

Aufgabe 3:

a.)  $1 + 3 + 5 + 7 + 9 + \dots = \sum_{k=0}^{\infty} (2k+1) = \sum_{k=1}^{\infty} (2k-1)$

b.)  $1 + 4 + 9 + 16 + 25 + \dots = \sum_{k=1}^{\infty} (k^2)$

c.)  $3 + 7 + 11 + 15 + \dots = \sum_{k=0}^{\infty} (3+4k) = \sum_{k=1}^{\infty} (4k-1)$

d.)  $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \dots = \sum_{k=0}^{\infty} \frac{1}{5^{k+1}} = \sum_{k=1}^{\infty} \frac{1}{5^k}$

e.)  $2 + 3 + 2 + 3 + 2 + \dots = \sum_{k=1}^{\infty} \frac{5}{2} + \frac{1}{2} \cdot (-1)^k$  „Alternierend“

f.)  $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \sum_{k=0}^{\infty} \frac{1}{2^k} \in \mathbb{C}$

g.)  $1 + \frac{1}{2^z} + \frac{1}{3^z} + \frac{1}{4^z} + \dots = \zeta(z) = \sum_{k=1}^{\infty} \frac{1}{k^z}$

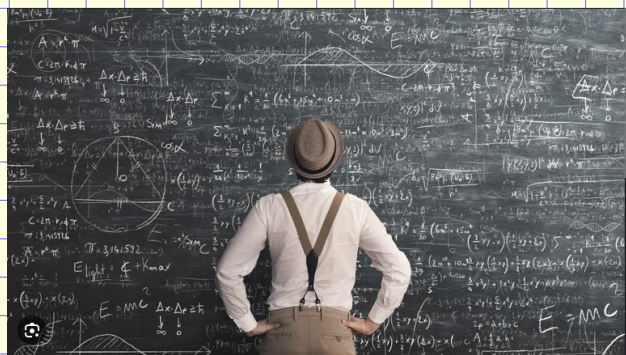
„Riemannsche Zeta-Funktion“

Riemannsche Vermutung: Alle nichttriviale NS haben den Realteil  $\frac{1}{2}$ .



↑  
Millennium Falke

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↑  
Millennium Probleme

Die Liste enthält die folgenden sieben Probleme:

1. der Beweis der Vermutung von Birch und Swinnerton-Dyer aus der Zahlentheorie,
2. der Beweis der Vermutung von Hodge aus der algebraischen Geometrie,
3. Analyse von Existenz und Regularität von Lösungen des Anfangswertproblems der dreidimensionalen inkompressiblen Navier-Stokes-Gleichungen.
4. die Lösung des P-NP-Problems der Informatik,
5. der Beweis der Poincaré-Vermutung in der Topologie (2002 gelöst von Grigori Jakowlewitsch Perelman, die Vermutung trifft zu),
6. der Beweis der Riemannschen Vermutung der Zahlentheorie,
7. die Erforschung der Gleichungen von Yang-Mills.

b.)

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

Allgemein / Ansatz:  $\sum_{k=1}^n (a_{k+1} - a_k) = a_{n+1} - a_1$

$$\stackrel{n=4}{=} a_{1+1} - a_1 + a_{2+1} - a_2 + a_{3+1} - a_3 + a_{4+1} - a_4$$

$$= \underline{a_2} - a_1 + \underline{a_3} - \underline{a_2} + \underline{a_4} - \underline{a_3} + \underline{a_5} - \underline{a_4}$$

$$= a_5 - a_1$$

$$(k+1)^2 - k^2 = \underbrace{k^2 + 2k + 1}_{1. \text{ Binom.}} - k^2 = 2k + 1$$

$$\sum_{k=1}^n (a_{k+1} - a_k) = \sum_{k=1}^n ((k+1)^2 - k^2) = \sum_{k=1}^n (2k+1)$$

$$= a_{n+1} - a_1 = (n+1)^2 - 1^2 = n^2 + 2n + 1^2 - 1^2$$

$$= n^2 + 2n$$

$$= n(n+2)$$

$$\star \sum_{k=1}^n (2k+1) = n(n+2)$$

$$\Leftrightarrow \sum_{k=1}^n (2k) + \sum_{k=1}^n (1) = n(n+2)$$

$$\Leftrightarrow 2 \sum_{k=1}^n k + \underbrace{\sum_{k=1}^n 1}_n = n(n+2)$$

$$\Leftrightarrow 2 \sum_{k=1}^n k + n = n(n+2) \quad | -n$$

$$\Leftrightarrow 2 \sum_{k=1}^n k = n(n+2) - n \quad | \cdot \frac{1}{2}$$

$$\Leftrightarrow \sum_{k=1}^n k = \frac{n(n+2) - n}{2}$$

$$\Leftrightarrow \sum_{k=1}^n k = \frac{n^2 + 2n - n}{2} = \frac{n^2 + n}{2} = \frac{n(n+1)}{2}$$

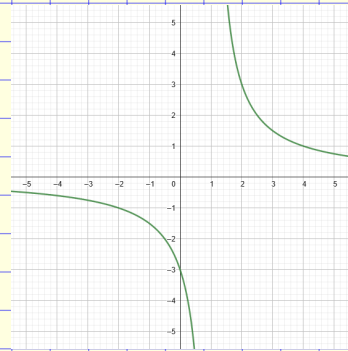
$$\Leftrightarrow \boxed{\sum_{k=1}^n k = \frac{n(n+1)}{2}} \quad \text{Gaußsche Summenformel}$$

# Aufgabe 1:

i.) a.)  $f(x) = \frac{3}{x-1}$ ,  $x \rightarrow \infty$

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{3}{x-1} = 0^+$$

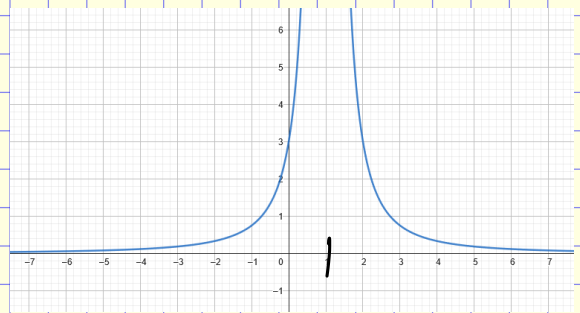
$$= \lim_{x \rightarrow \infty} \frac{\overset{\rightarrow 0}{3/x}}{\underset{\rightarrow 0}{1 - 1/x}} = \frac{0}{1} = 0$$



b.)  $f(x) = \frac{3}{(x-1)^2}$ ,  $x \rightarrow 1$

↑ Parabel um 1 nach rechts

$$\lim_{x \rightarrow 1} \frac{3}{(x-1)^2} = \infty$$



c.)  $\frac{2(x-2)}{(x^2-4)}$ ,  $x \rightarrow 2$

3. Binom.

$$\lim_{x \rightarrow 2} 2(x-2) = 0 = \lim_{x \rightarrow 2} (x^2-4)$$

$$\Leftrightarrow \lim_{x \rightarrow 2} \frac{2(x-2)}{(x^2-4)} = \lim_{x \rightarrow 2} \frac{2(x-2)}{(x+2)\cancel{(x-2)}} = \frac{2}{x+2} = \frac{2}{2+2} = \frac{2}{4} = \frac{1}{2}$$



d.)  $\frac{2(x-2)}{x^2-4}$ ,  $x \rightarrow -2$

$$\lim_{x \rightarrow -2} \frac{2}{x+2} \quad (\text{wegen oben}) = \pm \infty$$

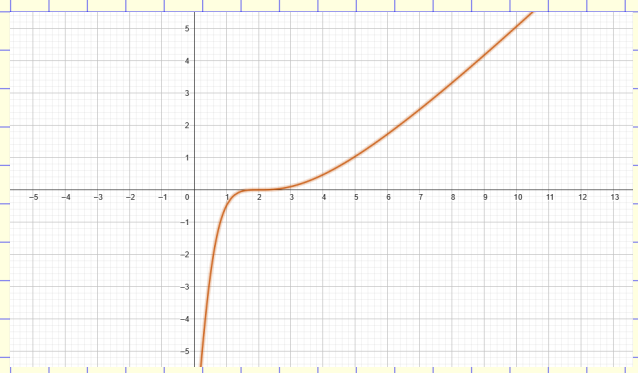
weil Definitionslücke  
(Nullstelle im Nenner)

e.)  $\lim_{x \rightarrow \infty} \frac{(x-2)^3}{x^2+1} \rightarrow \infty$ , „aber schneller“

$\Leftrightarrow \frac{(x-2)^3}{x^2+1} = \frac{x^3 - 6x^2 + 12x - 8}{x^2+1}$   $\triangleleft$  (Pascalsches Dreieck)

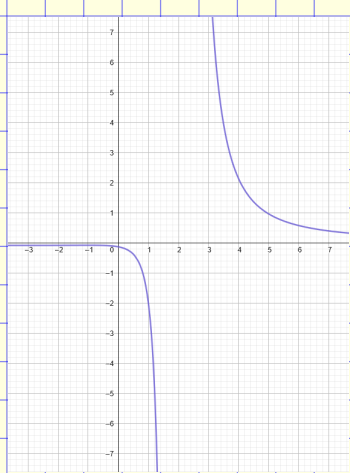
$\lim_{x \rightarrow \infty} \frac{1 - \frac{6}{x} + \frac{12}{x^2} - \frac{8}{x^3}}{\frac{1}{x} + \frac{1}{x^3}} = \infty$

$\begin{matrix} \nearrow 0 & \nearrow 0 & \nearrow 0 \\ \searrow 0 & \searrow 0 & \searrow 0 \\ \downarrow 0 & \downarrow 0 & \downarrow 0 \end{matrix}$



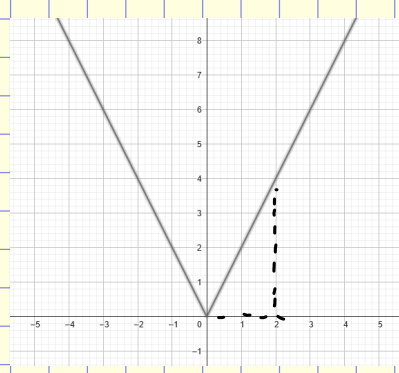
f.)  $\frac{x^2+1}{(x-2)^3}, x \rightarrow \infty$  „Kehrwert“ von f(x) bei e

$\lim_{x \rightarrow \infty} \frac{x^2+1}{(x-2)^3} = 0$



ii) a.)  $f(x) = 2|x| = \begin{cases} -2 \cdot x, & x < 0 \\ 0, & x = 0 \\ 2x, & x > 0 \end{cases} \Rightarrow$  Krit. Stelle bei 0

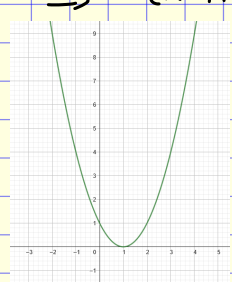
$\lim_{x \rightarrow 0^+} (2x) = 0^+$   
 $\lim_{x \rightarrow 0^-} (-2x) = 0^+$   
 $f(0) = 0$



$\sqrt{x^2} \neq (\sqrt{x})^2$

$\Rightarrow$  Stetig (rechterseitiger/linkerseitiger Grenzwert stimmen überein)

b.)  $f(x) = |(x-1)^2| = (x-1)^2$ , da  $x \in \mathbb{R} \Rightarrow (x-1)^2 \geq 0$   
 $\rightarrow$  Parabel um 1 nach rechts  
 $\Rightarrow$  Stetig auf  $\mathbb{R}$



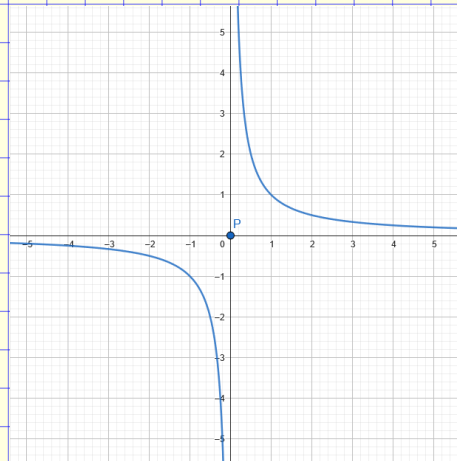
$$c.) f(x) = \begin{cases} 1/x & , x \neq 0 \quad (1) \\ 0 & , x = 0 \quad (2) \end{cases}$$

$$(1) \begin{cases} \lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty \\ \lim_{x \rightarrow 0^+} \frac{1}{x} = \infty \end{cases}$$

|      |      |    |      |     |
|------|------|----|------|-----|
| x    | -2   | -1 | -0,5 | ... |
| f(x) | -1/2 | -1 | -2   | ... |

$$(2) f(0) = 0 \Rightarrow \text{Polstelle}$$

$\Rightarrow$  nicht stetig

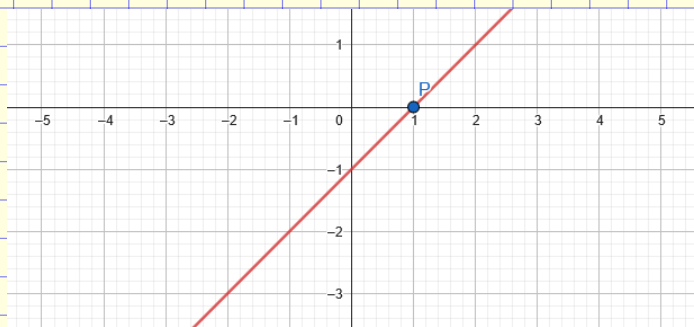


$$d.) f(x) = \begin{cases} \frac{x^2 - 2x + 1}{x - 1} & , x \neq 1 \quad (1) \\ 0 & , x = 1 \quad (2) \end{cases}$$

$$(1) \text{ 2. Binom. Formel (Zähler): } f(x) = \frac{x^2 - 2x + 1}{x - 1} = \frac{(x-1)^2}{x-1} = x-1$$

$$\lim_{x \rightarrow 1^\pm} f(x) = \lim_{x \rightarrow 1^\pm} (x-1) = 0^\pm \rightarrow \text{Gerade mit NS bei } x=1$$

$$(2) f(1) = 0 \quad \text{parat} \Rightarrow \text{stetig}$$

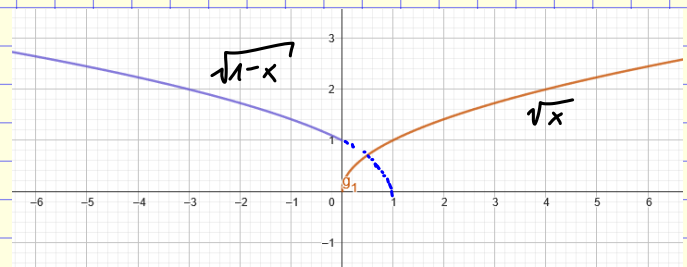


$$e.) f(x) = \begin{cases} \sqrt{x} & , x \geq 0 \quad (1) \\ \sqrt{1-x} & , x < 0 \quad (2) \end{cases}$$

$$(1) \lim_{x \rightarrow 0^+} (\sqrt{x}) = 0^+$$

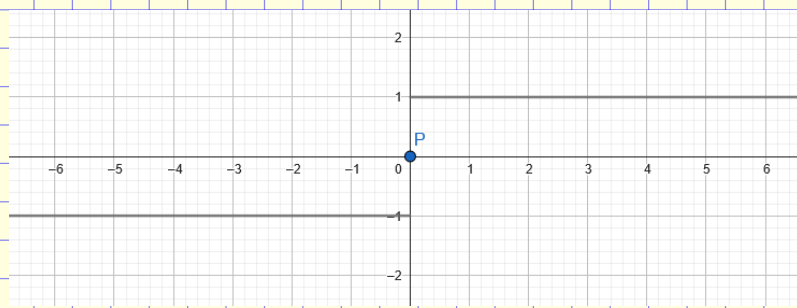
$$(2) \lim_{x \rightarrow 0^-} (\sqrt{1-x}) = 1^+ = \lim_{x \rightarrow 0^-} (\sqrt{-x+1})$$

$\uparrow$   $\rightarrow$  Spiegelung an y-Achse  
 Verschiebung um 1 nach links



$$\textcircled{3} \quad f(0) = \sqrt{0} = 0 \quad \Rightarrow \text{unstetigkeit, da } 0 \neq 1$$

$$f.) \quad f(x) = \text{Sign}(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$



$$\left. \begin{array}{l} \lim_{x \rightarrow 0^+} (f(x)) = 1 \\ \lim_{x \rightarrow 0^-} (f(x)) = -1 \\ f(0) = 0 \end{array} \right\} \Rightarrow \text{unstetig}$$

$$g.) \quad f(x) = \begin{cases} \frac{x+2}{x^2+x-2}, & x \neq -2 \quad \textcircled{1} \\ \frac{1}{3}, & x = -2 \quad \textcircled{2} \end{cases} \rightarrow \frac{-2+2}{4-2-2} = \frac{0}{0} \quad \text{⚡}$$

$$\begin{aligned} \textcircled{1} \quad \text{Nullstellen Nenner: } x^2+x-2 & \stackrel{!}{=} 0 \Rightarrow x = -\frac{1}{2} \pm \sqrt{\frac{1}{4}+2} \\ & \Leftrightarrow x = -\frac{1}{2} \pm \sqrt{\frac{9}{4}} \\ & \Leftrightarrow x = -\frac{1}{2} \pm \frac{3}{2} \\ & \Leftrightarrow x = 1 \quad \vee \quad x = -2 \end{aligned}$$

Linearfaktoren:

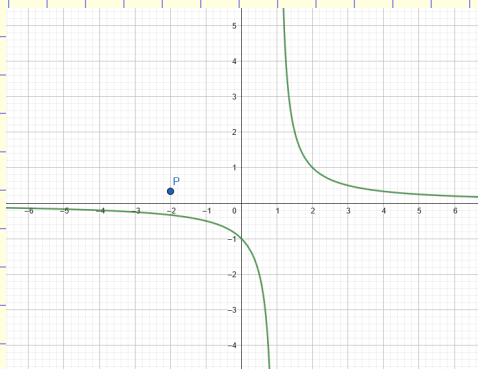
$$x^2+x-2 = (x-1)(x+2)$$

$$f(x) \textcircled{1} \Rightarrow \frac{x+2}{x^2+x-2} = \frac{x+2}{(x-1)(x+2)} = \frac{1}{x-1}$$

$\Rightarrow$  hebbare Def. Lücke bei  $x = -2$

$$\textcircled{1} \quad \lim_{x \rightarrow -2^+} \left( \frac{1}{x-1} \right) = -\frac{1}{3} \quad \textcircled{2} \quad f(-2) = \frac{1}{3}$$

$$-\frac{1}{3} \neq \frac{1}{3} \Rightarrow \text{unstetigkeit}$$



$$h.) f(x) = \frac{2x^2 + 4x - 6}{x^2 + x - 2} = \frac{2(x^2 + 2x - 3)}{x^2 + x - 2} \leftarrow \textcircled{1}$$

$$= \frac{2(x^2 + 2x - 3)}{x^2 + x - 2} \leftarrow \textcircled{2}$$

$$\textcircled{1} \quad 2x^2 + 4x - 6 \stackrel{!}{=} 0 \quad | \cdot \frac{1}{2}$$

$$x^2 + 2x - 3 = 0$$

$$\text{pq-Formel: } x = -1 \pm \sqrt{4} \Rightarrow x = -1 \vee x = 2$$

$$\Rightarrow (x+1)(x-2)$$

$$\textcircled{2} \quad x^2 + x - 2 = 0$$

$$x = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + 2} \Rightarrow x = 1 \vee x = -2$$

$$\Rightarrow (x-1)(x+2)$$

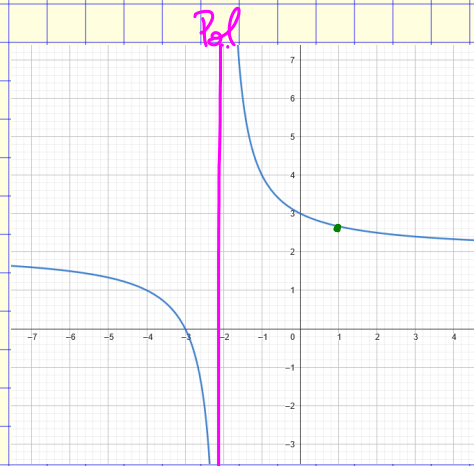
$$\Rightarrow f(x) = \frac{2 \cdot (x-1)(x+3)}{(x-1)(x+2)} = \frac{2(x+3)}{(x+2)}$$

• hebbare Def. Lücke bei  $x = 1$

• Polstelle bei  $x = -2$

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{2(x+3)}{(x+2)} = \frac{2(1+3)}{1+2} = \frac{2+6}{3} = \frac{8}{3}$$

stetig auf  $\mathbb{R} \setminus \{-2\}$

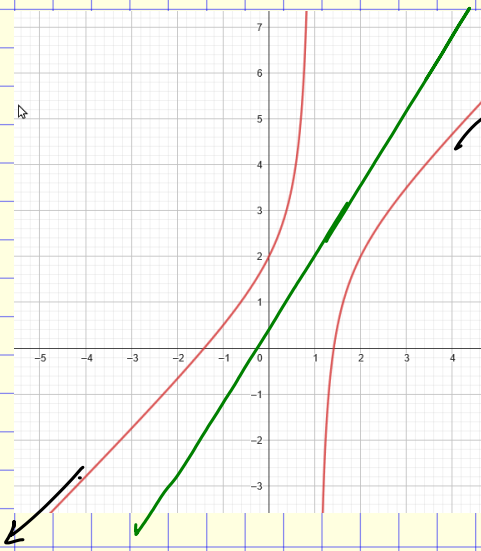


iii)

$$a.) \quad \frac{2-x^2}{1-x} \Rightarrow \text{Kritische Stelle bei } 1$$

$$\lim_{x \rightarrow \pm\infty} \frac{2-x^2}{1-x} \stackrel{! \cdot \frac{1}{x^2}}{=} \lim_{x \rightarrow \pm\infty} \left( \frac{\frac{2}{x^2} - 1}{\frac{1}{x^2} - \frac{1}{x}} \right) = \lim_{x \rightarrow \pm\infty} \left( \frac{-1}{\frac{1}{x^2} - \frac{1}{x}} \right) = \pm\infty$$

$\downarrow \quad \downarrow$   
 $0 \quad 0 < 0$  für  $+\infty$   
 $> 0$  für  $-\infty$



Zähler  $\geq$  Nenner:

$$\frac{2-x^2}{1-x} \stackrel{\cdot(-1)}{=} \frac{x^2-2}{x-1}$$

$$\begin{array}{r} (x^2-2) : (x-1) = x+1 \\ -(x^2-x) \\ \hline x-2 \\ -(x-1) \\ \hline -1 \end{array}$$

$$\Rightarrow \frac{x^2-2}{x-1} = \underbrace{x+1}_{\text{Asympt.}} - \frac{1}{x-1}$$

vernachlässigen

$$\lim_{x \rightarrow \pm\infty} \frac{2-x^2}{1-x} = \lim_{x \rightarrow \pm\infty} \left( x+1 - \frac{1}{x-1} \right) = \lim_{x \rightarrow \pm\infty} (x+1)$$

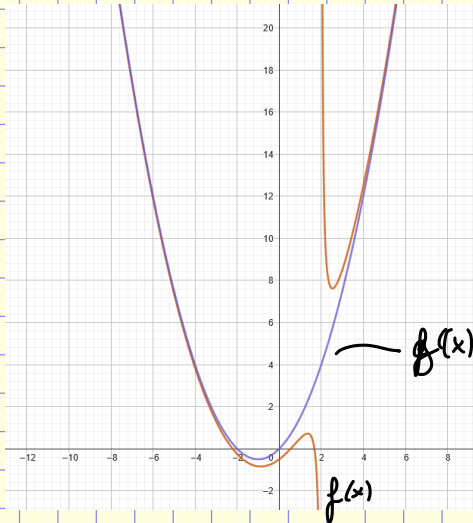
• NS:  $x = \pm\sqrt{2}$

$$= \lim_{x \rightarrow \pm\infty} (x)$$

• Polstelle  $x=1$

b.)  $f(x) = \frac{\frac{x^3}{2} - 2x + 1}{x-2} \rightarrow$  Asympt.  $x=2$

$$\lim_{x \rightarrow \pm\infty} f(x) = \frac{\frac{x^3}{2} - 2x + 1}{1 - \frac{2}{x}} \xrightarrow{\frac{0}{0}} \lim_{x \rightarrow \pm\infty} \frac{\frac{x^2}{2} - 2}{1} = \lim_{x \rightarrow \pm\infty} \left( \frac{x^2}{2} - 2 \right) = \infty$$



$f(x) = \frac{x^2}{2} - 2$

Polynomdivision:  $\left( \frac{x^3}{2} - 2x + 1 \right) : (x-2) = \frac{x^2}{2} + x + \frac{1}{x-2}$

$$\begin{array}{r} - \left( \frac{x^3}{2} - x^2 \right) \\ \hline x^2 - 2x + 1 \\ - (x^2 - 2x) \\ \hline 1 \end{array}$$

$$\lim_{x \rightarrow \pm\infty} f(x) = \lim_{x \rightarrow \pm\infty} \frac{x^2}{2} + x + \frac{1}{x-2} = \frac{x^2}{2} + x$$

$$\frac{x^2}{2} + x = \frac{1}{2} (x+1)^2 - \frac{1}{2}$$

$\uparrow$   
 $x = -1$

$\leftarrow y = -\frac{1}{2}$



• Polstelle bei  $x = 2$

$$\cdot \lim_{x \rightarrow \pm\infty} f(x) = \infty$$

Aufgabe 2:

i) a.)  $f(x) = \frac{1}{x^2 - 25}$

① Lfz:  $x^2 - 25 = 0 \xrightarrow{+25} x^2 = 25 \xrightarrow{\sqrt{\quad}} x = \pm 5$

$$\Rightarrow x^2 - 25 = (x-5)(x+5) \quad (3. \text{ Binom. Formel})$$

$$\begin{aligned} \textcircled{2} \text{ PBE: } f(x) &= \frac{1}{x^2 - 25} = \frac{1}{(x-5)(x+5)} \stackrel{!}{=} \frac{A}{x-5} + \frac{B}{x+5} \\ &= \frac{A(x+5) + B(x-5)}{(x-5)(x+5)} \\ &= \frac{Ax + 5A + Bx - 5B}{(x-5)(x+5)} \\ &= \frac{(A+B)x + 5(A-B)}{(x-5)(x+5)} \end{aligned}$$

$$\frac{1}{(x-5)(x+5)} \stackrel{!}{=} \frac{(A+B)x + 5(A-B)}{(x-5)(x+5)}$$

$$\Rightarrow \underbrace{0 \cdot x + 0 \cdot x^2 + 1}_{\text{Zähler}} \stackrel{!}{=} \underbrace{(A+B)x + 5(A-B)}_{\substack{0 & 1}} \quad (\text{Koeffizientenvergleich!})$$

$$\Rightarrow A+B=0 \xrightarrow{+A} B=-A \Rightarrow B = -\frac{1}{10}$$

$$5(A-B)=1 \Rightarrow 5 \cdot 2A = 1 \Rightarrow A = \frac{1}{10}$$

$$f(x) = \frac{1}{x^2 - 25} = \frac{1/10}{x-5} - \frac{1/10}{x+5}$$

b.)  $g(x) = \frac{x+29}{x^2+3x-28}$

$$\begin{aligned} \textcircled{1} \text{ Lfz: } x^2 + 3x - 28 &\stackrel{!}{=} 0 \Rightarrow x = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + 28} \\ &= -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{112}{4}} \\ &= -\frac{3}{2} \pm \sqrt{\frac{121}{4}} \\ &= -\frac{3}{2} \pm \frac{11}{2} \\ &\Rightarrow x = 4 \vee x = -7 \end{aligned}$$

$$\textcircled{2} \text{ PZ: } x^2 + 3x - 28 = (x-4)(x+7)$$

$$g(x) = \frac{x+29}{x^2+3x-28} = \frac{x+29}{(x-4)(x+7)} \stackrel{!}{=} \frac{A}{(x-4)} + \frac{B}{(x+7)} = \frac{A(x+7)+B(x-4)}{(x-4)(x+7)}$$

$$x+29 \stackrel{!}{=} A(x+7)+B(x-4) = Ax+7A+Bx-4B \\ = Ax+Bx+7A-4B$$

$$\Rightarrow x+29 \stackrel{!}{=} \underbrace{(A+B)}_{A+B=1}x + \underbrace{7A-4B}_{7A-4B=29}$$

$$\textcircled{1} A+B=1 \stackrel{!-A}{\Rightarrow} B=1-A$$

$$\textcircled{2} 7A-4B=29$$

$$\Rightarrow 7A-4(1-A)=29$$

$$\Rightarrow 7A-4+4A=29 \quad | +4$$

$$\Rightarrow 7A+4A=33$$

$$\Rightarrow 11A=33 \quad | :3$$

$$\Rightarrow A=3$$

$$g(x) = \frac{x+29}{x^2+3x-28} = \frac{3}{x-4} - \frac{2}{x+7}$$

$$\text{c.) } h(x) = \frac{3x^2+5x-2}{(x^2+1)(3x+1)} \stackrel{!}{=} \frac{Ax+B}{x^2+1} + \frac{C}{3x+1}$$

doppelte Ns

$$\Rightarrow (Ax+B)(3x+1)+C(x^2+1)=3x^2+5x-2$$

$$\Rightarrow \underline{3Ax^2} + \underline{Ax} + \underline{3Bx} + B + \underline{Cx^2} + C = 3x^2+5x-2$$

$$\Rightarrow \underline{(3A+C)}x^2 + \underline{(A+3B)}x + \underline{B+C} = \underline{3}x^2 + \underline{5}x - \underline{2}$$

$$\Rightarrow 3A+C=3 \quad ; \quad A+3B=5 \quad ; \quad B+C=-2$$

$$\Rightarrow A=2, B=1, C=-3$$

ii)

$$a.) f(x) = \frac{2x^3 - x^2 - 10x + 19}{x^2 + x - 6}$$

Zähler  $\geq$  Nenner $\rightarrow$  Polynomdivision

$$\Rightarrow (2x^3 - x^2 - 10x + 19) : (x^2 + x - 6) = 2x - 3 + \frac{5x + 1}{x^2 + x - 6}$$

$$- (2x^3 + 2x^2 - 12x)$$

$$\underline{-3x^2 + 2x + 19}$$

$$- (-3x^2 - 3x + 18)$$

$$\underline{5x + 1}$$

Rest, da  $x^1 < x^2$ 

$$\text{PBZ: } \frac{5x+1}{x^2+x-6} \rightarrow \text{LFZ}$$

$$x^2 + x - 6 \stackrel{!}{=} 0 \Rightarrow x = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + 6}$$

$$\Rightarrow x = 2 \vee x = -3$$

$$\begin{aligned} \frac{5x+1}{x^2+x-6} &= \frac{5x+1}{(x-2)(x+3)} \stackrel{!}{=} \frac{A}{x-2} + \frac{B}{x+3} \\ &= \frac{A(x+3) + B(x-2)}{(x-2)(x+3)} \end{aligned}$$

$$\Rightarrow A(x+3) + B(x-2) = 5x+1$$

$$\Rightarrow Ax + 3A + Bx - 2B = 5x + 1$$

$$\Rightarrow Ax + Bx + 3A - 2B = 5x + 1$$

$$\Rightarrow \underline{(A+B)x} + \underline{3A-2B} = \underline{5x+1}$$

$$\Rightarrow A+B = 5 \quad ; \quad 3A-2B = 1$$

NS  
ausnutzen:

$$\textcircled{1} \quad x = -3 \rightarrow -5B = -15 + 1 \Rightarrow B = \frac{14}{5}$$

$$\textcircled{2} \quad x = 2 \rightarrow 5A = 10 + 1 \Rightarrow A = \frac{11}{5}$$

$$\Rightarrow f(x) = \frac{2x^3 - x^2 - 10x + 19}{x^2 + x - 6} = \underbrace{2x - 3}_{\text{Poly.}} + \underbrace{\frac{\frac{11}{5}}{x-2} + \frac{\frac{14}{5}}{x+3}}_{\text{PBZ}}$$

$$b.) g(x) = \frac{x^4 - 5x^3 + 7x^2 - 13x - 10}{x^3 - 5x^2}$$

Zähler > Nenner  
 $\Rightarrow$  Polyn.

$$\Rightarrow \frac{(x^4 - 5x^3 + 7x^2 - 13x - 10) : (x^3 - 5x^2) = x + \frac{7x^2 - 13x - 10}{x^3 - 5x^2}}{\begin{array}{l} - (x^4 - 5x^3) \\ \hline 0 + 7x^2 - 13x - 10 \end{array} \rightarrow \text{Rest}}$$

PBZ

PBZ:  $x^3 - 5x^2 = 0$   $\xrightarrow{x^2 \text{ ausklammern}}$   $x^2(x-5) = 0$

↑ doppelte NS      ↑ einfache NS

$$\frac{7x^2 - 13x - 10}{x^3 - 5x^2} = \frac{7x^2 - 13x - 10}{x^2(x-5)}$$

$$\Rightarrow \frac{7x^2 - 13x - 10}{x^2(x-5)} \stackrel{!}{=} \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-5}$$

$$= \frac{Ax^2(x-5) + Bx(x-5) + Cx^2x}{x^2(x-5)}$$

$$= \frac{Ax^2(x-5) + Bx(x-5) + Cx^3}{x^3(x-5)}$$

$$\Rightarrow \frac{7x^2 - 13x - 10}{x^2(x-5)} = \frac{Ax(x-5) + B(x-5) + Cx^2}{x^2(x-5)}$$

$$\begin{aligned} \Rightarrow \underline{7x^2} - \underline{13x} - \underline{10} &= Ax(x-5) + B(x-5) + Cx^2 \\ &= \underline{Ax^2} - \underline{5Ax} + \underline{Bx} - \underline{5B} + \underline{Cx^2} \\ &= \underline{(A+C)x^2} + \underline{(B-5A)x} - \underline{5B} \end{aligned}$$

$$A+C = 7 \quad ; \quad B-5A = -13 \quad ; \quad -5B = -10$$

$$\Rightarrow C = 4$$

$$\Rightarrow A = 3$$

$$\Rightarrow B = 2$$

$$g(x) = \frac{x^4 - 5x^3 + 7x^2 - 13x - 10}{x^3 - 5x^2} = x + \underbrace{\frac{3}{x} + \frac{2}{x^2} + \frac{4}{x-5}}_{\text{PBZ}}$$

Poly