbb{R}$

bijektiv

ii) $f(x) = x^2 + \underbrace{2}_{p}x - \underbrace{15}_{q}$

a.) $D = \mathbb{R}$

$W = [-16;\infty)$ kommt von Rechnung:

① Hälfte von $p \rightarrow \frac{2}{2} = 1$

→ quadrieren & Addieren / Subtrahieren

$\Rightarrow \underbrace{x^2 + 2x + 1^2} - 1^2 - 15$

② Binom. Formel Rückwärts

$\Rightarrow (x+1)^2 \underbrace{-1^2 - 15}$

$\Rightarrow (x+1)^2 - 16$ Scheitelpunktform

↑ ↖ y-Koordinate

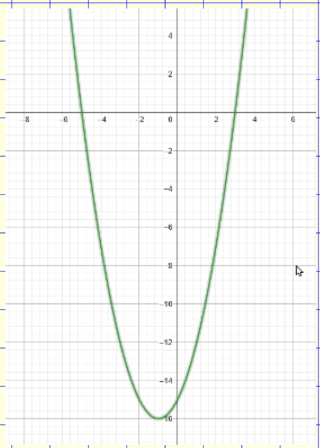
x-Koordinate (umgedrehtes VZ)

$f(x) = x^2 + 2x - 15 \overset{!}{=} 0$

$\longrightarrow$ pq-Formel $\Rightarrow x \in \{-5; 3\}$

b.) $f(x) = (x-3)(x+5)$

c.)



Mit Hand :)

d.) mo. wachsend: $x \in [-1; \infty)$

mo. fallend: $x \in (-\infty; -1]$

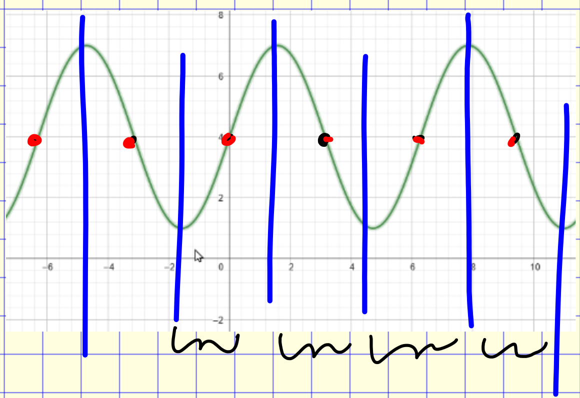
iii)

a.) $f(x) = 3\sin(x) + 4$

$D = \mathbb{R}$

$W = [1; 7]$

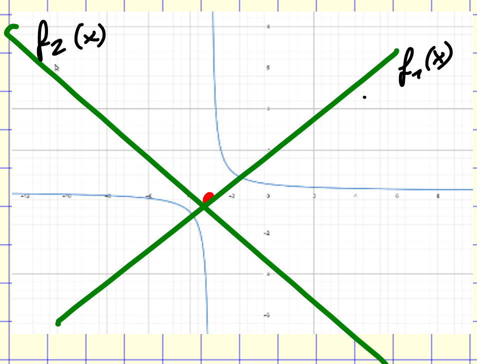
Symmetrie: - „Nullstellen": Punktsymmetrie



- Achsensymmetrie

$\rightarrow$ Maxima / Minima

b.) $f(x) = \frac{1}{x+3}$



bijektiver $D = \mathbb{R} \setminus \{-3\}$

$W = \mathbb{R} \setminus \{0\}$

Punktsymmetrie „dazwischen"

Achsensymmetrie:

$f_1(x) = x + 3$

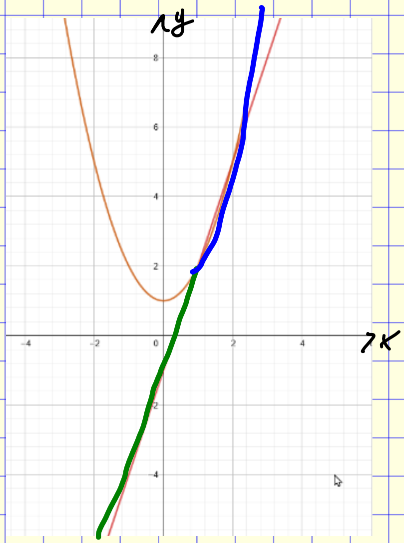
$f_2(x) = -x - 3$

c.) $f(x)=\begin{cases}3x-1 & , x<1\\ x^2+1 & , x\geq 1\end{cases}$

① affin linear Steigung 3, y-Achsenabschn. −1

② Parabelast nach oben verschoben

$D=\mathbb{R}$; $W=\mathbb{R}$



bijektiv ; keine einfache Symmetrie

iv)

a.) $f(x) = x\sin(x)$

für $x \Rightarrow -x$: $\Rightarrow -x\sin(-x)$

$\Rightarrow -x\cdot(-1)\sin(x)$

$\Rightarrow x\sin(x)$

$= f(x) \Rightarrow f(-x) = f(x) \Rightarrow$ AS

b.) $f(x) = \dfrac{e^x + e^{-x}}{2}$

$f(-x) = \dfrac{e^{-x}+e^{x}}{2} = \dfrac{e^x+e^{-x}}{2} = f(x) \Rightarrow$ AS

c.) $f(x) = \dfrac{(x^5+4x^3+2x)\sin^2(x)}{|x|\cos(x)}$

$f(-x) = \dfrac{((-x)^5+4(-x)^3+2(-x))\sin^2(-x)}{|-x|\cos(-x)}$

* da gerade, Symmetrie sin(x)

$= ((-x)^5 + 4(-x)^3 + \underbrace{2(-x)}\,)\underbrace{\sin^2(-x)}\cdot\dfrac{1}{\underbrace{|-x|}\cos(-x)}$

$= (-x^5 - 4x^3 - 2x)\ \sin^2(x)^{*}\ \dfrac{1}{|x|\cdot\cos(x)}$

$= -(x^5+4x^3+2x)\cdot\sin^2(x)\cdot\dfrac{1}{|x|\cos(x)}$

$= -\underbrace{\dfrac{(x^5+4x^3+2x)\sin^2(x)}{|x|\cos(x)}}_{f(x)} = -f(x) \Rightarrow$ P.S

$\Rightarrow f(-x) = -f(x)$

d.) $f(x) = (x+8)^3 - (x-8)^3$

$f(-x) = \underbrace{(-x+8)^3}_{} - (-x-8)^3$

$= -(x-8)^3 + (x+8)^3$

$= (x+8)^3 - (x-8)^3$

$= f(x)$

$\Rightarrow f(-x) = f(x) \Rightarrow$ A.S

$(-1)^{2,4,6} = 1$

$(-1)^{3,5,7} = -1$

Pascalsches Dreieck

$(a+b)^n$

1		$(a+b)^0$
1 1		$(a+b)^1$
1 2 1		$(a+b)^2$
1 3 3 1		$(a+b)^3$

e.) $f(x) = (x+8)^2 - (x-8)^2$

$f(-x) = (-x+8)^2 - (-x-8)^2$

$= \underbrace{(-(x-8))^2}_{*} - \underbrace{(-(x+8))^2}_{*}$

$= (x-8)^2 - (x+8)^2$

$= -[\underbrace{(x+8)^2 - (x-8)^2}_{f(x)}]$

$= -f(x)$ $\Rightarrow f(-x) = -f(x)$

$\Rightarrow$ P.S

*: Nur ≥ 0, da exponent gerade

f.) $f(x) = \ln\left(\sqrt{x^2+1} + x\right)$

$f(-x) = -f(x) \Rightarrow$ P.S

$\rightarrow$ Annahme $\Rightarrow$ Testen!

$f(-x) = -f(x) \quad | + f(x)$

$f(-x) + f(x) = 0$

$f(-x) + f(x) = \ln\left(\sqrt{(-x)^2+1} - x\right) + \ln\left(\sqrt{x^2+1} + x\right)$

$= \ln\left(\left(\sqrt{(-x)^2+1} - x\right)\left(\sqrt{x^2+1} + x\right)\right)$

$= \ln\left(x^2 + 1 - x\sqrt{x^2+1} + x\sqrt{x^2+1} - x^2\right)$

$= \ln(1)$

erfüllt $= \underline{\underline{0}}$

$\Rightarrow$ P.S

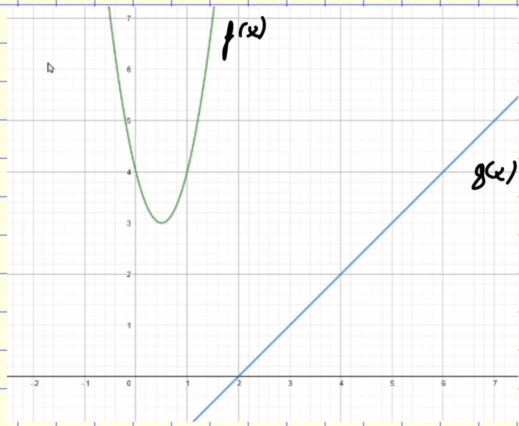
Aufgabe 2:

① $f(x) = 4x^2 - 4x + 4$

② $g(x) = x - 2$

③ $(f \circ g)(x)$

④ $(g \circ f)(x)$



① $f(x) = 4\left(x - \frac{1}{2}\right)^2 + 3$ $\quad D = \mathbb{R}$

$W = [3; \infty)$

② $\left.\begin{matrix} D = \mathbb{R} \\ W = \mathbb{R} \end{matrix}\right\}$ $\mathbb{R} \to \mathbb{R} : f(x) = \ldots$

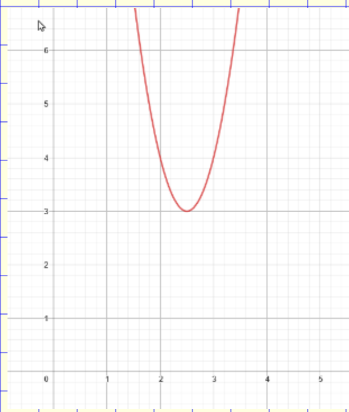
③ $(f \circ g)(x) = 4((x-2)^2) - 4(x-2) + 4$

$= 4(x^2 - 4x + 4) - 4x + 8 + 4$

$= 4x^2 - 16x + 16 - 4x + 8 + 4$

$= 4x^2 - 20x + 28$

$\Rightarrow 4\left(x - \frac{5}{2}\right)^2 + 3$

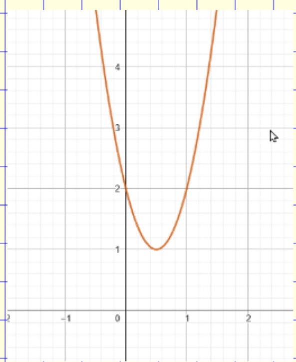


$\mathbb{R} \to [3; \infty)$

④ $(g \circ f)(x) = 4x^2 - 4x + 4 - 2$

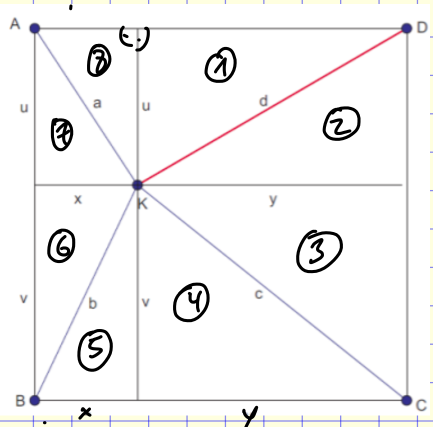
$= 4x^2 - 4x + 2$

$\Rightarrow 4\left(x - \frac{1}{2}\right)^2 + 1$



$\mathbb{R} \to [1; \infty)$

Aufgabe 3:



ges: $d(a, b, c) = ?$

⑦/⑧ $\Rightarrow a^2 = u^2 + x^2 \quad | -u^2$

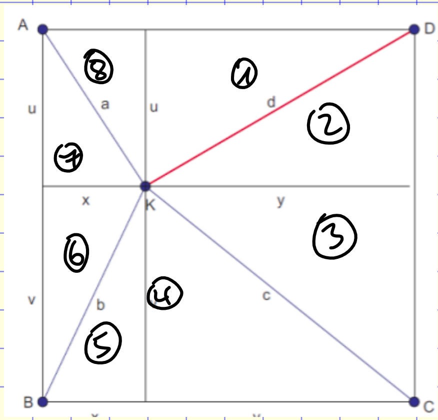
$x^2 = a^2 - u^2$

⑤/⑥ $\Rightarrow b^2 = v^2 + x^2 \quad | -x^2$

$v^2 = b^2 - x^2$ ⊛

③/④ $\Rightarrow c^2 = y^2 + v^2 \quad | -y^2$

$v^2 = c^2 - y^2$ ⊛



$v^2 = v^2$ (*)

$$b^2 - x^2 = c^2 - y^2$$

$$b^2 - (a^2 - u^2) = c^2 - y^2$$

$$b^2 - a^2 + u^2 = c^2 - y^2 \quad /+y^2 \; /+a^2 \; /-b^2$$

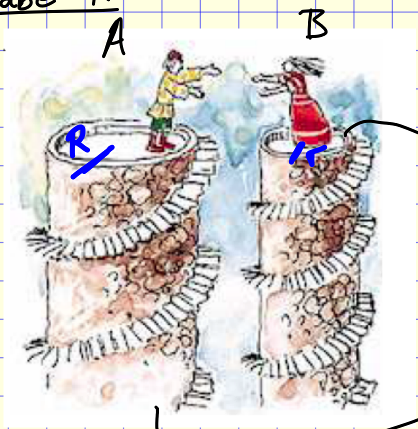
$$\rightarrow \quad u^2 + y^2 = c^2 + a^2 - b^2$$

$$(1)(2) \Rightarrow d^2 = y^2 + u^2$$

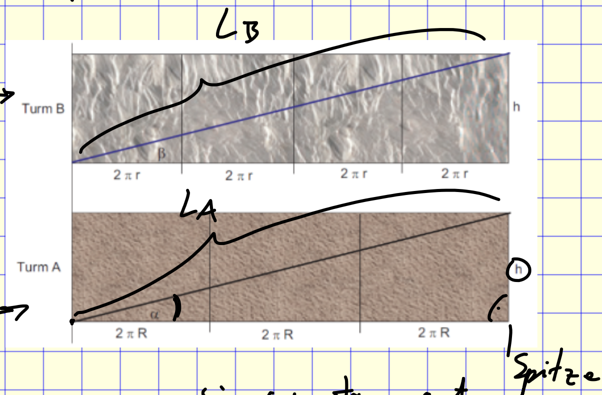
$$d^2 = c^2 + a^2 - b^2 \qquad /\sqrt{}$$

$$\Rightarrow d = \sqrt{c^2 + a^2 - b^2}$$

Aufgabe 4:



$R > r$



- $\sin(\alpha) = \frac{h}{L_A} \iff L_A = \frac{h}{\sin(\alpha)}$
- $\sin(\beta) = \frac{h}{L_B} \iff L_B = \frac{h}{\sin(\beta)}$

gleiche Steigung $\Rightarrow \alpha = \beta$

$\Rightarrow \sin(\alpha) = \sin(\beta)$

sin	cos	tan	cot
$\frac{G}{H}$	$\frac{A}{H}$	$\frac{G}{A}$	$\frac{A}{G}$

$\hookrightarrow \sin(\alpha) = \frac{\text{Gegen}}{\text{Hypo}}$

$\cos(\alpha) = \frac{\text{Ankat}}{\text{Hyp}}$

$\tan(\alpha) = \frac{\text{Geg}}{\text{An}}$; $\cot(\alpha) = \frac{\text{An}}{\text{Geg}}$

Nachtrag: Dezimalpräfixe