$\underline{a} \times \underline{b} \times \underline{c} \qquad \underline{b}(\underline{a}\cdot\underline{c}) - (\underline{a}\cdot\underline{b})\underline{c}$

$$\cdot \quad \operatorname{rot} \underline{B} = \underline{\nabla} \times (\underline{\nabla} \times \underline{A}) = \underline{\nabla}(\underline{\nabla} \cdot \underline{A}) - \underbrace{\Delta}_{(\underline{\nabla}\cdot\underline{\nabla})} \underline{A}$$

$$\text{mit } \underbrace{\underline{\nabla}_r}_{\text{bzgl. } \underline{r}!} \cdot \underline{A} = \frac{\mu_0}{4\pi} \int_{\mathbb{R}^3} d^3r' \, \underline{\nabla}_r \cdot \frac{\underline{j}(\underline{r}')}{|\underline{r} - \underline{r}'|}$$

$$= \frac{\mu_0}{4\pi} \int_{\mathbb{R}^3} d^3r' \, \underline{j}(\underline{r}') \left(- \underbrace{\underline{\nabla}_{r'}}_{\text{bzgl. } \underline{r}'!} \frac{1}{|\underline{r} - \underline{r}'|} \right)$$

Produkt-regel

$$= \frac{\mu_0}{4\pi} \int_{\mathbb{R}^3} d^3r' \left[- \underline{\nabla}_{r'} \left(\frac{\underline{j}(\underline{r}')}{|\underline{r} - \underline{r}'|} \right) + \frac{1}{|\underline{r} - \underline{r}'|} \underbrace{\underline{\nabla}_{r'} \underline{j}(\underline{r}')}_{=0 \text{ weil stationär}} \right]$$

$\left(\frac{\partial}{\partial t} \rho + \operatorname{div} \underline{j} = 0 \Rightarrow \frac{\partial}{\partial t} \rho = 0 \right)$

$$= - \frac{\mu_0}{4\pi} \int_{\mathbb{R}^3} d^3r' \, \underline{\nabla}_{r'} \left(\frac{\underline{j}(\underline{r}')}{|\underline{r} - \underline{r}'|} \right)$$

Satz v. Gauß

$$\stackrel{\downarrow}{=} - \frac{\mu_0}{4\pi} \int_{S_\infty} d\underline{f}' \, \frac{\underline{j}(\underline{r}')}{|\underline{r} - \underline{r}'|} = 0$$

S_∞: Sphäre mit Radius $\to \infty$

für hinreichend schnell abklingende ($r \to \infty$) Stromverteilung

(Coulomb-Eichung: $\underline{\nabla} \cdot \underline{A} = 0$)

$$\Rightarrow \operatorname{rot} \underline{B} = -\Delta \underline{A} = - \frac{\mu_0}{4\pi} \int_{\mathbb{R}^3} d^3r' \, \underline{j}(\underline{r}') \underbrace{\Delta \frac{1}{|\underline{r} - \underline{r}'|}}_{= -4\pi \delta(\underline{r} - \underline{r}')}$$

$$\Rightarrow \operatorname{rot} \underline{B} = \mu_0 \underline{j}(\underline{r})$$

(nur für stationäre Strom- und Ladungsverteilungen!)

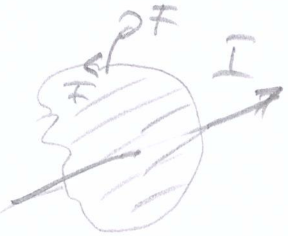
$$\Rightarrow \Delta \underline{A} = - \mu_0 \underline{j}(\underline{r})$$

vektorielle Poisson-Gleichung

(vgl. $\Delta \phi = - \frac{1}{\varepsilon_0} \rho$)

Satz v. Stokes

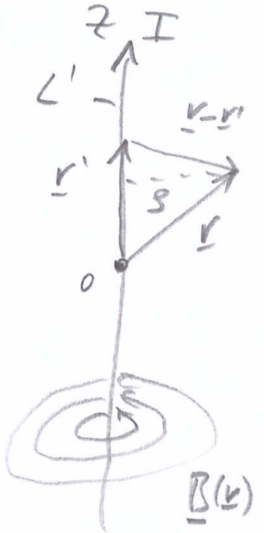
• $$\int_F d\underline{f}\cdot \mathrm{rot}\,\underline{B} \overset{!}{=} \oint_{\partial F} d\underline{s}\cdot\underline{B} = \mu_0 \underbrace{\int_F d\underline{f}\cdot \underline{j}(\underline{r})}$$



$$\Rightarrow \underbrace{\oint_{\partial F} d\underline{s}\cdot \underline{B}}_{\text{Zirkulation entlang geschlossenem Weg } \partial F} = \mu_0 \underbrace{I}_{\text{Strom durch die geschlossene Fläche}}$$

Ampère'sches Durchflutungsgesetz

Bsp.: (i) gerader Leiter (unendlich lang)



magnetische Induktion: Biot-Savart-Gesetz

$$\underline{B}(\underline{r}) = \frac{\mu_0}{4\pi} I \int_{L'} d\underline{r}' \times \frac{|\underline{r}-\underline{r}'|}{|\underline{r}-\underline{r}'|^3}$$

Zylinderkoordinaten: $\underline{r}' = z'\underline{e}_z \Rightarrow d\underline{r}' = dz'\underline{e}_z$

$$\underline{r}-\underline{r}' = \rho\,\underline{e}_\rho + (z-z')\,\underline{e}_z$$

$$\underline{e}_z\times\left[\rho\,\underline{e}_\rho + (z-z')\,\underline{e}_z\right] = \rho\,\underline{e}_\varphi$$

$$\Rightarrow \underline{B}(\underline{r}) = \frac{\mu_0 I}{4\pi}\rho\,\underline{e}_\varphi \int_{-\infty}^{\infty} \frac{dz'}{\left[\rho^2+(z-z')^2\right]^{3/2}}$$

$$= \underbrace{\frac{\mu_0}{4\pi}\cdot\frac{I}{\rho}}_{|\underline{B}| = B(\rho) = B_\varphi(\rho)}\underline{e}_\varphi \underbrace{\left.\frac{z'-z}{\left[\rho^2+(z-z')^2\right]^{1/2}}\right|_{-\infty}^{\infty}}_{=2}$$

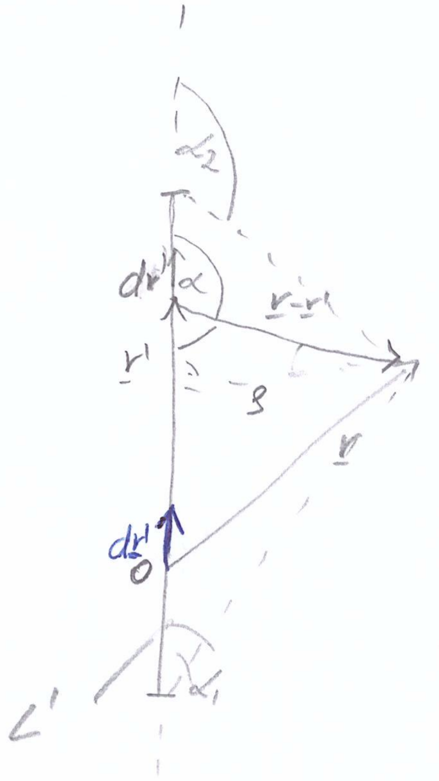
$$= \frac{\mu_0}{2\pi}\frac{I}{\rho}\underline{e}_\varphi = \frac{\mu_0}{2\pi}\frac{I}{\rho^2}\begin{pmatrix}-\rho\sin\varphi\\ \rho\cos\varphi\\ 0\end{pmatrix} = \frac{\mu_0}{2\pi}\frac{I}{\rho^2}\begin{pmatrix}-y\\ x\\ 0\end{pmatrix}$$

$\underline{B}$-Linien: konzentrische Kreise um L

• Zirkulation der magnetischen Induktion:

$$\oint \underline{B}\cdot d\underline{s} = \frac{\mu_0}{2\pi}\frac{I}{\rho}\underbrace{\int_0^{2\pi}\rho\, d\varphi}_{2\pi\rho} = \mu_0 I$$

(ii) gerader Leiter (endliche Länge)



$$|\underline{B}(\underline{r})| = \frac{\mu_0 I}{4\pi}\int_{L'} \frac{d\underline{r}' \times (\underline{r}-\underline{r}')}{|\underline{r}-\underline{r}'|^3}$$

$$= \frac{\mu_0 I}{4\pi}\int_{L'} \frac{dr'\,|r-r'|\sin\alpha}{|\underline{r}-\underline{r}'|^3}$$

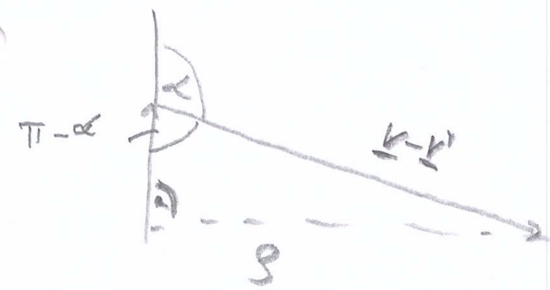
$$|d\underline{r}' \times (\underline{r}-\underline{r}')| = dr'\,|\underline{r}-\underline{r}'|\sin\alpha$$

$$= \frac{\mu_0 I}{4\pi}\int_{\alpha_1}^{\alpha_2} d\alpha\, \frac{\cancel{|\underline{r}-\underline{r}'|^2}}{|\underline{r}-\underline{r}'|^3}$$

$$dr'\sin\alpha = |\underline{r}-\underline{r}'|\,d\alpha$$

$$= \frac{\mu_0 I}{4\pi}\int_{\alpha_1}^{\alpha_2} d\alpha\, \frac{1}{|\underline{r}-\underline{r}'|}$$ ← hängt von α ab!

$$= \frac{\mu_0 I}{4\pi\rho}\int_{\alpha_1}^{\alpha_2} d\alpha\, \sin\alpha$$



$$\sin(\pi-\alpha) = \frac{\rho}{|\underline{r}-\underline{r}'|} = \sin\alpha$$

$$= \frac{\mu_0 I}{4\pi\rho}\left[-\cos\alpha\right]_{\alpha_1}^{\alpha_2}$$

$$= \frac{\mu_0 I}{4\pi\rho}\left(\cos\alpha_1 - \cos\alpha_2\right)$$

Test: unendlich langer Leiter: $\alpha_1 = 0$, $\alpha_2 = \pi$

$\cos\alpha_1 = 1$, $\cos\alpha_2 = -1$

$$\Rightarrow |\underline{B}(\underline{r})| = \frac{\mu_0 I}{4\pi\rho}\left(1-(-1)\right)$$

$$= \frac{\mu_0 I}{2\pi\rho} \quad (= B(\rho))$$