• Energiebilanz:

Test: Kontinuitätsgleichung: $\frac{\partial \rho}{\partial t} + \underline{\nabla} \cdot \underline{j} = 0$

mit $\rho = \underline{\nabla} \cdot \underline{D}$ $\quad \hookrightarrow \underline{\nabla} \cdot (\dot{\underline{D}} + \underline{j}) = \underline{\nabla} \cdot (\underline{\nabla} \times \underline{H}) = 0$ (Ladungserhaltung)

$$\uparrow$$
$$\underline{\nabla} \times \underline{H} - \dot{\underline{D}} = \underline{j}$$

Frage: Welche Erhaltungssätze in Maxwellgleichungen enthalten?

$$\underline{\nabla} \times \underline{E} + \frac{\partial}{\partial t} \underline{B} = 0 \quad | \cdot \underline{H} \quad \Rightarrow \underline{H} \cdot (\underline{\nabla} \times \underline{E}) + \underline{H} \cdot \frac{\partial}{\partial t} \underline{B} = 0 \qquad (1)$$

$$\underline{\nabla} \times \underline{H} - \frac{\partial}{\partial t} \underline{D} = \underline{j} \quad | \cdot \underline{E} \quad \Rightarrow \underline{E} \cdot (\underline{\nabla} \times \underline{H}) - \underline{E} \cdot \frac{\partial}{\partial t} \underline{D} = \underline{j} \cdot \underline{E} \qquad (2)$$

$$(1)-(2): \underbrace{\underline{H} \cdot (\underline{\nabla} \times \underline{E}) - \underline{E} \cdot (\underline{\nabla} \times \underline{H})} + \underline{H} \cdot \frac{\partial}{\partial t} \underline{B} + \underline{E} \cdot \frac{\partial}{\partial t} \underline{D} = -\underline{j} \cdot \underline{E}$$

$$= \underline{\nabla} \cdot (\underline{E} \times \underline{H}) = \underline{\nabla} \cdot (\underline{E} \times \underline{H}) + \underline{\nabla} \cdot (\underline{E} \times \underline{H}) \qquad \text{und } \underline{H} \cdot \frac{\partial}{\partial t} \underline{B} = \frac{1}{\mu\mu_0} \underline{B} \cdot \frac{\partial}{\partial t} \underline{B} = \frac{1}{2\mu\mu_0} \frac{\partial}{\partial t} \underline{B}^2$$

$$= \underline{H} \cdot (\underline{\nabla} \times \underline{E}) + \underline{E} \cdot (\underline{H} \times \underline{\nabla}) \qquad \underline{E} \cdot \frac{\partial}{\partial t} \underline{D} = \epsilon\epsilon_0 \underline{E} \cdot \frac{\partial}{\partial t} \underline{E} = \frac{1}{2} \epsilon\epsilon_0 \frac{\partial}{\partial t} \underline{E}^2$$

$$= \underline{H} \cdot (\underline{\nabla} \times \underline{E}) - \underline{E} \cdot (\underline{\nabla} \times \underline{H})$$

$$\Rightarrow \quad \underline{\nabla} \cdot \underbrace{(\underline{E} \times \underline{H})}_{= \underline{S}} + \frac{\partial}{\partial t} \underbrace{\left(\frac{1}{2\mu\mu_0} \underline{B}^2 + \frac{1}{2} \epsilon\epsilon_0 \underline{E}^2 \right)}_{= \frac{1}{2}(\underline{E} \cdot \underline{D} + \underline{B} \cdot \underline{H}) = w} = \underbrace{-\underline{j} \cdot \underline{E}}_{\sigma}$$

$$\Rightarrow \quad \frac{\partial}{\partial t} w + \underline{\nabla} \times \underline{S} = \sigma$$

w: Energiedichte des elektromagnetischen Feldes

$\underline{S}$: Energiestromdichte (Poynting-Vektor)

σ: Leistungsdichte (Quelldichte der Feldenergie)

$\sigma = -\underline{j} \cdot \underline{E} < 0$: Abnahme (Joule'sche Wärme)

> 0: Zunahme (Antennenabstrahlung)

der Feldenergie