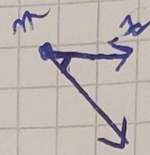
Kapitel 2 D'Alembert



$$\underline{F} = \underline{F}_E + \underline{F}_Z$$

$$\delta\underline{r}\cdot\underline{F}_Z = \delta\underline{r}\cdot(\underline{F}-\underline{F}_E)$$

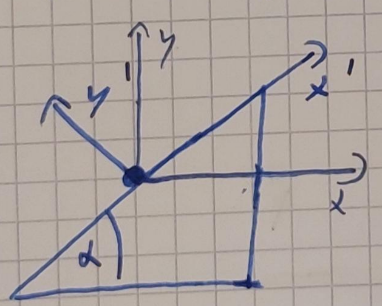
$$\sum_i \delta\underline{r}_i\cdot\underline{F}_Z = \sum_i \delta\underline{r}\cdot(\underline{F}-\underline{F}_E)=0$$

$$\sum_i \delta\underline{r}_i\cdot\underline{F}_Z = 0$$

$$\sum_i \delta\underline{r}_i\cdot(\underline{F}-\underline{F}_{E_i})=0$$

$$\boxed{\sum_i \delta\underline{r}_i\cdot(m\ddot{\underline{r}}-\underline{F}_E)=0}$$

Bsp: Körper auf schiefer Ebene



$$y/x = \tan\alpha$$

$$f(x,y) = y - x\tan\alpha = 0$$

$$\delta\underline{r} = \begin{pmatrix}\delta x\\ \delta y\end{pmatrix}$$

z.B. $\delta f = \frac{\partial f}{\partial x}\delta x + \frac{\partial f}{\partial y}\delta y = -\tan\alpha\,\delta x + \delta y = 0$

$$\Rightarrow \delta\underline{r} = \delta x\begin{pmatrix}1\\ \tan\alpha\end{pmatrix}$$

$$\delta W = (m\ddot{\underline{r}} - F)\cdot\delta\underline{r} = 0$$

$$\Rightarrow \left[m \begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} - m g \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right] \cdot \delta x \begin{pmatrix} 1 \\ \tan\alpha \end{pmatrix} = 0$$

$$\Leftrightarrow m\, \delta x \begin{pmatrix} \ddot{x} \\ \ddot{x} \tan\alpha + g \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \tan\alpha \end{pmatrix} = 0$$

$$\Rightarrow \left[m \ddot{x} (1 + \tan^2\alpha) + m g \tan\alpha \right] \delta x = 0$$

$$\Rightarrow \quad \ddot{x} = - \frac{g \tan\alpha}{1 + \tan^2\alpha}$$

mit $1 + \tan^2\alpha = \frac{1}{\cos^2\alpha}$

$$\Rightarrow \frac{\tan\alpha}{1 + \tan^2\alpha} = \sin\alpha \cos\alpha$$

$$\Rightarrow \ddot{x} = - g \sin\alpha \cos\alpha$$