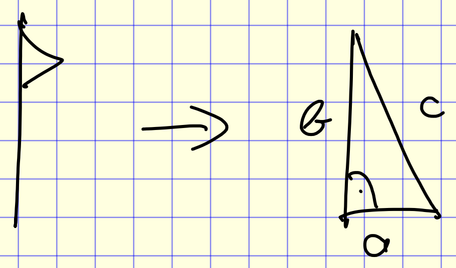
Vorrechnen Blatt 7

A1|



$a = 3m$

$b + c = 9m \iff c = 9m - b$

$a^2 + b^2 = c^2$

$9m^2 + b^2 = 81m^2 - 18m\, b + b^2$

$b = 4m$

2|

$M = 2(T_a + T_j)$ (1)

$T_a = 2 \cdot T_j$ (2)

$M + 9 = 3(T_j + 9)$ (3)

(2) in (1):

$M = 2 \cdot (2 \cdot T_j + T_j)$

(4) $M = 6 \cdot T_j$

~~(4) in (2)~~ (4) in (3): $6T_j + 9 = 3T_j + 27$

$3T_j = 18$

$T_j = 6 \qquad T_a = 12$

$M = 36$

3| i)

$2x + 3y = 8$

$x - y = -1$

$$\longrightarrow \begin{array}{cc} x & y \\ \end{array}$$
$$\left(\begin{array}{cc|c} 2 & 3 & 8 \\ 1 & -1 & -1 \end{array}\right)$$

$$\xrightarrow{2\mathrm{II}} \left(\begin{array}{cc|c} 2 & 3 & 8 \\ 2 & -2 & -2 \end{array}\right) \xrightarrow{\mathrm{I}-\mathrm{II}} \left(\begin{array}{cc|c} 0 & 5 & 10 \\ 2 & -2 & -2 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 0 & \overset{y}{1} & 2 \\ 2 & -2 & -2 \end{array}\right) \begin{array}{l} 0x + 1y = 2 \Rightarrow y = 2 \\ \Rightarrow 2x - 2\cdot 2 = -2 \Rightarrow x = 1 \end{array}$$

b) $$\left(\begin{array}{cc|c} 1 & -2 & -7 \\ 2 & 3 & 0 \end{array}\right) \xrightarrow{II - 2I} \left(\begin{array}{cc|c} 1 & -2 & -7 \\ 0 & 7 & 14 \end{array}\right)$$

$$\xrightarrow[II/7]{} \left(\begin{array}{cc|c} 1 & -2 & -7 \\ 0 & 1 & 2 \end{array}\right) \Rightarrow y = 2$$

$$1x - 2\cdot 2 = -7 \Leftrightarrow x = -3$$

c) $$\left(\begin{array}{ccc|c} 5 & 1 & 2 & 3 \\ -2 & 0 & 1 & -1 \\ 1 & 1 & 1 & 0 \end{array}\right) \xrightarrow[II + 2III]{I - 5III} \left(\begin{array}{ccc|c} 0 & -4 & -3 & 3 \\ 0 & 2 & 3 & -1 \\ 1 & 1 & 1 & 0 \end{array}\right)$$

$$\xrightarrow[2III - II]{I + 2II} \left(\begin{array}{ccc|c} 0 & 0 & 3 & 1 \\ 0 & 2 & 3 & -1 \\ 2 & 0 & -1 & 1 \end{array}\right) \xrightarrow[3III + I]{II - I} \left(\begin{array}{ccc|c} 0 & 0 & 3 & 1 \\ 0 & 2 & 0 & -2 \\ 6 & 0 & 0 & 4 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 0 & 0 & 1 & 1/3 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 2/3 \end{array}\right) \begin{array}{l} \Rightarrow x = \frac{2}{3} \\ \Rightarrow y = -1 \\ \Rightarrow z = 1/3 \end{array}$$

ii)

$$\left(\begin{array}{cc|c} 2 & 3 & b \\ 1 & a & 4 \end{array}\right) \xrightarrow{\mathrm{I}-2\mathrm{II}} \left(\begin{array}{cc|c} 0 & 3-2a & b-8 \\ 1 & a & 4 \end{array}\right)$$

$$\xrightarrow{2\mathrm{II}+\mathrm{I}} \left(\begin{array}{cc|c} 0 & 3-2a & b-8 \\ 2 & 3 & b \end{array}\right)$$

$$\xrightarrow[(3-2a)\,\mathrm{II}-3\mathrm{I}]{} \left(\begin{array}{cc|c} 0 & 3-2a & b-8 \\ 2\cdot(3-2a) & 0 & \underbrace{(3-2a)\,b-3(b-8)}_{-2ab+24} \end{array}\right)$$

$$\xrightarrow[\mathrm{II}/2]{} \left(\begin{array}{cc|c} 0 & 3-2a & b-8 \\ 3-2a & 0 & -ab+12 \end{array}\right)$$

Fall 1: $3-2a=0 \iff 2a=3 \Rightarrow \underline{\underline{a=\frac{3}{2}}}$

$0=b-8 \Rightarrow \underline{\underline{b=8}}$

Fall 2: $3-2a\neq 0$

$$\left(\begin{array}{cc|c} 0 & 1 & \frac{b-8}{3-2a} \\ 1 & 0 & \frac{-ab+12}{3-2a} \end{array}\right) \quad \begin{array}{l} \Rightarrow \underline{y=\frac{b-8}{3-2a}} \\ \Rightarrow \underline{x=\frac{-ab+12}{3-2a}} \end{array}$$

$$\frac{x}{y} = \frac{12 - ab}{b - 8}$$

b) $$\left(\begin{array}{ccc|c} 1 & 2 & -1 & s \\ 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{array}\right)$$

$$\xrightarrow{I - II} \left(\begin{array}{ccc|c} 0 & 1 & -1 & s-1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{array}\right) \xrightarrow{I - III} \left(\begin{array}{ccc|c} 0 & 0 & 0 & s-3 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{array}\right)$$

$$0 = s - 3 \Rightarrow s = 3$$

$$1x + 1y = 1 \Leftrightarrow \boxed{y = 1 - x}$$

$$0 + 1y - 1z = 2 \Leftrightarrow \boxed{z = -x - 1}$$

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ 1-x \\ -x-1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + x \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

$$x + 2 - 2x + x + 1 = 3$$

$$3 = 3$$

4

	X	Y	Z
Schubladen	6	4	6
Böden	12	12	14
Türen	2	3	4

$$\rightarrow \begin{pmatrix} 6 & 4 & 6 \\ 12 & 12 & 14 \\ 2 & 3 & 4 \end{pmatrix}$$

$$\left.\begin{array}{l} x = 15 \\ y = 9 \\ z = 6 \end{array}\right\} \Rightarrow \begin{pmatrix} 15 \\ 9 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} 6 & 4 & 6 \\ 12 & 12 & 14 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 15 \\ 9 \\ 6 \end{pmatrix} = \begin{array}{c} \begin{pmatrix} 162 \\ 372 \\ 81 \end{pmatrix} \end{array} \begin{array}{l} \leftarrow \text{Schubladen} \\ \leftarrow \text{Böden} \\ \leftarrow \text{Türe} \end{array}$$

$$\begin{pmatrix} 6 \\ 4 \\ 6 \end{pmatrix} \begin{pmatrix} 15 \\ 9 \\ 6 \end{pmatrix} = 90 + 36 + 36 = 162$$

$$\begin{pmatrix} 12 \\ 12 \\ 14 \end{pmatrix} \begin{pmatrix} 15 \\ 9 \\ 6 \end{pmatrix} = 12 \cdot 15 + 12 \cdot 9 + 14 \cdot 6 = 372$$

$$\begin{array}{c} x \\ y \\ z \end{array} \begin{pmatrix} 6 & 12 & 2 \\ 4 & 12 & 3 \\ 6 & 14 & 4 \end{pmatrix}$$

$$\left.\begin{array}{l} x = 15 \\ y = 9 \\ z = 6 \end{array}\right\} \begin{pmatrix} 15 \\ 9 \\ 6 \end{pmatrix} \begin{array}{c} x \\ y \\ z \end{array}$$

$$\Rightarrow \begin{pmatrix} 15 & 9 & 6 \end{pmatrix}$$

$$m\uparrow \begin{pmatrix} 15 & 9 & 6 \end{pmatrix} \underset{\xrightarrow{\quad n \quad}}{\begin{pmatrix} 6 & 12 & 2 \\ 4 & 12 & 3 \\ 6 & 14 & 4 \end{pmatrix}} = m\uparrow \underset{\xrightarrow{\quad n \quad}}{\begin{pmatrix} 162 & 372 & 81 \end{pmatrix}}$$

ü) a)

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 \\ 4 & -3 & -2 \end{pmatrix} = \begin{pmatrix} 6 & -2 & -1 \\ 8 & -6 & -4 \\ 6 & 8 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \end{pmatrix} = 2 + 4 = 6$$

b)

$$\begin{pmatrix} -2 & 3 & 1 \\ 1 & 1 & 1 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} -3 & 5 & -2 \\ 1 & 2 & -3 \\ 2 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix}$$

$$= 11 \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 11 \cdot \mathbb{1}$$

$$\underline{\mathbb{1}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\underline{\mathbb{1}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(iv) $A\vec{x} = \vec{b}$ $\qquad \vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$

$$A = \begin{pmatrix} 2 & 3 \\ 4 & -3 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\vec{b} = (3,1)^T = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\begin{matrix} 2x + 3y = 3 \\ 4x - 3y = 1 \end{matrix} \longrightarrow \left(\begin{array}{cc|c} 2 & 3 & 3 \\ 4 & -3 & 1 \end{array}\right)$$

$$\xrightarrow{I \cdot 2} \left(\begin{array}{cc|c} 4 & 6 & 6 \\ 4 & -3 & 1 \end{array}\right) \xrightarrow{I - II} \left(\begin{array}{cc|c} 0 & 9 & 5 \\ 4 & -3 & 1 \end{array}\right)$$

$$0x + 9y = 5$$

$$\underline{y = \frac{5}{9}}$$

$$4x - \frac{3 \cdot 5}{9} = 1 \iff 4x - \frac{15}{9} = 1$$

$$4x = \frac{24}{9}$$

$$x = \frac{6}{9} = \underline{\underline{\frac{2}{3}}}$$

b) $A \vec{x} = \vec{b}$ $\qquad \vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 2 & 4 & 5 & 1 \\ 3 & 5 & 6 & -2 \end{array}\right) \xrightarrow{\mathrm{III} - 3\mathrm{I}} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 2 & 4 & 5 & 1 \\ 0 & -1 & -3 & -11 \end{array}\right)$$

$$\xrightarrow{\mathrm{II} - 2\mathrm{I}} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 0 & -1 & -5 \\ 0 & -1 & -3 & -11 \end{array}\right) \begin{array}{l} \Rightarrow \underline{z = 5} \\ \Rightarrow 0x - y - 15 = -11 \end{array}$$

$$-y = 4$$

$$\underline{y = -4}$$

$$x - 8 + 15 = 3$$

$$x + 7 = 3$$

$$\underline{x = -4}$$

$$A \vec{x} = \vec{b}$$

5 $a + b + c + d = 2000$ ①

$$\left.\begin{aligned} x &= e + a \\ x &= b - e \\ x &= c \cdot e \\ x &= \frac{d}{e} \end{aligned}\right\} \quad e + a = b - e = c \cdot e = \frac{d}{e}$$

$a < 100 \qquad a \in \mathbb{Z}$

$$a = c \cdot e - e$$
$$b = c \cdot e + e$$
$$d = c \cdot e \cdot e$$

$$☆ = c \cdot (e + e + e^2 + 1)$$
$$= c \cdot (c^2 + 2e + 1)$$

Einsetzen in ①:

$$2000 = \overbrace{(c \cdot e - e)}^{a} + \overbrace{(c \cdot e + e)}^{b} + c + \overbrace{c \cdot e \cdot e}^{d} \Rightarrow e^2$$

$$2000 = c \cdot e + c \cdot e + c \cdot e^2 + c \quad ☆$$

$$2000 = c \cdot (e+1) \cdot (e+1) \leftarrow$$

$$\boxed{2000 = c \cdot (e+1)^2} \Leftrightarrow c = \frac{2000}{(e+1)^2} \quad ☆$$

$$a < 100 \Leftrightarrow c \cdot e - e < 100 \Rightarrow \boxed{e(c-1) < 100} \quad \triangle$$

$$\Rightarrow ☆ \text{ in } \triangle: \quad e\left(\frac{2000}{(e+1)^2} - 1\right) < 100 \qquad a \in \mathbb{Z}$$

$$\Rightarrow e + 1 = 20, \quad c = 5, \quad a = e(c-1) = 19 \cdot 4$$

$$a = 76$$

$100 - 76 = \underline{\underline{24}}$ Knöpfe fehlen!

iii) $$A = \begin{pmatrix} 2 & 3 \\ 4 & -3 \end{pmatrix}$$

$$A \cdot A^{-1} = A^{-1} \cdot A = \mathbb{1}$$

$$\left(\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 4 & -3 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{II - 2I} \left(\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 0 & -9 & -2 & 1 \end{array}\right) \xrightarrow{3I + II} \left(\begin{array}{cc|cc} 6 & 0 & 1 & 1 \\ 0 & -9 & -2 & 1 \end{array}\right)$$

$$\xrightarrow[II/-9]{I/6} \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{6} & \frac{1}{6} \\ 0 & 1 & \frac{2}{9} & -\frac{1}{9} \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} \\ \frac{2}{9} & -\frac{1}{9} \end{pmatrix}$$

$$A \cdot A^{-1} = \mathbb{1}$$

Bei 2×2 Matrix:

$$A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$