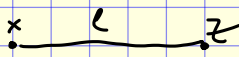


Blatt 4

1) a) $\frac{1}{4}$ d) -1
b) $\frac{1}{2}$ e) 1
c) 2 f) $x = -3$

2) a) $W = \mathbb{R}$ b) $W = [-1, \infty)$

3) 

$$\begin{aligned} l + l + 2 &= 12 \\ l &= 5 \text{ km} \end{aligned}$$

• y

4) i) a) $f(x) = x - 2$; $g(x) = 1 - 2x$

$$(f+g)(x) = -x - 1, D_{f+g} = \mathbb{R}$$

$$(f-g)(x) = 3x - 3, D_{f-g} = \mathbb{R}$$

$$(f \cdot g)(x) = -2x^2 + 5x - 2, D_{f \cdot g} = \mathbb{R}$$

$$\left(\frac{f}{g}\right)(x) = \frac{x-2}{1-2x}, D_{f/g} = \mathbb{R} \setminus \left\{\frac{1}{2}\right\}$$

b) $f(x) = \sin(x)$; $g(x) = \cos(x)$

$$(f+g)(x) = \sin(x) + \cos(x), D_{f+g} = \mathbb{R}$$

$$(f-g)(x) = \sin(x) - \cos(x), D_{f-g} = \mathbb{R}$$

$$(f \cdot g)(x) = \sin(x) \cos(x), D_{f \cdot g} = \mathbb{R}$$

$$\left(\frac{f}{g}\right)(x) = \frac{\sin(x)}{\cos(x)}, D_{f/g} = \left\{x \in \mathbb{R}, x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$$

$$k \in \mathbb{Z}$$

ii) a) $g(x) = \ln(x)$, $f(x) = x + 1$

b) $g(x) = x^2$, $f(x) = \frac{x+2}{x+1}$

c) $g(x) = x^2$, $f(x) = \cos(x)$

iii) $f(x) = x^2$, $g(x) = \sqrt{x}$, $h(x) = \frac{1}{x}$

a) $(g \circ f)(x) = \sqrt{x^2} = |x|$, $D_{g \circ f} = \mathbb{R}$
 $(f \circ g)(x) = (\sqrt{x})^2 = x$, $D_{f \circ g} = [0, \infty)$

b) $f \circ (g+h) = (\sqrt{x} + \frac{1}{x})^2 = x + \frac{2}{\sqrt{x}} + \frac{1}{x^2}$
 $D = (0, \infty)$

c) $h \circ (f \cdot g) = \frac{1}{x^2 \sqrt{x}} = \frac{1}{x^{\frac{5}{2}}}$, $D = (0, \infty)$

d) $f \circ (g \circ h) = \sqrt{\frac{1}{x^2}} = \frac{1}{x}$, $D = (0, \infty)$

5) b) $y = |\sin(x)|$ $W = [0, 1]$

c) $y = x + (x-1)$ $W = [1, \infty)$

d) $y = e^{\cos(x)}$ $W = [1/e, e]$

a) $y = e^x$ & $y = e^{-x}$ $W = (-\infty, 0) \cup (0, \infty)$

ii) a) $\frac{5}{6} \sin(7x+8)$

$$\sin(7x+8) = \sin(7x+8+2\pi) = \sin(7(x+\frac{2}{7}\pi)+8)$$

$$\frac{5}{6} \sin(7x+8) = \frac{5}{6} \sin(7(x+\frac{2}{7}\pi)+8)$$

$$f(x) = f(x+\frac{2}{7}\pi) : \text{periodisch mit kleinster Periode } \frac{2\pi}{7}$$

$$-1 \leq \sin(7x+8) \leq 1$$

$$-\frac{5}{6} \leq f(x) \leq \frac{5}{6}$$

$$|f(x)| \leq \frac{5}{6} : \text{beschränkt, kleinster Fktu-Wert } -\frac{5}{6}, \text{ größter Fktu-Wert } \frac{5}{6}$$

b) $\frac{1}{4+\sin(x)}$

$$f(x) = \frac{1}{4+\sin(x+2\pi)} = \frac{1}{4+\sin x} = f(x) \rightarrow \text{periodisch mit kleinster Periode } 2\pi$$

$$-1 \leq \sin x \leq 1$$

$$3 \leq 4+\sin x \leq 5$$

$$\frac{1}{5} \geq \frac{1}{4+\sin x} \geq \frac{1}{3}$$

$$|f(x)| \leq \frac{1}{3}, \text{ beschränkt, kleinster Fktu-Wert } \frac{1}{5}, \text{ größter Fktu-Wert } \frac{1}{3}$$

iii) $f(x) = x^3 - 39x - 70$

$$x = -2 \Rightarrow -8 + 78 - 70 = 0$$

$$(x^3 - 39x - 70) : (x+2) = x^2 - 2x - 35 = (x+5)(x-7)$$

$$\text{Extrema: } 3x^2 - 39 = 0 \rightarrow \pm \sqrt{\frac{39}{3}} = \pm 3,61$$

Wendepunkt: 0

6) $\overline{T}(t) \quad T_0 = T(t=0s) \quad T_\infty$

$$\Delta T(t) = \Delta T_0 \cdot e^{-kt} = (T_0 - T_\infty) e^{-kt}$$

$$\Delta T(t) = T(t) - T_\infty$$

$$\Delta T_0(t) = T_0 - T_\infty$$

g: $T_\infty = 20^\circ\text{C}$, $\overline{T}(t = \frac{1}{3}h) = 80^\circ\text{C}$, $T(t = 3h) = 30^\circ\text{C}$

$$\frac{\Delta T(3h)}{\Delta T(\frac{1}{3}h)} = \frac{30^\circ\text{C} - 20^\circ\text{C}}{80^\circ\text{C} - 20^\circ\text{C}} = \frac{10}{60} = \frac{1}{6} = \frac{e^{k \cdot 3}}{e^{-k \cdot \frac{1}{3}}} = e^{k(\frac{1}{3}-3)} = \frac{1}{6} \quad | \ln()$$

$$k \cdot (\frac{1}{3} - 3) = \ln(\frac{1}{6})$$

$$-\frac{8}{3} \cdot k = \ln(\frac{1}{6}) \quad | \cdot (-\frac{3}{8})$$

$$k = -\frac{3}{8} \cdot \ln(\frac{1}{6}) ; \quad [k] = \frac{1}{h}$$

$$\Delta T_0 = \Delta T(3h) \cdot e^{k \cdot 3h} = \Delta T(3h) \cdot e^{\ln(6 \frac{5}{3}) \cdot 3}$$

$$= \Delta T(3h) = 6 \frac{5}{3}$$

$$T_0 - T_A = \Delta T_0 = 10 \cdot 6 \frac{5}{3} \text{ } ^\circ\text{C} = 75,06 \text{ } ^\circ\text{C}$$

$$T_0 = 95,06 \text{ } ^\circ\text{C}$$

$$\Delta T(t) = 75,06 \text{ } ^\circ\text{C} \cdot e^{-0,672 \frac{t}{h}}$$

$$\Delta T(t) = 5 \text{ } ^\circ\text{C} ; \Delta T(t) \stackrel{!}{=} 25$$

$$t = \frac{\ln(\frac{5}{75})}{-0,672} \text{ h} \approx 4 \text{ h}$$