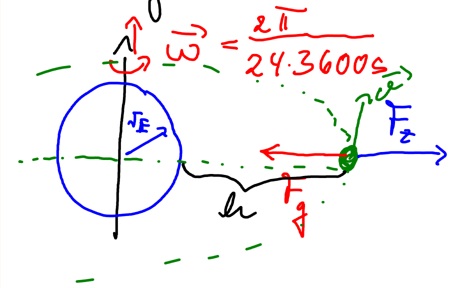
Vorlesung 18.12.20

30) ges. ϑ und h von Satellit



$$|\vec{F}_g| = |\vec{F}_z|$$

$$F_g = G\frac{mM}{(r_E+h)^2}\ ;\quad F_z = \frac{m\vartheta^2}{(r_E+h)}\ ;\quad \vartheta = \frac{u}{T} = \frac{2\pi(r_E+h)}{24\cdot 3600\,s}$$

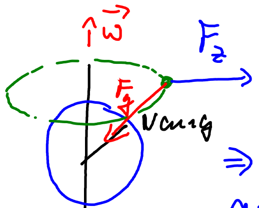
$$F_z = \frac{\cancel{m}\,4\pi^2(r_E+h)^2}{(r_E+h)\,T^2} = G\frac{\cancel{m}M}{(R+h)^2}$$

$$\frac{4\pi^2}{T^2}(r_E+h)^3 = G\cdot M\ ;\quad \text{aus } \cancel{m}\cdot g = G\frac{\cancel{m}M}{r_E^2}$$

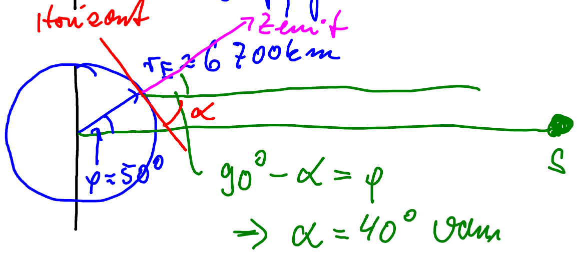
$$\frac{4\pi^2}{T^2}(r_E+h)^3 = r_E^2\, g$$

$$h = \sqrt[3]{\frac{g\, r_E^2\, T^2}{4\pi^2}} - r_E\ ;\quad T = 86000\,s$$

$$\underline{h = 35\,800\,km} \Rightarrow \vartheta = 3{,}1\,km/s$$



$\Rightarrow$ Kräfte heben sich nicht auf, geht nicht



$90° - \alpha = \varphi$

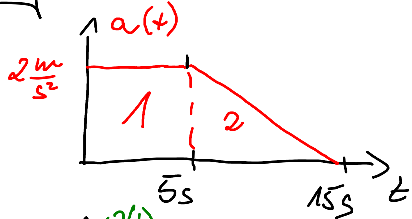
$\Rightarrow \alpha = 40°$ vom Horizont

Was ist die kleinste Umlaufzeit
$\hat{=}\ h \to 0$

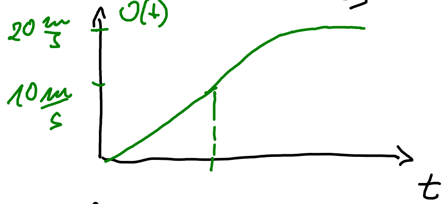
$$\Rightarrow T = 1{,}4\,h\ ;\quad \vartheta = r_E\cdot\omega_E = 7{,}9\,\frac{km}{s}$$

$\hat{=}$ 1. kosmische Geschwindigkeit

31) konst. und variable Beschleunigung



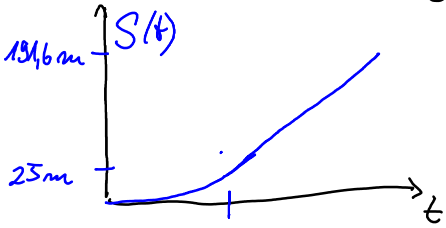
$$a(t) = \begin{cases} 2\,m\,s^{-2} & 0 \le t \le 5\,s \quad (1) \\ 2\,m\,s^{-2} - 0.2\,\frac{m}{s^3}(t-5\,s) & 5\,s \le t \le 15\,s \end{cases}$$



Phase 1

$$\vartheta = a\cdot t\ ;\quad \vartheta(5\,s) = 10\,m\,s^{-1} \mathrel{\hat{=}} \vartheta_0 \text{ für Phase 2}$$

$$s = \frac{a}{2}t^2\ ;\quad s(5\,s) = 25\,m \mathrel{\hat{=}} s_0 \text{ für Phase 2}$$



Phase 2

$$t' = t - 5\,s\ ;\quad a(t) = \underbrace{a_0 - \beta t'}\ ;\quad a_0 = 2\,m/s^2$$
$$\beta = 0{,}2\,m\,s^{-3}$$

$$\vartheta(t') = \vartheta_0 + \int_0^{t'} a(t')\,dt' = \vartheta_0 + a_0 t' - \beta\frac{t'^2}{2}\ ;\quad \vartheta_0 = 10\,m\,s^{-1}$$

$$\vartheta(t' = 10\,s) = 10\,m\,s^{-1} + 20\,m\,s^{-1} - 0{,}2\,m\,s^{-3}\cdot\frac{100\,s^2}{2} = 20\,m\,s^{-1}$$

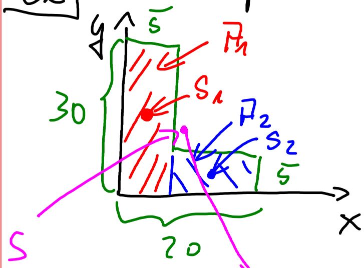
$$S(t') = S_0 + \int_0^{t'} v(t)\,dt' = S_0 + v_0 t' + \frac{a_0}{2} t'^2 - \beta \frac{t'^3}{6} ;$$

$$S(t' = 10\,s) = 25\,m + 100\,m + 100\,m - 0.2\,m s^{-3} \cdot \frac{1000\,s^3}{6}$$

$$= 191{,}6\,m$$

32| Schwerpunkte

$$\vec{r}_S = \frac{\sum \vec{r}_i \cdot m_i}{\sum m_i}$$



Schwerpunkt in x und y-Richtung

$$x_S = \frac{A_1 \cdot x_1 + A_2 x_2}{A_1 + A_2} ; \quad A_1 = 150 ; \; x_1 = 2{,}5 \qquad A_2 = 75 \quad x_2 = 12{,}5$$

$$x_S = 5{,}8$$

analog y

$$y_S = \frac{A_1 \cdot y_1 + A_2 y_2}{A_1 + A_2} ; \quad y_1 = 15 \qquad y_2 = 2{,}5$$

$$y_S = 10{,}8$$ S ist außerhalb Körper

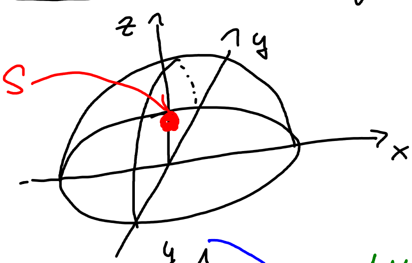
Schwerpunkt Halbkugel

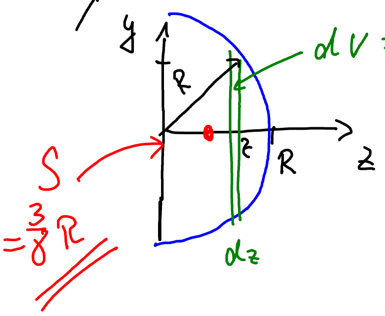
wo auf der z-Achse?

$$dm = \rho\, dV$$

$$\vec{r}_S = \frac{\int \vec{r}\, dm}{\int dm} = \frac{\int \vec{r}\, dV}{\int dV}$$

$$\vec{r}_S = \begin{pmatrix} r_{S,x} \\ r_{S,y} \\ r_{S,z} \end{pmatrix} \begin{matrix} \to 0 \\ \to 0 \\ \to ?? \end{matrix}$$





$$dV = A_y \cdot dz = \pi \cdot y(z)^2 dz = \pi\,(R^2 - z^2)\,dz$$

$$y(z) = \sqrt{R^2 - z^2}$$

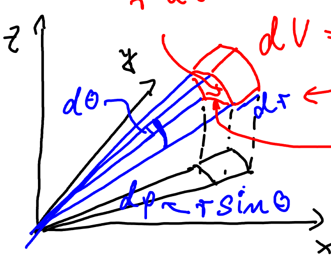
$$\vec{r}_{S,z} = \frac{\int_0^R z\,\pi\,(R^2 - z^2)\,dz}{V_{HK} = \frac{2}{3}\pi R^3} = \frac{3}{2R^3} \int_0^R (R^2 z - z^3)\,dz$$

$$\vec{r}_{S,z} = \frac{3}{2R^3} \left[\frac{R^2 z^2}{2} - \frac{z^4}{4} \right]_0^R = \frac{3}{8} R$$

Vorlesung zu Volumenintegrale

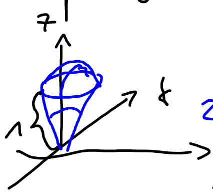
g) Kugelkoor $(x,y,z) \to (r,\theta,\varphi)$

$dx\cdot dy\cdot dz \to r^2 \sin\theta \, dr \, d\varphi \, d\theta$



$dV = dr \cdot r\sin\theta \, d\varphi \cdot r \, d\theta$

Bsp. Volumen Kegelsegment



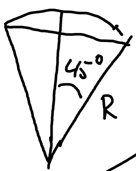
z.B. 90° Öffnung

$\Rightarrow \theta = 0 \ldots \frac{\pi}{4}$ ← 45°

$$V = \int_0^1 dr \, r^2 \int_0^{2\pi} d\varphi \int_0^{\pi/4} d\theta \sin\theta = 2\pi \left[-\cos\theta\right]_0^{\frac{\pi}{4}} = \frac{2\pi}{3}\left(1-\frac{1}{\sqrt{2}}\right)$$

Tafelwerk

$$V = \frac{2\pi R^2 \cdot h}{3}$$



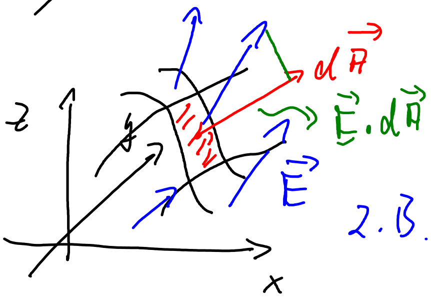
$h = R(1-\cos 45°) = 1 - \frac{1}{\sqrt{2}}$

$$V = \frac{2\pi}{3}\left(1 - \frac{1}{\sqrt{2}}\right) \checkmark \text{ OK}$$

Bem: es gibt noch andere Integrale z.B. Oberflächenintegrale

A = ?

$\Rightarrow$ Kugelkoor. $dA = r\sin\theta \, d\theta \, R \, d\varphi$ ← kein dr weil Fläche



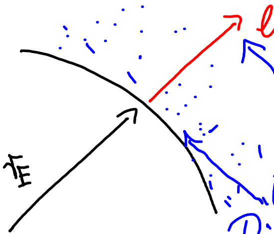
$\vec{E}\cdot d\vec{A}$

: z.B. in Elektrodynamik, gerichtete Oberflächenintegrale

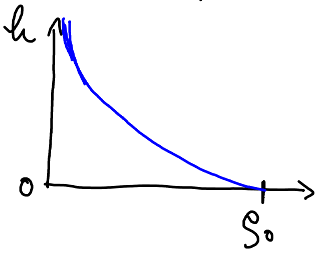
z.B. ges $\iint \vec{E}\cdot d\vec{A} = \ldots$

Anwendung Volumenintegrale

Was ist die Masse der Erdatmosphäre?



$\Rightarrow \rho(h), p(h)$



Luft = ideales Gas, & $T = $ const (Annahme)

$p\cdot V = n R \cdot T$; $R = 8{,}3 \frac{J}{mol\,K} = k_B \cdot N_A$; $n = \frac{m}{M}$

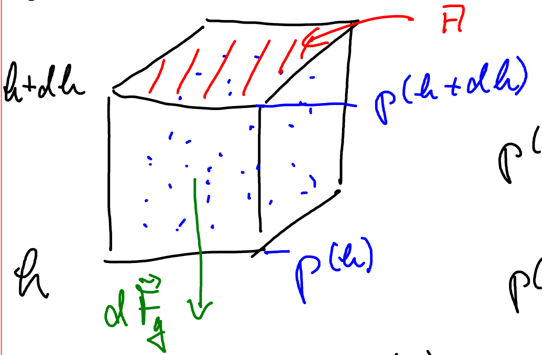
Boltzmannkonst. (k_B) ; M molare Masse

$$M_{Luft} = \frac{4}{5} M_{N_2} + \frac{1}{5} M_{O_2} = 29 \, g/mol$$

$\rho = \frac{m}{V}$ $\qquad p = \frac{m}{M} \cdot R \cdot T / V$

$p = \rho \cdot \frac{RT}{M}$; Check $\left.\begin{array}{l} T = 293\,K = {}^{0}C \\ p = 1\,bar = 10^5\,Pa \end{array}\right\} \rho = \rho_0 = 1{,}2\,\frac{kg}{m^3}$ ✓ Vorlag

Druckabnahme mit der Höhe aus Differentialgleichung herleiten



$$p(h) = p(h+dh) + \frac{dF_g}{A} \; ; \; dF_g = \underset{\downarrow\;\frac{p(h)\cdot M}{RT}}{\rho(h)} \cdot \underbrace{A \cdot dh}_{dV} \cdot g$$

$$p(h) = p(h+dh) + p(h)\cdot \frac{M}{RT} \cdot g\,dh$$

$$\frac{p(h) - p(h+dh)}{dh} = p(h)\frac{M}{RT}g$$

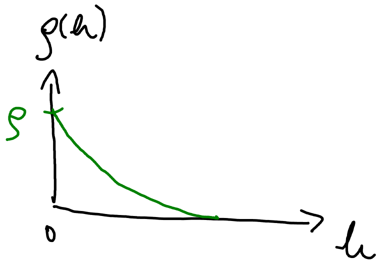
$$\frac{dp(h)}{dh} = -p(h)\frac{M}{RT}g$$; Dgl., 1. Ord., Trennung der Variablen

$$\frac{1}{p}dp = -\frac{Mg}{RT}dh \quad \Big| \int_0^h dh \; ; \; \int_{p(h=0)}^{p(h)} dp$$

$$[\ln p]_{p_0}^{p(h)} = -\frac{Mg}{RT}h$$

$$p(h) = p_0 e^{-\frac{Mg}{RT}h}$$

$$\Rightarrow \boxed{\rho(h) = \rho_0 e^{-\frac{Mg}{RT}h}} \overset{!}{=}$$



Masse der gesamten Atmosphäre

$$M = \int \rho\, dV = \int_{r_E}^{\infty} dr\, r^2 e^{-\frac{(r-r_E)Mg}{RT}} \rho_0 \underbrace{\int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta}_{=4\pi} = \underline{5{,}8 \cdot 10^{18}\,kg}$$

Typ $\int r^2 e^{-r} dr$

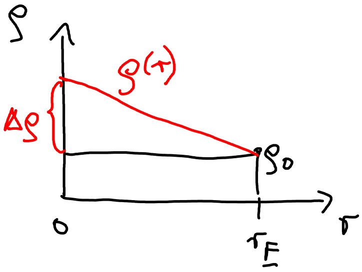
taucht immer auf, wenn bei Kugelkoor. keine Winkelabhängigkeit vorliegt

exakt nach Wiki $\quad M_{A\,exakt} = 5{,}15 \cdot 10^{18}\,kg$

33) ziemlich schwer

$M_♁ = 5{,}97 \cdot 10^{24}\,\mathrm{kg}$; $R_♁ = 6370\,\mathrm{km}$

$$\bar{\rho} = \frac{M_♁}{V} = 5500\,\mathrm{kg/m^3} > \rho_0 = 3500\,\mathrm{kg/m^3}$$



$$M_♁ = \underbrace{\int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta}_{4\pi} \cdot \int_0^{r_E} dr\, r^2 \cdot \rho(r) \; ; \quad \leftarrow \quad \rho(r) = \rho_0 + \Delta\rho \cdot \left(1 - \frac{r}{r_E}\right)$$

$$= 4\pi \cdot \left(\rho \frac{r_E^3}{3} + \Delta\rho \cdot \left(\frac{r_E^3}{3} - \frac{r_E^4}{4 r_E} \right) \right) = 5{,}97 \cdot 10^{24}\,\mathrm{kg}$$

$$\Downarrow$$

$$\Delta\rho = 8200\,\mathrm{kg/m^3}$$

$\rho(r=0) = 11700\,\mathrm{kg/m^3}$, $\rho_{Eisen} = 7800\,\mathrm{kg/m^3}$

$\uparrow$

$\hat{=}$ gedrücktes Eisen

das war es