IMPULSES AND PHYSIOLOGICAL STATES IN THEORETICAL MODELS OF NERVE MEMBRANE

Biophys. J 1, 445 (1961)

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The following linear differential equation describes an oscillating quantity x with damping constant k (the dots represent differentiation with respect to time t):

$$\ddot{x} + k\dot{x} + x = 0$$

Van der Pol (1926) replaced the damping constant by a damping coefficient which depends quadratically on x:

$$\ddot{x} + c(x^2 - 1)\dot{x} + x = 0$$

where c is a positive constant. It is convenient to use Liénard's transformation (Liénard, 1928; Minorsky, 1947):

$$y = \dot{x}/c + x^3/3 - x$$

and obtain the following pair of differential equations:

$$\dot{x} = c(y + x - x^3/3)$$

$$\dot{y} = -x/c$$

The BVP model is obtained by adding terms to these equations as follows:—

$$\dot{x} = c(y + x - x^3/3 + z) \quad (1)$$

$$\dot{y} = -(x - a + by)/c \quad (2)$$

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An Active Pulse Transmission Line Simulating Nerve Axon*

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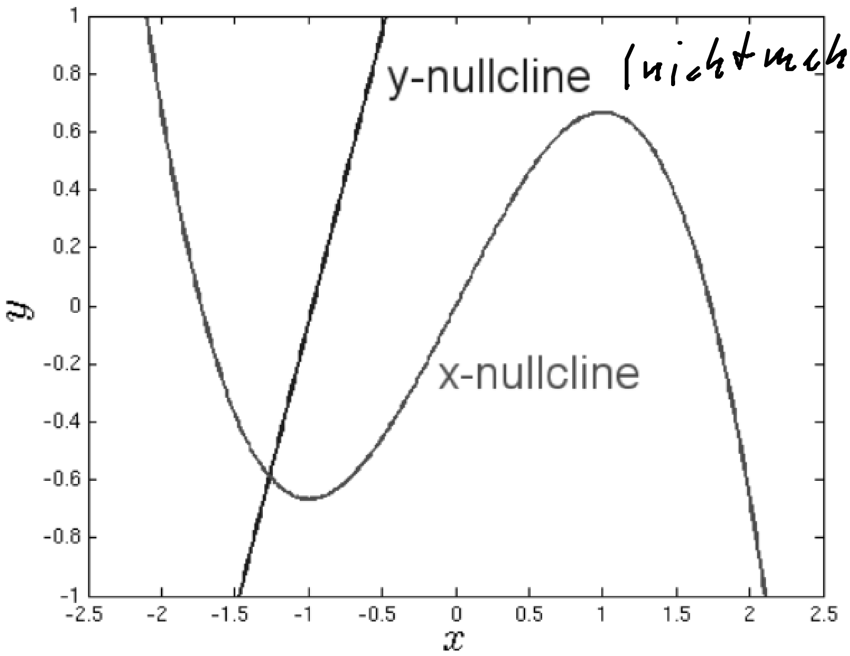
Recently, FitzHugh ingeniously simplified the H-H equations in case of a "space clamp," making use of an analog computer, and proposed the following BVP model (Bonhoeffer-van der Pol model).[9]

$$\begin{cases} J = \dfrac{1}{c}\,\dfrac{du}{dt} - w - \left(u - \dfrac{u^3}{3}\right), \\ c\,\dfrac{dw}{dt} + bw = a - u, \end{cases} \tag{2}$$



(nicht mehr senkrecht für $\gamma \neq 0$)

$\dot{y} = 0 \Rightarrow y = \frac{x}{\gamma} + \frac{a}{\gamma}$

hier: $a = 0.97$

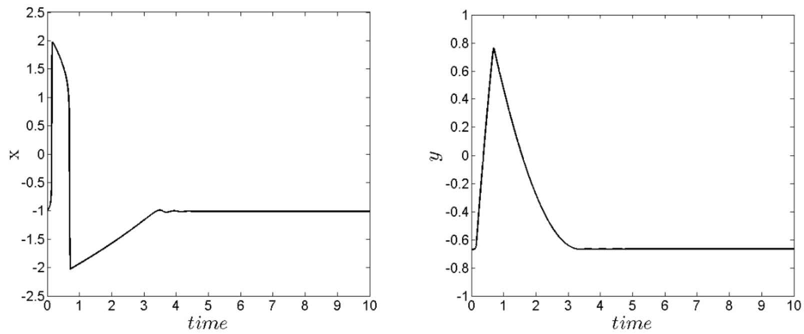
$\gamma = 0.5$

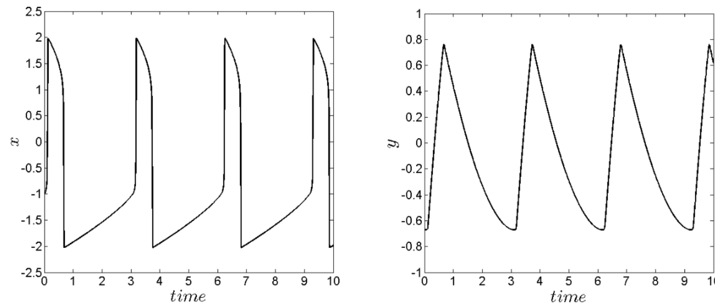
Bestimmung von (x^*, y^*): $\quad 0 = -\frac{a}{\gamma} + \left(1 - \frac{1}{\gamma}\right) x^* - \frac{(x^*)^3}{3}$

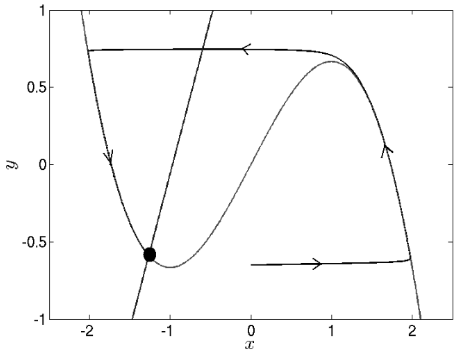
Nullstellen einer kubischen Gleichung

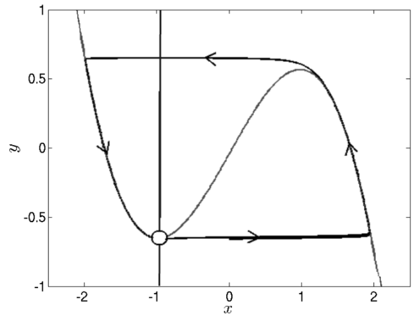
$$\begin{pmatrix} \delta\dot{x} \\ \delta\dot{y} \end{pmatrix} = \begin{pmatrix} \frac{1-(x^*)^2}{\varepsilon} & -\frac{1}{\varepsilon} \\ 1 & -\gamma \end{pmatrix} \begin{pmatrix} \delta x \\ \delta y \end{pmatrix} \Rightarrow \lambda_{1,2} = \frac{\mathrm{tr}\,\underline{\underline{J}} \pm \sqrt{(\mathrm{tr}\,\underline{\underline{J}})^2 - 4\det \underline{\underline{J}}}}{2}$$

Rolle von Parameter γ:



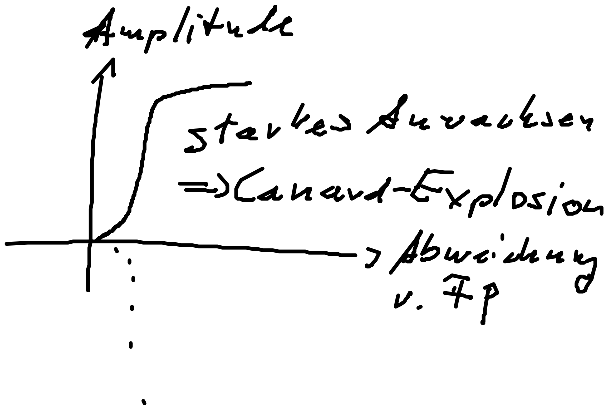


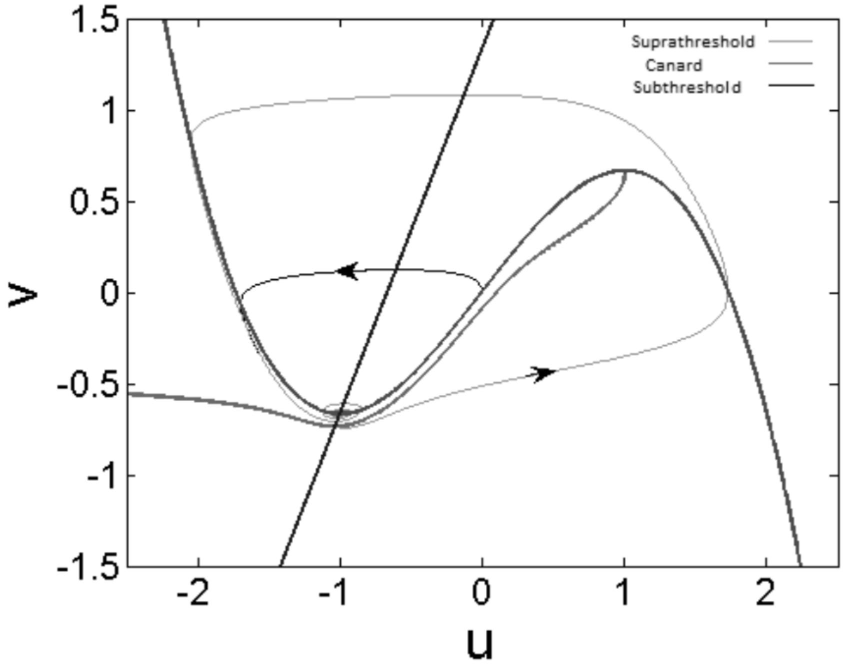




oszillierend: $\varepsilon = 0.005$, $a = 0.97$
$\gamma = 0.005$

anregbar: $\varepsilon = 0.005$, $a = 0.97$, $\gamma = 0.5$





$a = 0.67$
$\gamma = 0.5$