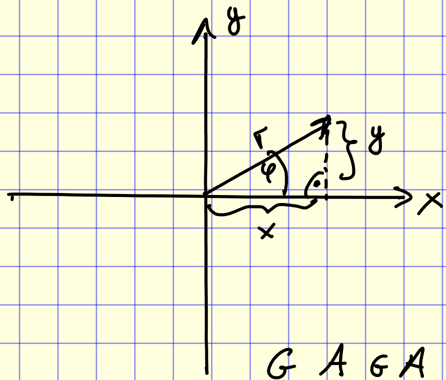
# Ergänzung Polar- & Kugelkoordinaten




$r :=$ Abstand zum Ursprung

Satz des Pythagoras:

$$x^2 + y^2 = r^2 \quad / \sqrt{\ }$$

$$\Rightarrow \boxed{r = \sqrt{x^2+y^2}} \qquad 1. \text{ Koordinate}$$

Winkel:

G A G A
H H A G

$$\Rightarrow \sin(\varphi) = \frac{G}{H} = \frac{y}{r} \quad / \cdot r \qquad \Rightarrow \cos(\varphi) = \frac{A}{H} = \frac{x}{r} \quad / \cdot r$$

$$\Rightarrow r \cdot \sin(\varphi) = y \qquad \Rightarrow r \cdot \cos(\varphi) = x$$

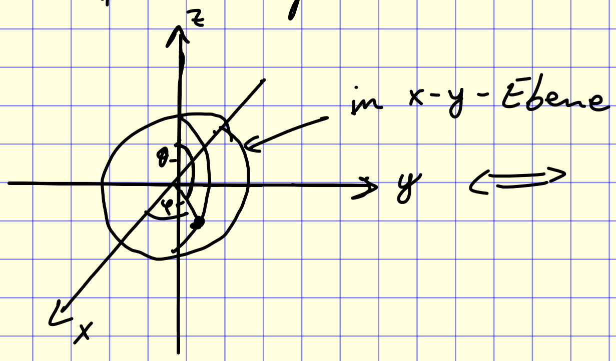
$$\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} r \cdot \cos(\varphi) \\ r \cdot \sin(\varphi) \end{pmatrix} = r \cdot \begin{pmatrix} \cos(\varphi) \\ \sin(\varphi) \end{pmatrix}$$

„Hintransformation" $x/y \to r/\varphi$

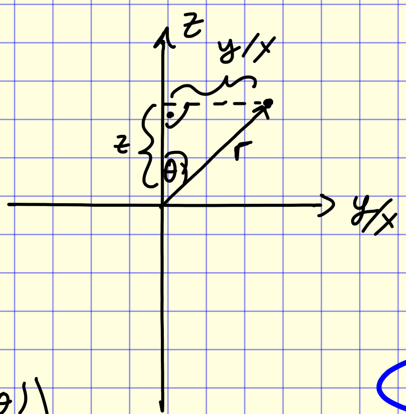
$$r = \sqrt{x^2+y^2} \qquad r \in (0; \infty]$$

$$\varphi = \arccos\left(\frac{x}{r}\right) \quad (\text{z.B.}) \qquad \varphi \in [0; 360)$$

„Rücktransformation" $r/\varphi \to x/y$ $\qquad \Rightarrow \varphi \in [0; 2\pi)$




$\Longleftrightarrow$




$$\sin(\theta) = \frac{G}{H} = \frac{y}{r} \quad / \cdot r$$

$$\Leftrightarrow r \cdot \sin(\theta) = y$$

$$\cos(\theta) = \frac{A}{H} = \frac{z}{r} \quad / \cdot r$$

$$r \cdot \cos(\theta) = z$$

$$\vec{r} = \begin{pmatrix} r \cdot \sin(\theta) \\ r \cdot \sin(\theta) \\ r \cdot \cos(\theta) \end{pmatrix} = r \cdot \begin{pmatrix} \sin(\theta) \\ \sin(\theta) \\ \cos(\theta) \end{pmatrix}$$

Da x-y-Ebene: $\sin(\theta) = \frac{x}{r} \quad / \cdot r$

$$r \cdot \sin(\theta) = x$$

Beschreibt nicht Position innerhalb der x-y-Ebene!

„Hintransformation" $x/y/z \to r, \theta, \varphi$: $\qquad \vec{r} = r \begin{pmatrix} \cos(\varphi)\sin(\theta) \\ \sin(\varphi)\sin(\theta) \\ \cos(\theta) \end{pmatrix}$

mit $\theta \in [0; \pi]$

$$r = \sqrt{x^2+y^2+z^2}, \quad r \in [0; \infty)$$

$$\varphi = \arccos\left(\frac{x}{r}\right), \quad \varphi \in [0; 2\pi)$$

$$\theta = \arccos\left(\frac{z}{r}\right), \quad \theta \in [0; \pi]$$