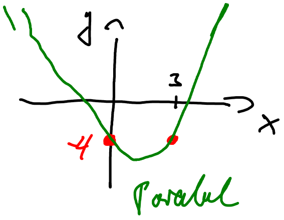
Vorlesung 13.11.2020

i) $f(x)=2x(x-3)-4$;



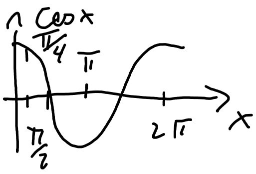
$x=0 \Rightarrow y=-4$

$x=3 \Rightarrow y=-4$

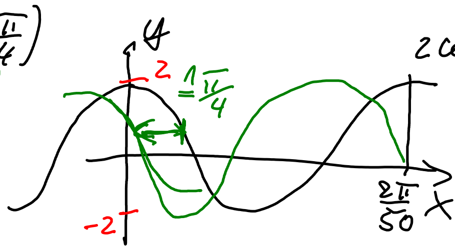
$x\to\infty \Rightarrow y\to+\infty$

$x\to-\infty \Rightarrow y\to+\infty$

ii) $f(x)=2\cos(50x+\frac{\pi}{4})$



Phase



$2\cos(50x)$ $\quad \cos(x)=\cos(x+2\pi)$

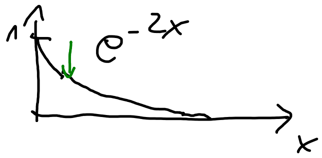
$50 \hat{=} \omega \quad \omega=2\pi f$

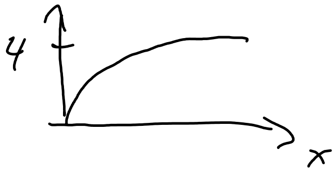
$50=2\pi f$

$f=\frac{50}{2\pi}$; $\quad T=\frac{1}{f}$

$T=\frac{2\pi}{50}$

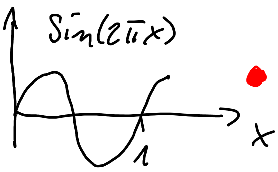
iii) $f(x)=4\cdot(1-e^{-2x})$

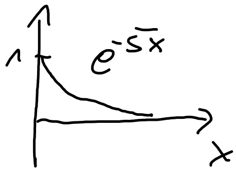


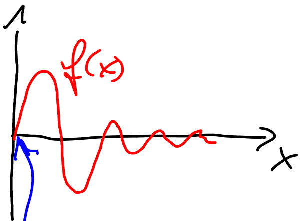


z.B. bei Zerfallsprozessen

iv) $f(x)=\sin(2\pi x)e^{-5x}$

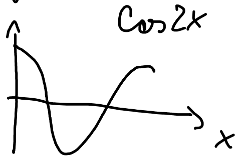




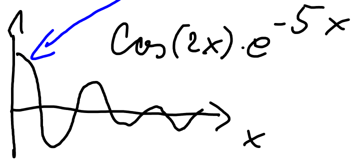


$\hat{=}$ gedämpften Schwingung

Vgl.



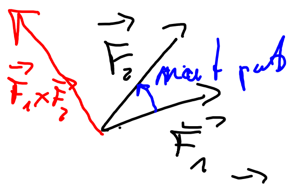
$\leadsto$



2) $\vec{a}=5\vec{e}_1+x\vec{e}_2+3\vec{e}_3$; $\vec{b}=1\vec{e}_1-2\vec{e}_2-7\vec{e}_3$; $\vec{a}\perp\vec{b} \Rightarrow \vec{a}\cdot\vec{b}=0$

$\Rightarrow 5\cdot1-2x-3\cdot7=0 \Rightarrow x=-8$

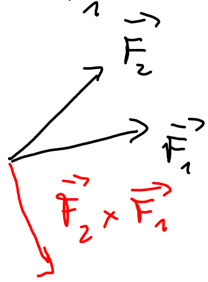
3b) $\vec{F}_1=(2,1,-3)$; $\vec{F}_2=(1,-2,1)$;



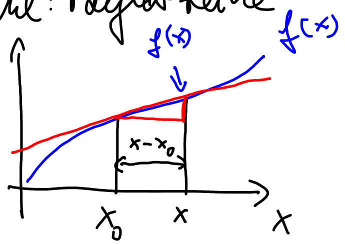
$\vec{F}_1\times\vec{F}_2 \perp \vec{F}_1$ & $\perp \vec{F}_2$

Einheitsvektor $\frac{\vec{F}_1\times\vec{F}_2}{|\vec{F}_1\times\vec{F}_2|}$

auch



Wdhl: Taylorreihe



$$f(x)=f(x_0)+\frac{df(x)}{dx}\Big|_{x_0}\frac{(x-x_0)^1}{1!}+\frac{d^2f(x)}{dx^2}\Big|_{x_0}\cdot\frac{(x-x_0)^2}{2!}+\dots$$

$$\boxed{f(x)=\sum_{k=0}^{\infty}\frac{d^kf(x)}{dx^k}\Big|_{x_0}\frac{(x-x_0)^k}{k!}}$$

$$\frac{d}{dx}x^n=n\,x^{n-1}$$

oft $x_0=0$

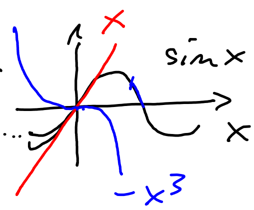
oder $x_0=1$

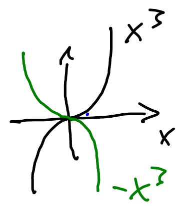
→ nur bis $k=N \Rightarrow \left|f(x)-\sum^N\dots\right|\le\left|\frac{d^{N+1}f}{dx^{N+1}}\cdot x^{N+1}\right|$

Bsp. $f(x)=\sin x\;;\;x_0=0\;;\;n=4$

$$f(x+0)=\sin(0)+\underbrace{\cos(0)}_{\frac{d\sin x}{dx}\big|_{x=0}}\cdot(x-0)^1-\sin(0)\frac{(x-0)^2}{2!}-\cos(0)\frac{(x-0)^3}{3!}\dots$$

$$\Rightarrow \sin(x)=0+\underline{x}-0-\frac{x^3}{3!}=x-\frac{x^3}{3!}+\frac{x^5}{5!}+\dots$$





Bsp. $\sin(0{,}1)=0{,}0998334$

$$\sin(0+0.1)=0.1-\frac{0.1^3}{6}=0.0998$$

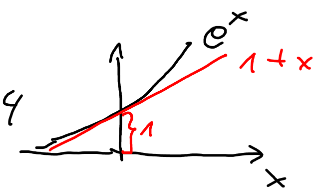
Bsp. $f(x)=\cos x\;;\;x_0=0\;;\;n=4$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\dots$$

$\cos(0.1)=0.995004$

$1-\frac{0.1^2}{2}+\frac{0.1^4}{4!}=0{,}995$

Bsp. $f(x)=e^x\;;\;x_0=0\;;\;n=4$



$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}\dots\;;\;\text{speziell } e^1=1+\frac{1}{1}+\frac{1}{2}+\frac{1}{3!}+\dots=2.718281828\dots$$

Bsp. $e^{0.1}=1.10517\;;\;1+0.1+\frac{0.1^2}{2}\dots=1.1052$

Einschub $\boxed{e^{i\varphi}=\cos\varphi+i\sin\varphi}$

$$\cos\varphi=\underline{1}-\underline{\frac{\varphi^2}{2}}+\underline{\frac{\varphi^4}{4!}}\dots$$

$$\sin\varphi=\underline{\varphi}-\underline{\frac{\varphi^3}{3!}}+\underline{\frac{\varphi^5}{5!}}\dots$$

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\dots$$

$\varphi \in \mathbb{R}$

$$e^{i\varphi}=1+i\varphi+\frac{(i\varphi)^2}{2!}+\frac{(i\varphi)^3}{3!}+\frac{(i\varphi)^4}{4!}+\dots$$

$$=\underline{1}+\underline{i\varphi}-\underline{\frac{\varphi^2}{2!}}-\underline{i\frac{\varphi^3}{3!}}+\underline{\frac{\varphi^4}{4!}}+\underline{\dots}=\boxed{\cos\varphi+i\sin\varphi=e^{i\varphi}}=e^{i(\varphi+2k\pi)}$$

etwas Spaß $\cos z = 2$?z=? ; $z \in \mathbb{R}$ $\hat{=}$ arccos 2 = z ?

$$\cos z = \frac{e^{iz}+e^{-iz}}{2} \text{ ; einsetzen von } e^{iz} \text{ ; } \quad \cos\varphi = \cos(-\varphi), \quad \sin\varphi = -\sin(-\varphi)$$

$$2 = \frac{e^{iz}+e^{-iz}}{2}$$

$$e^{iz}+e^{-iz} = 4 \qquad | \; e^{iz} = t$$

$$t + \frac{1}{t} - 4 = 0 \Rightarrow t^2 - 4t + 1 = 0 \Rightarrow t_{1/2} = 2 \pm \sqrt{3} = e^{iz_{1/2}} \quad | \ln$$

$$iz = \ln(2 \pm \sqrt{3}) + 2\pi i \cdot k ; \; \frac{1}{i} = -i$$

$$\boxed{z = -i\ln(2\pm\sqrt{3}) + 2\pi k}$$

noch mehr Spaß :-)

$i^i \hat{=}$ in Worten $(\text{ilch})^{(\text{ilch})}$ = Supereckelig

komplexe Zahl

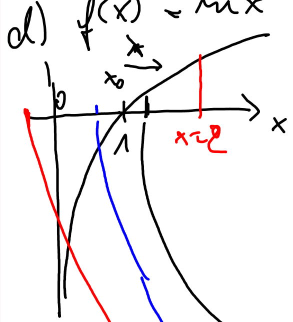
$$i\hbar \frac{d}{dx}\psi(x) = \left(\frac{\hbar^2}{2m} + V(x)\right)\psi(x)$$

$$\left(e^{i\frac{\pi}{2}}\right)^i = e^{i\frac{\pi}{2}\cdot i} = e^{-\frac{\pi}{2}} = \frac{1}{\sqrt{e^\pi}} = i^i = 0.2079\ldots$$

$\searrow \in \mathbb{R}$

Einschub Ende

d) $f(x) = \ln x$ an $x_0 = 1$; $n = 4$



$$\ln(x+1) = \ln x_0 + \frac{1}{x_0} x - \frac{1}{x_0^2}\cdot\frac{x^2}{2!} + \ldots$$

x_0 $\quad x_0 = 1 \quad x_0 = 1$

$$\ln(x+1) = 0 + x - \frac{1}{2}x^2 + \frac{1}{3!}x^3 - \frac{1}{4!}x^4 + \ldots$$

wenn $x = 0 \Rightarrow \ln(1) = 0$ ✓

$x = 0.1 \Rightarrow \ln(1.1) > 0$

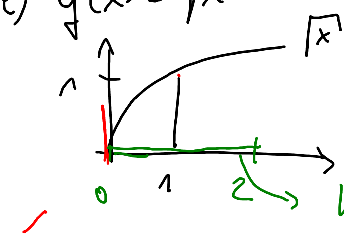
$x = -0.1 \Rightarrow \ln(0.9) < 0$ ausprobieren

$x = -1.1 \Rightarrow \ln(-0.1) = ?$ wird Reihe divergiert, ausprobieren

ebenfalls $x = 1.1 \hat{=} \ln(2.1)$ okay

aber Reihe divergiert auch,

e) $f(x) = \sqrt{x}$



$\sqrt{x}$; ? $\left.\frac{d}{dx}\sqrt{x}\right|_{x=0} \Rightarrow \infty$

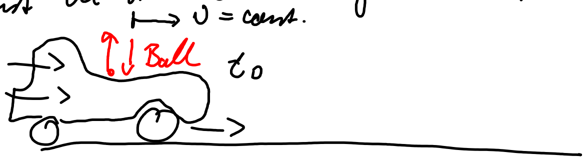
Konvergenzradius der Reihe

4.2 Inertialsysteme

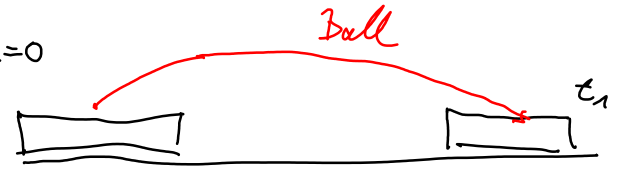
≙ nur ruhend oder gleichförmig bewegte Systeme ($v \neq 0$) also $a \doteq 0$

sonst nicht-Inertialsysteme ⇒ es treten Scheinkräfte auf

Bsp.

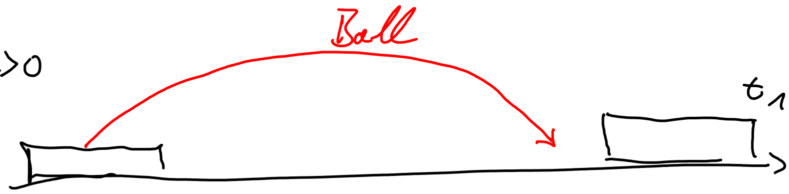


$a = 0$



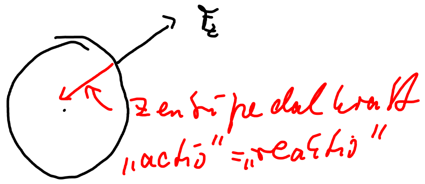
Ball + Auto gleich

$a > 0$



Auto scheinbar nach vorne mit Kraft „geschoben"

Scheinkräfte immer wenn $\vec{a} \neq 0$ ⇒ • Trägheitskraft $|\vec{a}| \neq 0$



• Zentrifugalkraft $|\vec{a}| = \text{const.}$ Richtung ändert sich (auf Kreisbahn $R = \text{const.}$)

• Corioliskraft & Eulerkraft $R \neq \text{const.}$

Herleitung: Zentrifugal (Zentripedal) & Corioliskraft

$$\vec{r}(t) = \begin{pmatrix} r(t)\cos\varphi(t) \\ r(t)\sin\varphi(t) \end{pmatrix}$$ in kart. Koor.

$r(t) = $ allg. ; $\varphi(t) = \omega \cdot t \Rightarrow \omega = \text{const.}$; sonst $\omega \neq \text{const.} \mathrel{\hat=}$ Eulerkraft.

$$\dot{\vec{r}}(t) = \begin{pmatrix} -r\omega\sin\omega t + \dot{r}\cos\omega t \\ r\omega\cos\omega t + \dot{r}\sin\omega t \end{pmatrix} = \vec{v}(t)$$

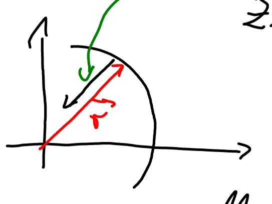
$$\ddot{\vec{r}}(t) = \vec{a}(t) = \begin{pmatrix} -\dot{r}\omega\sin\omega t - r\omega^2\cos\omega t + \ddot{r}\cos\omega t - \dot{r}\omega\sin\omega t \\ \dot{r}\omega\cos\omega t - r\omega^2\sin\omega t + \ddot{r}\sin\omega t + \dot{r}\omega\cos\omega t \end{pmatrix}$$

$$\vec{e}_r = \begin{pmatrix} \cos\varphi(t) \\ \sin\varphi(t) \end{pmatrix} \quad \forall t$$

$$= (-r\omega^2 + \ddot{r})\vec{e}_r + 2\dot{r}\omega\vec{e}_\varphi \, ; \qquad \vec{e}_\varphi = \begin{pmatrix} -\sin\varphi(t) \\ \cos\varphi(t) \end{pmatrix}$$

Zentripedalkomponente

Corioliskomponente



wenn $\omega \neq \text{const.} \mathrel{\hat=}$ variable Drehrate ⇒ auch Eulerkraft.

allg. $\vec{a} = \ddot{\vec{r}} = \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \dot{\vec{\omega}} \times \vec{r} + 2\vec{\omega} \times \dot{\vec{r}}$

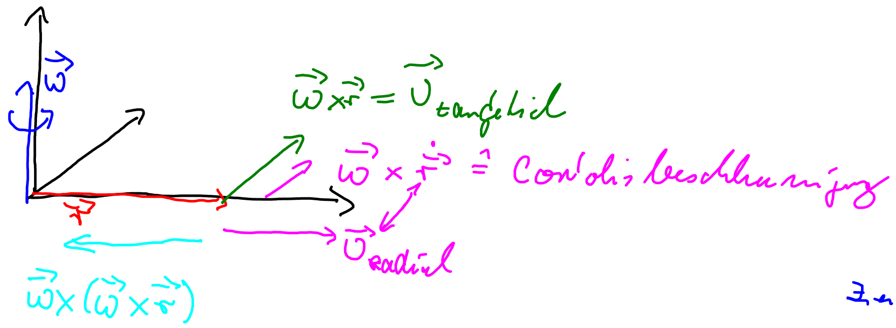
Zentripedal

$\mathrel{\hat=} \vec{v}_t$

Eulerkraft

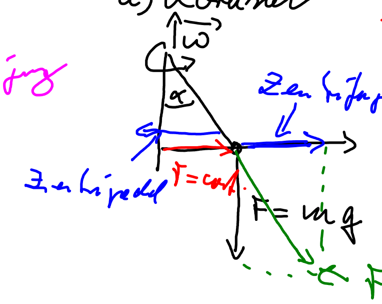
Coriolis

$\mathrel{\hat=} \vec{v}_{radial}$



Bsp. Zentripedalkraft

a) Karussel



$r = \text{const.} \Rightarrow v_r = 0$

$\Rightarrow$ Corioliskraft = 0

$\vec F_{zf} = -\vec F_{zp} = -m\omega^2 r$

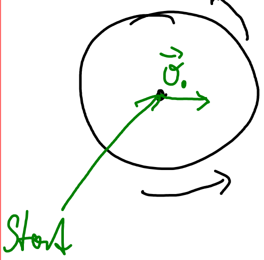
$\uparrow$

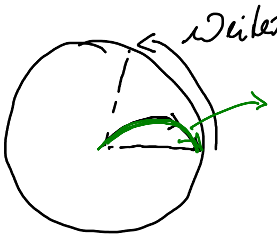
$F = m\cdot a$

b) $r \neq$ const in Ebene ≙ Corioliskraft

drehende Scheibe $\omega > 0$

$t<0$ $\quad$ $t>0$

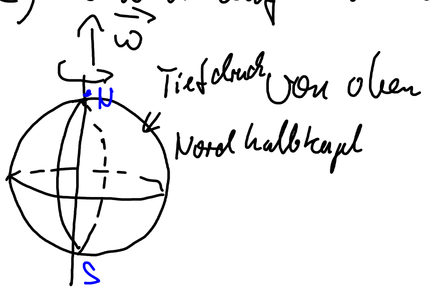


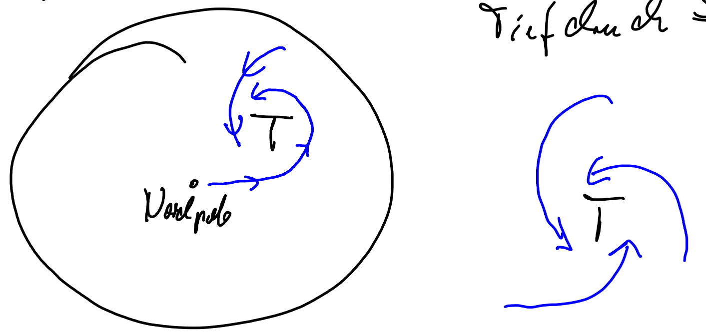


c) Coriolis auf der Erde; Erde zu einer Scheibe gedreht

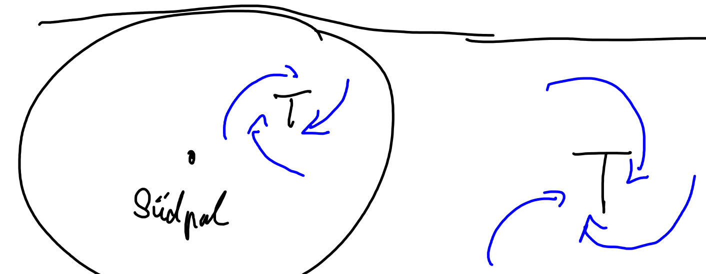
Tiefdruck $\Rightarrow p < \bar p_L$ mittlerer Luftdruck

ansaugen Luft





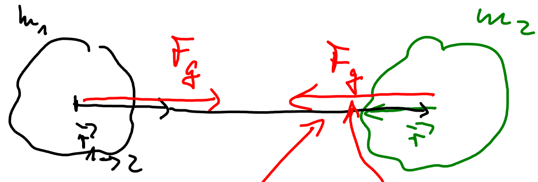
analog



4.3 Gravitationskraft (echte, keine Scheinkraft)

$$F_g = G\,\frac{m_1 m_2}{r^2}$$

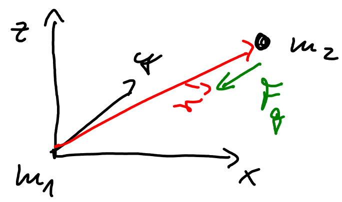
$$\vec F_{1\to2} = -G\,\frac{m_1 m_2}{r^2}\cdot\frac{\vec r_{1\to2}}{|\vec r|}$$



auf Erdoberfläche $|\vec r| = 6300\,km$

$$F_g = m\cdot g\ ;\ g = 9{,}81\,m/s^2\ ;\ g = G\,\frac{m_E}{r_E^2}$$

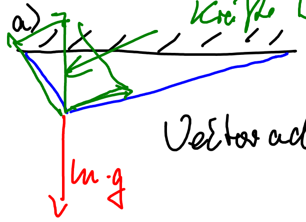
In kartesischen Koor.



$$\vec{F}_g = -G \cdot \frac{m_1 m_2}{x^2+y^2+z^2} \begin{pmatrix} \frac{x}{\sqrt{x^2+y^2+z^2}} \\ \frac{y}{\sqrt{x^2 \ldots}} \\ \frac{z}{\sqrt{\ldots}} \end{pmatrix}$$

4.4. Seilkräfte

Bsp.



Kräfte von dem Seil

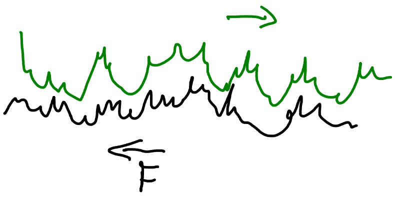
Vektoraddition

Eigenschaften: i) immer am Seil entlang
ii) immer gleich groß entlang eines Stückes

4.5. Reibungskräfte

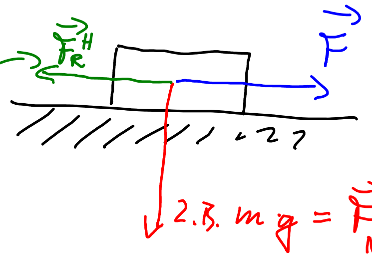
Def: zwischen Grenzflächen Körper / Körper oder Körper / Flüssigkeit / Gas

Entstehung: mikroskopische Rauhigkeiten / Haften an Oberfläche



z.B. durch Haftreibung ist $F > 0$ erforderlich für gegenseitige Bewegung

a) Haftreibung



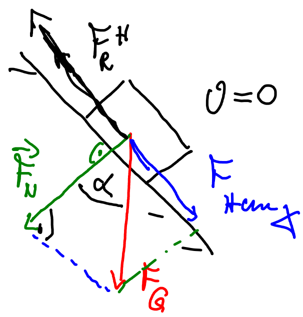
$\vec{F}_R^H \perp \vec{F}_N$

wenn $|\vec{F}_R^H| > |\vec{F}|$ dann Ruhe, sonst gleiten

F_N Normalkraft zur Oberfläche

$$F_R^H = \mu_H \cdot F_N$$

μ_H: Materialkonst. zw. Oberfläche und Körper



$F_N = m \cdot g \cdot \cos\alpha$

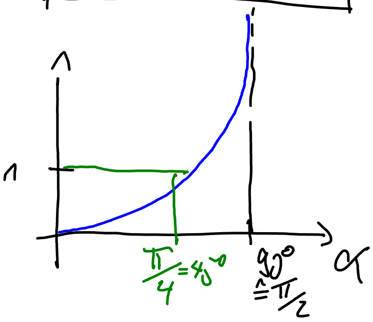
kein Gleiten

$F_{Hang} = m \cdot g \sin\alpha < F_R^H = \mu_H \cdot m g \cos\alpha$

$$\Rightarrow \boxed{\tan\alpha < \mu}$$

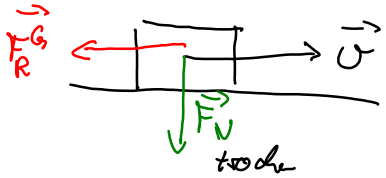
Bsp. $\mu_H = 0{,}1 \ldots 0{,}2$ Stahl / Stahl

$0.9 \ldots 1{,}3$ Gummi auf Asphalt



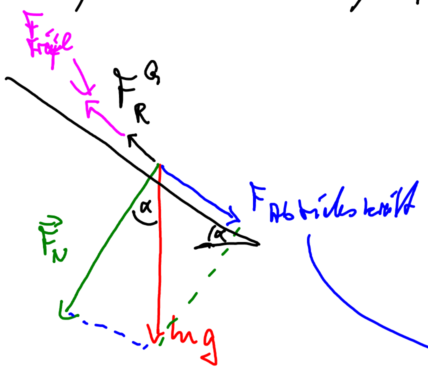
• Gleitreibung, $v > 0$

$\vec{F}_R^G \uparrow\downarrow \vec{v}$



trocken   mit öl

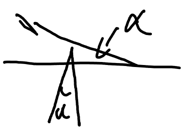
$|\vec{F}_R^G| = \mu_G \cdot F_N$ $\mu_G = 0{,}1 \ldots 0{,}01$ Stahl / Stahl



Körper wird beschleunigt => es entsteht nach actio = reactio eine Trägheitskraft entgegen der Beschleunigung

$F_{Träge}^G = m \cdot a$

$mg \cdot \sin\alpha = \mu \cdot mg\cos\alpha + m \cdot a$



$a = g\,(\underbrace{\sin\alpha - \mu\cos\alpha}_{\text{wenn dies} > 0 \text{ sonst Haften}})$