Bisher: Körperfestes System! S^*

Nun: Raumfestes System: S

Euler-Winkel:
$$\begin{pmatrix}\omega_1^*\\ \omega_2^*\\ \omega_3^*\end{pmatrix} = \dot\varphi\begin{pmatrix}\sin\psi\sin\vartheta\\ \cos\psi\sin\vartheta\\ \cos\vartheta\end{pmatrix} + \dot\vartheta\begin{pmatrix}\cos\psi\\ -\sin\psi\\ 0\end{pmatrix} + \dot\psi\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$\left[\begin{pmatrix}\omega_1\\ \omega_2\\ \omega_3\end{pmatrix} = \dot\varphi\begin{pmatrix}0\\0\\1\end{pmatrix} + \dot\vartheta\begin{pmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{pmatrix} + \dot\psi\begin{pmatrix}\sin\varphi\sin\vartheta\\ -\cos\varphi\sin\vartheta\\ \cos\vartheta\end{pmatrix}\right]$$

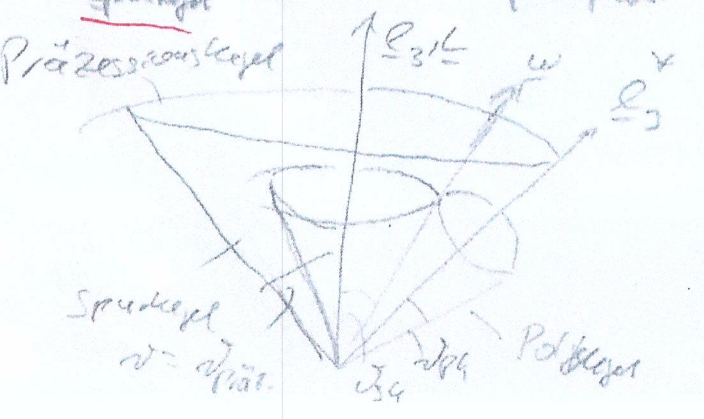
hier: $\dot\vartheta = 0 \Rightarrow$
$$\begin{pmatrix}\omega_1^*\\ \omega_2^*\\ \omega_3^*\end{pmatrix} = \dot\varphi\begin{pmatrix}\sin\psi\sin\vartheta\\ \cos\psi\sin\vartheta\\ \cos\vartheta\end{pmatrix} + \dot\psi\begin{pmatrix}0\\0\\1\end{pmatrix}$$
$$= \begin{pmatrix}\omega_\perp^*\sin(\Omega t)\\ \omega_\perp^*\cos(\Omega t)\\ \mathrm{const}\end{pmatrix}$$

$\Rightarrow \psi = \Omega t \Rightarrow \dot\psi = \Omega = \mathrm{const}$

$$\dot\varphi = \frac{\omega_\perp^*}{\sin\vartheta} = \mathrm{const} = \frac{\Theta_3\,\omega_3^*}{\Theta\cos\vartheta} = \Omega' = \frac{L}{\Theta}$$

$\Rightarrow$ Euler-Winkel: $\vartheta(t) = \vartheta_0$, $\psi(t) = \Omega t + \psi_0$, $\varphi(t) = \Omega' t + \varphi_0$

Spurkegel: $\alpha_{\text{Spurkegel}} = \arctan\dfrac{\dot\psi\sin\vartheta}{\dot\varphi + \dot\psi\cos\vartheta} = \arctan\dfrac{\Omega\sin\vartheta}{\Omega' + \Omega\cos\vartheta}$

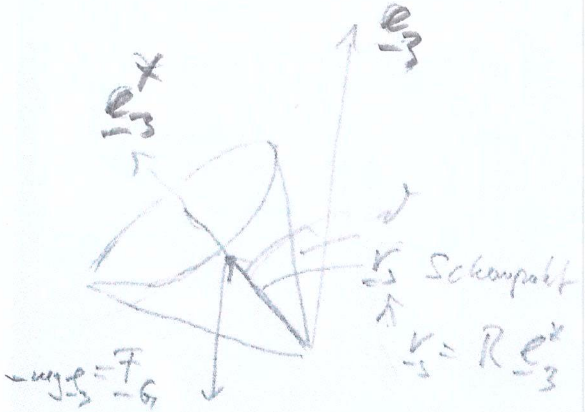


wäre die Erde kräftefreier symm. Kreisel:

$$\frac{\Theta - \Theta_3}{\Theta} \approx \frac{1}{300}$$

in Realität: $\frac{1}{430}$ (Gezeiten...)

2) Schwerer, symmetrischer Kreisel:



$\underline{F}_G = -mg\,\underline{e}_3$ greift am Schwerpunkt an

Drehmoment: $\underline{M} = \underline{r}_S \times \underline{F}$

$= -mgR\,\underline{e}_3^* \times \underline{e}_3$

$\Rightarrow \underline{M} \perp$ Figurenachse

Gesamtenergie: $E = T + V = const$

$V = mg\,\underline{e}_3 \cdot \underline{r}_S = mgR\,\underline{e}_3 \cdot \underline{e}_3^* = mgR\cos\vartheta$

$T = \frac{1}{2}\Theta(\omega_1^2 + \omega_2^2) + \frac{1}{2}\Theta_3\omega_3^2$

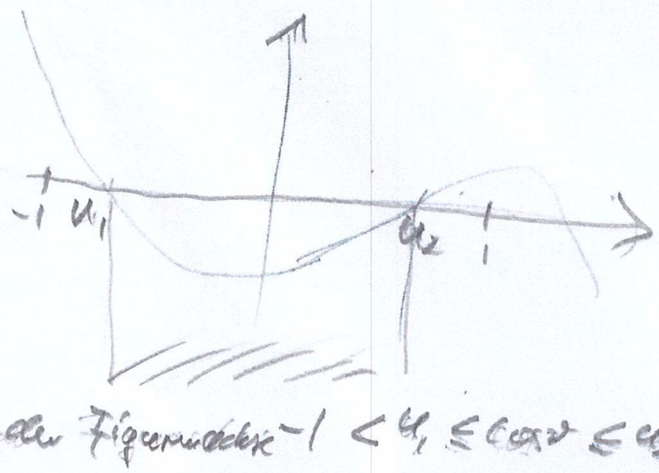
$$\Rightarrow E = \frac{\Theta}{2}(\dot\vartheta^2 + \dot\varphi^2\sin^2\vartheta) + \frac{\Theta_3}{2}(\dot\psi + \dot\varphi\cos\vartheta)^2 + mgR\cos\vartheta = const$$

$$\hookrightarrow E = \frac{\Theta}{2}\dot\vartheta^2 + \frac{(L_3 - L_3^*\cos\vartheta)^2}{2\Theta\sin^2\vartheta} + \frac{L_3^{*2}}{2\Theta_3} + mgR\cos\vartheta$$

$$\underset{u=\cos\vartheta}{\Leftrightarrow} \quad \dot u^2 + V_{eff}(u) = 0, \quad \dot u = -\dot\vartheta\sin\vartheta$$

$$\Rightarrow V_{eff}(u) = \frac{2}{\Theta}\left(\frac{L_3^{*2}}{2\Theta_3} + mgRu - E\right)(1-u^2) + \left(\frac{L_3 - L_3^* u}{\Theta}\right)^2$$

$$= (C_1 + C_2 u)(1-u^2) + (C_3 - C_4 u)^2$$



Bewegung der Figurenachse $-1 < u_1 \le \cos\vartheta \le u_2 < 1$ (Nutation)

3) Kräftefreier unsymmetrischer Kreisel!

$$0 < \Theta_1 < \Theta_2 < \Theta_3$$

$$\dot{\omega}_1 = -\frac{\Theta_3-\Theta_2}{\Theta_1}\,\omega_2\omega_3 = -\alpha_1\omega_2\omega_3$$

$$\dot{\omega}_2 = \frac{\Theta_3-\Theta_1}{\Theta_2}\,\omega_3\omega_1 = \alpha_2\omega_3\omega_1$$

$$\dot{\omega}_3 = -\frac{\Theta_2-\Theta_1}{\Theta_3}\,\omega_1\omega_2 = -\alpha_3\omega_1\omega_2$$

Fixpunkte: $\underline{\omega}^* = \begin{pmatrix}\omega\\0\\0\end{pmatrix}$ oder $\begin{pmatrix}0\\\omega\\0\end{pmatrix}$ oder $\begin{pmatrix}0\\0\\\omega\end{pmatrix}$

(Rotation um x- y- z- Achse)

Linearisierung: $\delta\underline{\omega} = \underline{\omega} - \underline{\omega}^*$

$$\begin{pmatrix}\delta\dot{\omega}_1\\\delta\dot{\omega}_2\\\delta\dot{\omega}_3\end{pmatrix} = \underbrace{\begin{pmatrix}0 & -\alpha_1\omega_3 & -\alpha_1\omega_2\\ \alpha_2\omega_3 & 0 & \alpha_2\omega_1\\ -\alpha_3\omega_2 & -\alpha_3\omega_1 & 0\end{pmatrix}}_{=A}\begin{pmatrix}\delta\omega_1\\\delta\omega_2\\\delta\omega_3\end{pmatrix}$$

Eigenwerte für:

$\underline{\omega}^{*(1)} = \begin{pmatrix}\omega\\0\\0\end{pmatrix}$: $0 = \det(\underline{A} - \lambda\mathbb{1}) = \begin{vmatrix}-\lambda & 0 & 0\\ 0 & -\lambda & \alpha_2\omega\\ 0 & -\alpha_3\omega & -\lambda\end{vmatrix} = -\lambda(\lambda^2 + \alpha_2\alpha_3\omega^2)$

$\Rightarrow \lambda_1^{(1)} = 0, \quad \lambda_{2,3}^{(1)} = \pm i\omega\sqrt{\alpha_2\alpha_3}$ stabil / Zentrum

$\underline{\omega}^{*(2)} = \begin{pmatrix}0\\\omega\\0\end{pmatrix}$: $0 = \begin{vmatrix}-\lambda & 0 & -\alpha_1\omega\\ 0 & -\lambda & 0\\ -\alpha_3\omega & 0 & -\lambda\end{vmatrix} = -\lambda(\lambda^2 - \alpha_1\alpha_3\omega^2)$

$\Rightarrow \lambda_1^{(2)} = 0, \quad \lambda_{2,3}^{(2)} = \pm\omega\sqrt{\alpha_1\alpha_3}$ instabil (Sattelpunkt)

$\underline{\omega}^{*(3)} = \begin{pmatrix}0\\0\\\omega\end{pmatrix}$: $0 = \begin{vmatrix}-\lambda & -\alpha_1\omega & 0\\ \alpha_2\omega & -\lambda & 0\\ 0 & 0 & -\lambda\end{vmatrix} = -\lambda(\lambda^2 + \alpha_1\alpha_2\omega)$

$\Rightarrow \lambda_1^{(3)} = 0, \quad \lambda_{2,3}^{(3)} = \pm i\omega\sqrt{\alpha_1\alpha_2}$ stabil / Zentrum

$\Rightarrow$ Dynamische Systeme