Vorrechnen Selbsteinschätzung

1 a) $$\frac{(d^2-4d+4)\,(4+d^2+4d)}{(d^2-4)} = \frac{(d-2)^{\cancel{2}}\cdot(d+2)^2}{(d\cancel{-2})\,(d\cancel{+2})}$$

$$= (d-2)\cdot(d+2) = d^2-4$$

b) $$\frac{13}{9}-\frac{1}{63}-\frac{21}{49} = \frac{91}{63}-\frac{1}{63}-\frac{21}{49} = \frac{90}{63}-\frac{21}{49}$$

$$= \frac{30}{21}-\frac{21}{49} = \frac{10}{7}-\frac{3}{7} = \frac{7}{7} = \underline{\underline{1}}$$

c) $$\frac{3a}{b} : \frac{a}{2b} \cdot \frac{2a}{3b} = \frac{3\cancel{a}\cdot 2\cancel{b}}{\cancel{b}\cdot\cancel{a}} \cdot \frac{2a}{3b} = \frac{12a}{3b} = 4\,\frac{a}{b}$$

d) $$\left(\frac{x^3\cdot y}{n^2\cdot m^3}\right)^5 : \left(\frac{x\cdot y^2}{n\cdot m^5}\right)^2 = \frac{x^{15}\cdot y^5}{n^{10}\cdot m^{15}} \cdot \frac{n^2\cdot m^{10}}{x^2\cdot y^4} = \frac{x^{13}\cdot y}{n^8 m^5}$$

e) $$\sqrt{\left(\sqrt{\sqrt[5]{a^2}}\right)^4 \sqrt{b^3}} = \left(\left(\left(\left(a^2\right)^{\frac{1}{5}}\right)^{\frac{1}{2}}\right)^4 \left(b^3\right)^{\frac{1}{2}}\right)^2$$

$$= \left(a^{2\cdot\frac{1}{5}\cdot\frac{1}{2}\cdot 4}\; b^{3\cdot\frac{1}{2}}\right)^{\frac{1}{2}} = \left(a^{\frac{4}{5}}\cdot b^{\frac{3}{2}}\right)^{\frac{1}{2}} = a^{\frac{2}{5}}\cdot b^{\frac{3}{4}}$$

f) $$\left(\frac{\sqrt[5]{\sqrt{7}\,x}}{\sqrt[4]{x^3}}\right)^{-2} = \frac{\left(\left(7^{\frac{1}{2}}\cdot x\right)^{\frac{1}{5}}\right)^{-2}}{\left(\left(x^3\right)^{\frac{1}{4}}\right)^{-2}} = \frac{\left(7^{\frac{1}{2}}\cdot x\right)^{-\frac{2}{5}}}{\left(x^3\right)^{-\frac{1}{2}}}$$

$$= \frac{7^{-\frac{2}{10}}\cdot x^{-\frac{2}{5}}}{x^{-\frac{3}{2}}} = \frac{7^{-\frac{1}{5}}\cdot x^{-\frac{2}{5}}}{x^{-\frac{3}{2}}} = \frac{x^{-\frac{2}{5}}\cdot x^{\frac{3}{2}}}{7^{\frac{1}{5}}}$$

$$= \frac{x^{\frac{3}{2}-\frac{2}{5}}}{7^{\frac{1}{5}}} = \frac{x^{\frac{15}{10}-\frac{4}{10}}}{7^{\frac{1}{5}}} = \frac{x^{\frac{11}{10}}}{7^{\frac{1}{5}}}$$

2/ a) $8x - (5x+2) = 3 - (5-2x)$

$$8x - 5x - 2 = 3 - 5 + 2x$$

$$3x - 2x = 3 - 5 + 2$$

$$x = 0$$

b) $x^4 - 2x^2 + 1 = 0$ $\qquad z = x^2$

$$z^2 - 2z + 1 = 0$$

$$\Rightarrow \quad (z-1)\cdot(z-1) = 0$$

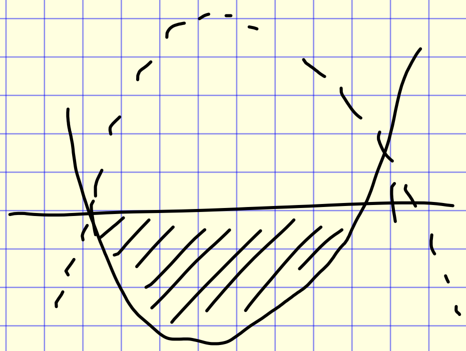
$$(x^2-1)\cdot(x^2-1) = 0$$

$$x = \pm 1$$

c) $x^2 - 2x - 10 \leq 5$

$$x^2 - 2x - 15 \leq 0$$

$$(x+3)(x-5) \leq 0$$



$$x \in [-3, 5]$$

d) $-2x^4 + 2x^3 - 14x + 10 > 4 - 7x + x^3(1-2x)$

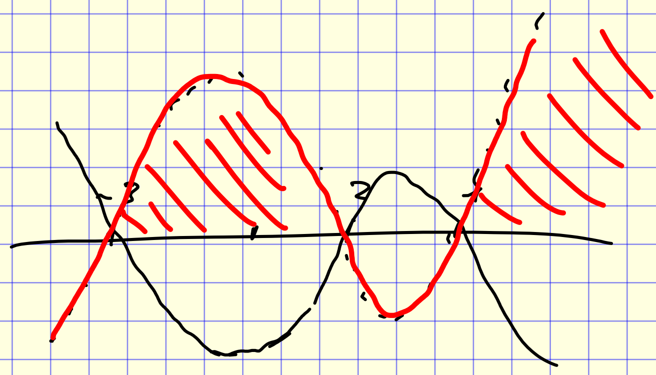
$-\cancel{2x^4} + 2x^3 - 14x + 10 > 4 - 7x + x^3 - \cancel{2x^4}$

$-x^3 + 2x^3 - 7x + 6 > 0$

$x^3 - 7x + 6 > 0 \qquad \Rightarrow NS = 1$

$(x^3 - 7x + 6) : (x-1) = x^2 + x - 6 \Rightarrow (x+3)(x-2)$

$\Rightarrow (x+3)(x-2)(x-1) > 0$



$x=0: \quad 3 \cdot (-2) \cdot (-1) > 0$

$x \in]-3, 1[\; \cup \;]2, +\infty[$

e) $\dfrac{x-1}{|x+4|} < 1 \iff x - 1 < |x+4|$

$\Rightarrow \overline{|x \in \mathbb{R} \setminus \{-4\}|}$

$\swarrow \quad x > -4$

Fall 1 $\qquad x + 4 > 0 \Rightarrow |x+4| = x+4$

$\dfrac{x-1}{x+4} < 1 \iff x - 1 < x + 4$

$x < x + 5$

Fall 2 $\qquad x + 4 < 0 \Rightarrow |x+4| = -(x+4) = -x-4$

$x < -4$

$\dfrac{x-1}{-x-4} < 1 \iff x - 1 < -x - 4 \iff x < -x - 3$

3)

Volumen: 6 Pyramiden → Würfel

$$V_{\square} = a^3$$

$$V_{\circ} = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi a^3$$

$$V_{\square} - V_{\circ} = a^3\left(1 - \frac{1}{6}\pi\right)$$

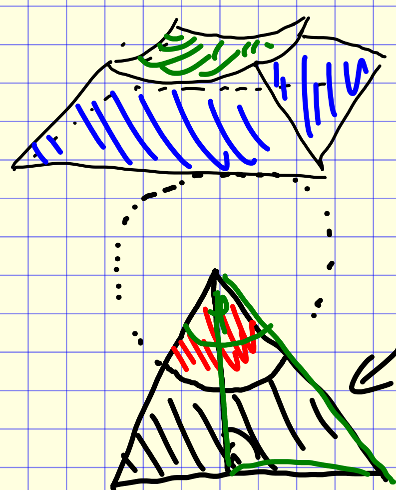
$$V_x = \frac{V_{\square} - V_{\circ}}{6} = \frac{a^3}{6}\left(1 - \frac{1}{6}\pi\right) \approx 0{,}079\, a^3$$

Oberfläche:

$$4\pi \cdot \left(\frac{a}{2}\right)^2 = \frac{4}{4} \cdot \pi a^2$$

$$G = a^2$$

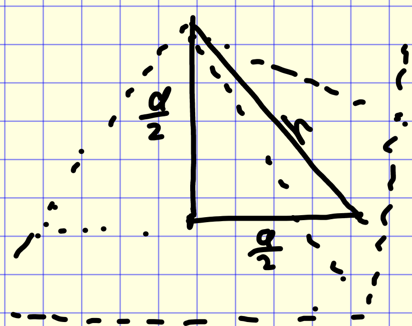
$$\frac{O_{\circ}}{6} = \frac{1}{6} \cdot 4\pi \cdot r^2 = \frac{1}{6} \cdot \pi a^2$$



$$A_{\Delta} = \frac{1}{2} h \cdot a$$

$$h = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2}$$

$$h = \sqrt{2 \cdot \left(\frac{a}{2}\right)^2} = \sqrt{2}\,\frac{a}{2}$$



$$A^*_{\Delta} = \frac{1}{2} \cdot \sqrt{2} \cdot \frac{a}{2} \cdot a = \frac{1}{4}\sqrt{2} \cdot a^2$$

GAGA
HHAG

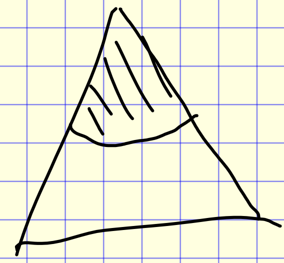
$$\tan\left(\frac{\varphi}{2}\right) = \left(\frac{a\,2}{2\sqrt{2}\,a}\right) = \left(\frac{1}{\sqrt{2}}\right)$$

$$\cancel{\tan^{-1}}\ \frac{\varphi}{2} = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) = 35{,}264°$$

$$a^3$$

$$\frac{1}{3} G \cdot h = \frac{1}{3} a^2 \cdot \frac{a}{2} = \frac{1}{6} \cdot a^3$$

$\Rightarrow \varphi = 70{,}53°$



$$A_0 = \pi \cdot r^2 = \frac{70{,}53°}{360°} \cdot \pi \cdot r^2$$

$$= \frac{70{,}53°}{360°} \cdot \pi \cdot \left(\frac{a}{2}\right)^2$$

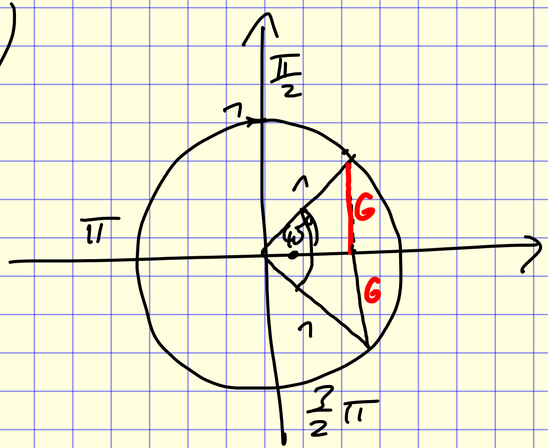
$$O_{ges} = a^2 + \frac{1}{6}\pi \cdot a^2 + 4 \cdot \left(\frac{1}{4}\sqrt{2}\, a^2 - \frac{70{,}53°}{360°} \cdot \pi \cdot \left(\frac{a}{2}\right)^2\right)$$

$$= a^2 + \frac{1}{6}\pi a^2 + \sqrt{2}\, a^2 - \frac{70{,}53°}{360°}\pi a^2$$

$$\approx \underline{\underline{2{,}322\, a^2}}$$

$$\sin(\alpha) = \frac{G}{H}$$

4/ a) $\sin\left(-\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right)$

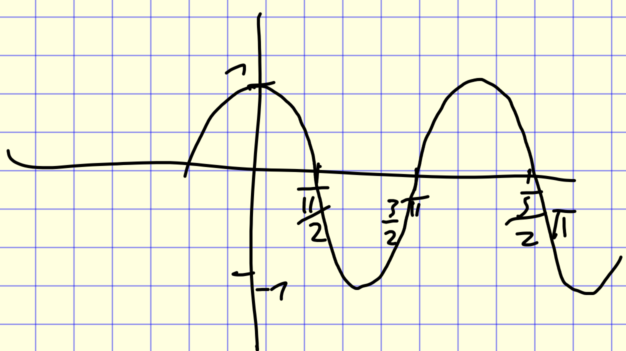


$$2G = \sqrt{1^2+1^2} = \sqrt{2}$$

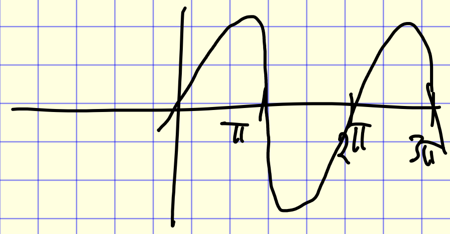
$$G = \frac{\sqrt{2}}{2}$$

$$\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

b) $\cos\left(\frac{11}{2}\pi\right) = 0$



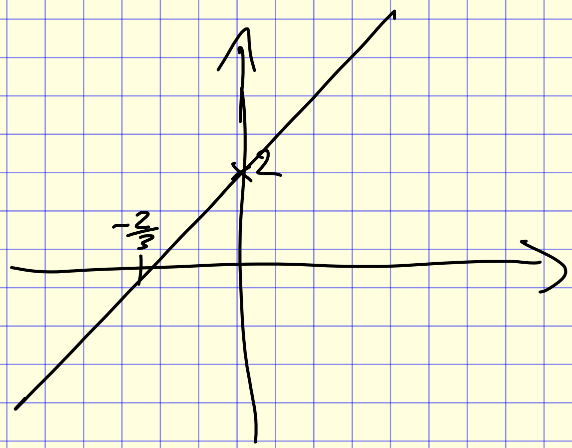
c) $\sin\left(\frac{104}{13}\pi\right) = \sin(8\pi) = 0$



5) a) $y(x) = 3x + 2$

$3x + 2 = 0$

$x = -\frac{2}{3}$

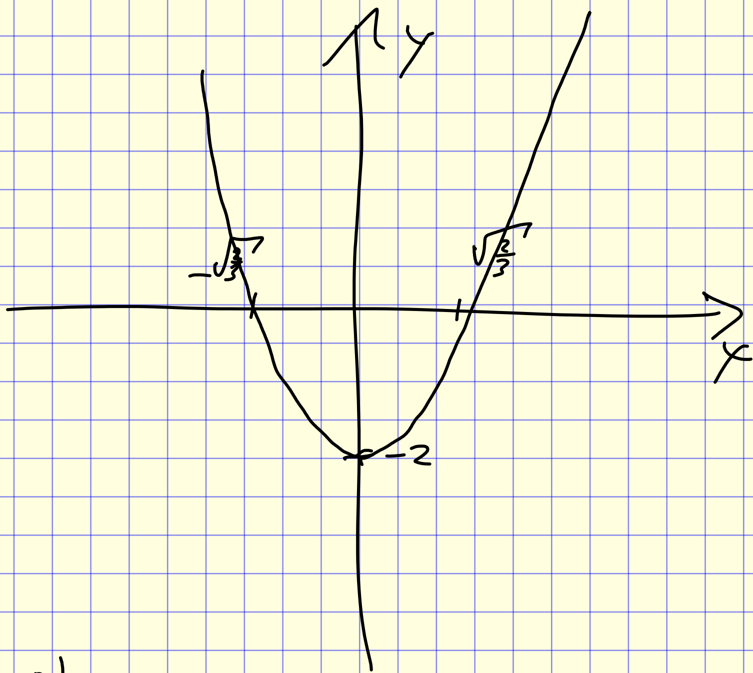


b) $y(x) = 3x^2 - 2$

$3x^2 - 2 = 0$

$x^2 = \frac{2}{3}$

$x = \pm\sqrt{\frac{2}{3}}$

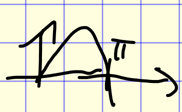


c) $A(t) = 5 \cdot \sin(0{,}5t + 135°)$

$\frac{135°}{360°} \cdot 2\pi \mathrel{\hat=} \frac{3}{4}\pi$

$A(t) = 5 \cdot \sin\left(0{,}5t + \frac{3}{4}\pi\right)$

$A(0) = 5 \cdot \sin\left(\frac{3}{4}\pi\right)$ $\quad 0{,}5t + \frac{3}{4}\pi = n \cdot \pi \quad n \in \mathbb{Z}$

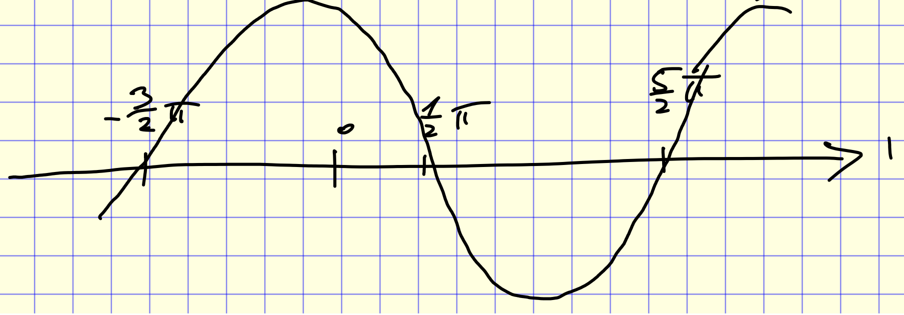


$t = 2\pi n - \frac{3}{2}\pi$

$t_0 = -\frac{3}{2}\pi$

$t_1 = 2\pi - \frac{3}{2}\pi$

$= \frac{1}{2}\pi$



6/ $x - 2y - z = 0$
$x + 3y + z = -1$
$-x + 2y + 2z = 1$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 1 & 3 & 1 & -1 \\ -1 & 2 & 2 & 1 \end{array}\right)$$

$$\xrightarrow[\text{II}-\text{I}]{\text{II}+\text{III}} \left(\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & 5 & 2 & -1 \\ 0 & 5 & 3 & 0 \end{array}\right) \overset{\text{II}-\text{III},\ \cancel{\text{III}-\text{II}}}{\Rightarrow} \left(\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 5 & 3 & 0 \end{array}\right)$$

$$\Rightarrow \underline{\underline{z = 1}} \quad , \quad 5y + 3 = 0 \Rightarrow \underline{\underline{y = -\frac{3}{5}}}$$

$$x + \frac{6}{5} - 1 = 0 \quad \Rightarrow \quad \underline{\underline{x = -\frac{1}{5}}}$$

7/ a) $f(x) = \frac{1}{x^3}$ $\quad f'(x) = \frac{-3}{x^4}$

b) $f(x) = \sqrt{x^5} = x^{\frac{5}{2}}$ $\quad f'(x) = \frac{5}{2} \cdot x^{\frac{3}{2}}$

c) $f(x) = \cos\left(\frac{1-x^2}{x}\right)$

$$f'(x) = -\sin\left(\frac{1-x^2}{x}\right) \cdot \left[\frac{-2x \cdot x - 1 \cdot (1-x^2)}{x^2}\right]$$

$$= -\sin\left(\frac{1-x^2}{x}\right) \cdot (-)\left[\frac{2x^2 + (1-x^2)}{x^2}\right] \quad \to 2x^2 + 1 - x^2 = x^2 + 1$$

$$= \left(\frac{1+x^2}{x^2}\right) \cdot \sin\left(\frac{1-x^2}{x}\right)$$

8) a) $\int (x^3 - 3x + 4)\,dx = \frac{1}{4}x^4 - \frac{3}{2}x^2 + 4x + C$

b) $\int \sin\left(\frac{x}{4} + 3\right) dx = -\cos\left(\frac{x}{4}+3\right) \cdot 4 + C$

$= -4\cos\left(\frac{x}{4}+3\right) + C$

c) $\int_1^e \frac{\cos(\ln x)}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

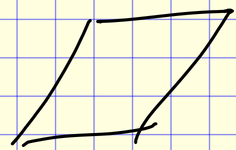
$dx = x \cdot du$

$\int_1^e \frac{\cos(u)}{\cancel{x}} \cdot \cancel{x}\,du = \int_1^e \cos(u)\,du$

$= \left[\sin(u)\right]_1^e = \left[\sin(\ln(x))\right]_1^e$

$= \sin(\underbrace{\ln(e^1)}_{1}) - \underbrace{\sin(\underbrace{\ln(1)}_{0})}_{0} + C = \sin(1) + C$

9) $\overrightarrow{AB} = \vec{B} - \vec{A} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ ☆



$\overrightarrow{BC} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ *

$|\overrightarrow{AB}| = \sqrt{4^2 + 1^2} = \sqrt{17} = |\overrightarrow{CD}|$

$\overrightarrow{CD} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$ ☆

$|\overrightarrow{BC}| = \sqrt{4+9} = \sqrt{13} = |\overrightarrow{DA}|$

$\overrightarrow{DA} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$ *

$\Rightarrow \overrightarrow{AB} \parallel \overrightarrow{CD}, \quad \overrightarrow{BC} \parallel \overrightarrow{DA} \quad \Rightarrow$ ▱